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Flow of a Second-Grade Visco-Elastic Fluid in a Porous Converging Channel

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Abstract

The problem dealing with the two dimensional and steady flow of a second-grade fluid in a converging channel has been analyzed. It is assumed that the fluid is injected into the channel through one wall and sucked from the channel through the other wall at the same velocity, whose magnitude varies in inverse proportion to the distance along the wall from the origin of the channel. The flow and heat transfer phenomena have been characterized by non-dimensional parameters Re (Reynolds number), R (cross-flow Reynolds number), N (elastic number), and Pr (Prandtl number). The basic equations governing the flow and heat transfer are reduced to a set of ordinary differential equations by using the appropriate transformations for the velocity components and temperature. These equations have been solved approximately for small values of N subject to the relevant boundary conditions by employing a numerical technique. The effects of the above-mentioned parameters on the velocity and temperature distributions have been discussed.

Key words: Second-grade fluid, Converging channel, Viscoelasticity, Suction/Injection.

Introduction

The flow of Newtonian and non-Newtonian fluids in a porous surface channel has attracted the interest of many investigators in view of its applications in engineering practice, particularly in chemical industries. Examples of these are the cases of boundary layer control, transpiration cooling and gaseous diffusion. Theoretical research on steady flow of this type was initiated by Berman (1953) who found a series solution for the two-dimensional laminar flow of a viscous incompressible fluid in a parallel-walled channel for the case of a very low cross-flow Reynolds number. After his pioneering work, this problem has been studied by many researchers considering various variations in the problem, e.g., Choi *et al.* (1999) and references cited therein. For the case of a converging or diverging channel with a permeable wall, if the Reynolds number is large and if there is suction or injection at the walls whose magnitude is inversely proportional to the distance along the wall from the origin of the channel, a solution for laminar boundary layer equations can be obtained (Rosenhead, 1963). Terrill (1965) considered the symmetric problem where the rate of fluid injection at one wall is taken to be equal to the rate of fluid suction at the other wall, and obtained an analytical solution valid for slow flow through the channel. Terrill's solution contains several errors, as was also pointed out by Roy and Nayak (1982). The converging and diverging flow with wall suction or injection was studied by Balmer and Kauzlarich (1971). Their titles refer to elastic fluids, but they actually used only the power-law fluid. Roy and Nayak (1982) re-examined Terrill's problem by replacing Newtonian fluid by Walter's B' viscoelastic fluid. In their study, perturbation solutions of the velocity field have been obtained by taking the elastic parameter and the Reynolds number as the perturbation parameters, respectively. Recently, Öztürk *et al.* (1998) have analyzed the same problem by using the Reiner-Rivlin fluid model and they obtained an analytical solution in the case of slow flow.

The aim of this work is to study Terrill's problem by introducing a second-grade fluid and to assess qualitatively the effect of the elasticity of the fluid (via the elastic number N) on the velocity and temperature distributions for different values of Reynolds number, cross-flow Reynolds number, and Prandtl number. Unlike previous studies that were restricted to the case of slow flow, we carried out our calculations at higher values of the Reynolds number; in addition, we examined the heat transfer aspect of the problem under consideration.

Formulation of the problem

Consider the steady, two-dimensional, incompressible laminar flow of the second-order viscoelastic fluid in a converging channel, the side walls of which are situated at $\theta = \pm\alpha$ (see Figure 1). We assume that there is suction at one wall and equal injection at the other wall, and the velocities of injection and suction at the walls vary in inverse proportion to the distance along the wall from the origin of the channel.

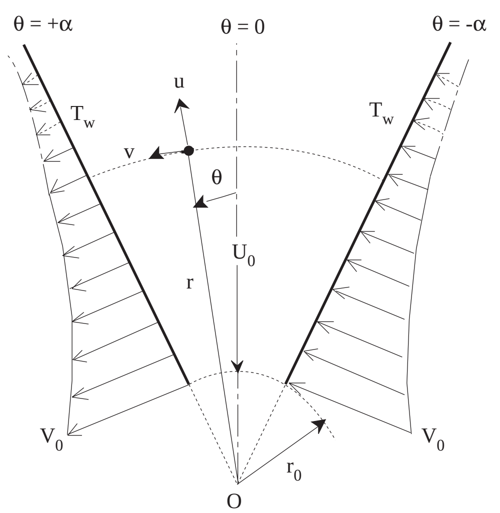

Figure 1. Sketch of flow geometry and coordinate system.

The Cauchy stress tensor $\mathbf{T}$ for the second-order viscoelastic fluid is given as follows (Rivlin and Ericksen, 1955):

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S},$$

$$\mathbf{S} = \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{1}$$

Here, $-p\mathbf{I}$ is the constitutively indeterminate part of the stress due to the constraint of incompressibility, $\mathbf{S}$ the extra stress tensor, μ the coefficient of viscosity, α_1 and α_2 the normal stress moduli, and kinematical tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined through

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \tag{2}$$

$$\mathbf{A}_2 = \frac{D}{Dt}\mathbf{A}_1 + \mathbf{L}\cdot\mathbf{A}_1 + \mathbf{A}_1\cdot\mathbf{L}^T, \tag{3}$$

$$\mathbf{L} = \nabla\mathbf{v}(L_{ij} = v_{j;i}), \tag{4}$$

where $\mathbf{v}$ is the velocity vector, ∇ is the gradient operator, the semicolon stands for covariant differentiation and D/D t is the material time derivative. We notice that $\alpha_1 = \alpha_2 = 0$, the model (1) reduces to the classical linearly viscous fluid model.

If the fluid modeled by Eq. (1) is to be compatible with thermodynamics in the sense that all motions of the fluid meet Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid is a minimum in equilibrium, then (Dunn and Fosdick, 1974).

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0. \tag{5}$$

The fluids characterized by the above restrictions have come to be known as second-grade fluids as opposed to second-order fluids (Ariel, 2001). The restrictions in Eq. (5) have been the subject of much controversy. Experimental results made under the assumption that the fluid being tested is a second-order fluid are in contradiction with the restrictions in Eq. $(5)_{2,3}$. On the other hand, Dunn and Fosdick (1974) demonstrated that if $\alpha_1 < 0$ while the other two restrictions hold, the fluid exhibits unacceptable instability characteristics. Later Fosdick and Rajagopal (1979) showed that if $\alpha_1 < 0$, the fluid exhibited anomalous behaviour not expected in materials of rheological interest. A thorough discussion of the issues involved can be found in the recent critical review of Dunn and Rajagopal (1995). We shall not get into a more detailed discussion of these issues. In this study we shall assume that the model under consideration meets Eq. (5), as is compatible with the present literature. For this case the constitutive equation (1) can be written as

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \beta(\mathbf{A}_2 - \mathbf{A}_1^2), \beta = \alpha_1 = -\alpha_2. \quad (6)$$

In addition to Eq. (6), the basic equations of the problem are the following:

Continuity equation:

$$\nabla \cdot \mathbf{v} = 0, \quad (7)$$

Equations of motion:

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla \cdot T, \quad (8)$$

Energy equation:

$$\rho c_p(\mathbf{v} \cdot \nabla T) = k\Delta T + tr(S \cdot \nabla \mathbf{v}), \quad (9)$$

where ρ the density, T the temperature, c_p the specific heat at constant pressure, k the thermal conductivity, tr denotes the trace of a second-order tensor, and Δ is the Laplacian operator. Note that the second term on the right hand side of the energy equation represents the viscous dissipation. The assumptions made in the above equations are as follows: (a) the flow is steady and laminar; (b) the fluid is incompressible; (c) the body forces are negligible; (d) all the physical properties, e.g. viscosity, specific heat and thermal conductivity of the fluid, remain invariable throughout the fluid; (e) the heat flux vector can be represented by Fourier's law; and (f) the effect of radiant heating is negligible.

We shall seek a velocity field, compatible with the continuity equation, of the form (Terrill, 1965; Roy and Nayak, 1982)

$$u(r,\theta) = \frac{U_0 r_0}{r} f(\theta), \quad v(r) = \frac{V_0 r_0}{r} \quad (10)$$

In the above representation, u and v are the velocity components in the directions of r and θ, respectively (see Fig. 1), r_0 is a typical length in which the velocities of injection at $\theta = -\alpha$ and of suction at $\theta = +\alpha$ are V_0, and the magnitude of the velocity component in the direction of r at the centre line to be assumed is U_0.

The boundary conditions for the velocity field are

$$u(r,-\alpha) = 0, \; u(r,+\alpha) = 0, \; u(r_0, 0) = -|U_0| \quad (11)$$

The first two conditions are no-slip and the other relates to the quantity of fluid entering the channel. Note that the minus sign in the third condition denotes the flow of a convergent channel.

It follows from Eq. (6), (10) and the equations of motion (8), that

$$\begin{aligned}\frac{\partial p}{\partial r} &= \frac{\rho}{r^3} r_0^2 (V_0^2 + U_0^2 f^2 - U_0 V_0 f' + \frac{\nu U_0}{r_0} f'') \\ &+ \frac{\beta}{r^5} r_0^2 (-8V_0^2 - 8U_0^2 f^2 + 4U_0 V_0 f' \\ &- 2U_0^2 f'^2 - 4U_0^2 f f'' + U_0 V_0 f'''), \quad (12)\end{aligned}$$

$$\frac{\partial p}{\partial \theta} = \frac{2\mu U_0 r_0}{r^2} f' + \frac{2\beta r_0^2 U_0^2}{r^4} f' f'', \quad (13)$$

where $\nu = \mu/\rho$ called the kinematic viscosity, and the prime denotes the differentiation with respect to θ. By cross-differentiating Eqs.(12) and (13) we eliminate the pressure and obtain the following governing equation:

$$\begin{aligned}& f''' - Rf'' + 2Reff' + 4f' + \\ & N(-16Reff' + 4Rf'' - 4Reff''' + Rf^{IV}) = 0, \quad (14)\end{aligned}$$

where the cross-flow Reynolds number R, the Reynolds number Re, and the elastic number N are defined through, respectively

$$R = \frac{V_0 r_0}{\nu}, \; Re = \frac{U_0 r_0}{\nu}, \; N = \frac{\beta}{\rho r_0^2} \quad (15)$$

Integrating Eq. (14) once we get

$$\begin{aligned}& f'' - Rf' + Ref^2 + 4f + N(-8Ref^2 + 4Rf' \\ & -4Reff'' + 2Ref'^2 + Rf''') = C, \quad (16)\end{aligned}$$

where C is a constant to be determined.

The boundary conditions (11) are re-written as follows:

$$f(-\alpha) = 0, \quad f(0) = -1, \quad f(+\alpha) = 0. \quad (17)$$

It is recorded that for a Newtonian fluid (N = 0) Eq. (16) together with the associated boundary conditions (17) are the same as those obtained by Terrill (1965). To calculate the value of constant C, using the condition $f(-\alpha) = 0$ in Eq. (16) we have

$$\begin{aligned} &f^{''}(-\alpha) - Rf^{'}(-\alpha) + N\{4Rf^{'}(-\alpha) + \\ &2Ref^{'2}(-\alpha) + Rf^{'''}(-\alpha)\} = C. \end{aligned} \tag{18}$$

The term in Eq. (14) having the N factor represents the non-Newtonian character of the fluid. It is noted that Eq. (14) is one order higher than Navier-Stokes' equations due to the presence of elasticity in the fluid. It would thus appear that the additional boundary condition must be imposed to obtain a solution. In order to overcome this diffucuity, we seek a solution of Eq. (14) of the form

$$f = f_0 + Nf_1 + O(N^2), \tag{19}$$

valid for sufficiently small N. Inserting f from Eq. (19) into Eq. (14) and equating the corresponding coefficient of N up to first order, we get the following differential equations

$$f_0^{'''} - Rf_0^{''} + 2Ref_0f_0^{'} + 4f_0^{'} = 0, \tag{20}$$

$$\begin{aligned} &f_1^{'''} - Rf_1^{''} + (2Ref_0 + 4)f_1^{'} + 2Ref_0^{'}f_1 \\ &-16Ref_0f_0^{'} + 4Rf_0^{''} - 4Ref_0f_0^{'''} + \\ &Rf_0^{IV} = 0. \end{aligned} \tag{21}$$

In a similar manner the higher order terms can be obtained. However the calculations will become complicated. Moreover, the solutions considered are valid for small values of N. Therefore, we retain up to first-order terms. From Eqs. (17) and (19) it follows that the boundary conditions for Eqs. (20) and (21) are

$$f_0(-\alpha) = 0,\ f_0(0) = -1,\ f_0(+\alpha) = 0, \tag{22}$$

$$f_1(-\alpha) = 0,\ f_1(0) = 0,\ f_1(+\alpha) = 0. \tag{23}$$

The integration of Eqs. (20) and (21) subject to the related boundary conditions (22) and (23) has been performed numerically.

The pressure distribution can be obtained by integrating Eqs. (12) and (13). Hence

$$\begin{aligned} &p(r,\theta) = p_0 + \tfrac{\mu\nu}{2r^2}(4Ref - R^2 - \\ &ReC + 8NR^2 + 2NRe^2f^{'2}). \end{aligned} \tag{24}$$

where p_0 a constant reference pressure and the constant C takes the value in Eq. (18). In the absence of N, Eq. (24) is the same as that obtained by Terrill (1965).

Next, we introduce a temperature field of the form

$$T(r,\theta) = \frac{\nu^2}{c_p}\frac{g(\theta)}{r^2} + T_w, \tag{25}$$

where T_w is the constant temperature of the walls. Substituting Eqs. (10) and (25) into Eq. (9) leads to the ordinary differential equation

$$\begin{aligned} &g^{''} - RPrg^{'} + 2(2 + RePrf)g + \\ &+Pr(4Re^2f^2 + Re^2f^{'2} - 4RRef^{'} + 4R^2) \\ &+NPrRe(-8Re^2f^3 - 8R^2f + 12ReRff^{'} \\ &-2Re^2ff^{'2} - 2R^2f^{''} + ReRf^{'}f^{''} = 0, \end{aligned} \tag{26}$$

where $Pr = \mu c_p/k$ is the Prandtl number. Equation (26) is to be solved subject to the boundary conditions

$$g(-\alpha) = 0,\ g(+\alpha) = 0. \tag{27}$$

In order to solve Eq.(26) , f_0 and f_1 functions are first determined from Eqs.(20) and (21) and it can be then solved numerically.

Numerical Results and Discussion

At the outset it is necessary to mention that the constitutive equation used in this paper describes a viscoelastic fluid for slow and slowly varying processes. It should be pointed out clearly that the sole purpose of using Eq. (6), the practical limitations of the second-grade model notwithstanding, is to examine qualitatively at least how fluid elasticity (via the material constant β) affects velocity and temperature distributions. In addition, as a result of singularity at r = 0, the solutions are expected to be valid only in converging nozzles rather than in between two intersecting planes having a sink at r = 0.

Several numerical methods can be used to solve the above differential equations. One convenient and accurate method that we will use here is the so-called shooting method for Eq. (20) subject to (22). Equation (20) together with the associated boundary conditions (22) are reduced to three first-order differential equations. For the given values of the parameters, the conditions $f_0^{'}(-\alpha)$ and $f_0^{''}(-\alpha)$ are roughly estimated and the differential equation is processed by using the fourth-order Runge-Kutta procedure. The mathematical problem is to find the correct values of $f_0^{'}(-\alpha)$ and $f_0^{''}(-\alpha)$, which yield the known values of $f_0(0)$ and $f_0(+\alpha)$. Since for Re = 0 the analytical solution provides exact initial values for $f_0^{'}(-\alpha)$ and $f_0^{''}(-\alpha)$ then a successive numerical solution can be generated as Re is increased. The accuracy of the assumed missing initial conditions is checked by comparing the calculated values of $f_0(0)$ and $f_0(+\alpha)$ with their given values in Eq. (22). If a difference exists, the computations with new and improved values for $f_0^{'}(-\alpha)$ and $f_0^{''}(-\alpha)$ are repeated. This process is continued until the agreement between the calculated and known values of $f_0(0)$ and $f_0(+\alpha)$ is within the specified degree of accuracy. The accuracy of missing initial conditions that yield the known values of $f_0(0)$ and $f_0(+\alpha)$ is 10^{-7} at least. The results have been summarized in Table 1 for several values of the parameters.

Linear differential equations (21) and (26) subject to the boundary conditions (23) and (27) have been solved numerically with the help of the finite difference method. We subdivide the interval $[-\alpha, \alpha]$ into M equal subintervals and select the mesh points $\theta_k = -\alpha + hk$ for $k = 0, 1, 2, \ldots, M$, where the step size $h = 2\alpha/M$. Numerical solutions are given for a converging channel with a total opening of 60^0 for the possible combinations of two Reynolds numbers, three elastic numbers, two cross-flow Reynolds numbers and four Prandtl numbers. The step size h is chosen as $h = \pi/3000$. An approximation of the solution at the nodal points has been obtained by solving the resulting linear algebraic equations with the aid of a computer.

Table 1. Missing initial conditions for Eq. (20).

Re	R	$f_0^{'}(-\pi/6)$	$f_0^{''}(-\pi/6)$
0.5	2	-2.47492899	0.10427840
0.5	5	-1.87198189	-1.02146310
1	2	-2.53243459	0.34211371
1	5	-1.91774450	-0.91886505
3	2	-2.75874606	1.32153836
3	5	-2.09783530	-0.48380740
5	2	-2.97974560	2.34505833
5	2	-2.27422537	-0.00935354

The numerical solutions provided can be best used as benchmark solutions to test more complex computational fluid dynamics codes. For this reason, as well as for determining the accuracy to which the numerical solutions have been computed, we should demonstrate that the numerical solutions presented are not strongly step size dependent. This can be done by repeating the algorithm using different step sizes and comparing results. We solved Eqs. (20), (21) and (26) choosing four different step sizes, when Re = 3, R = 5, N = 0.015 and Pr = 1.5, and presented the results at some mesh points (see Tables 2 and 3). It is clear from these tables that the difference among the numerical solutions is reasonably small.

Since our perturbation analysis is valid only for small values of the the elastic number N, the variation of N is limited to a range from 0.0 to 0.03. In addition, the numerical solutions obtained for the problem under consideration point to the conclusion that the perturbation solutions, even though obtained without making any assumption as to the sizes of Reynolds number Re and cross-flow Reynolds number R, give acceptable results only when $Re \leq 5$ and $R \leq 5$. For N > 0.03, Re > 5 and R > 5, since the effects of successive terms in the perturbation expansion are more significant, i.e. $|Nf_1| > |f_0|$, the perturbation solutions fail to give satisfactory results, that is, the solutions cannot be trusted to be meaningful.

Predictions based on the foregoing analysis are displayed graphically in Figures 2 to 9. In Figures 2 to 5, the function $f(\theta)$, which corresponds to the velocity component in the direction of r, is plotted versus θ for two different values of Reynolds number Re and cross-flow Reynolds number R, with the

Table 2. Comparison of the numerical solutions with different step sizes for $f(\theta)$ over $[-\pi/6, \pi/6]$ with Re = 3, R = 5 and N = 0.015.

	$f(\theta)$			
θ	$h = \pi/6000$	$h = \pi/3000$	$h = \pi/1500$	$h = \pi/750$
- $\pi/6$	-0.00000000	-0.00000000	-0.00000000	-0.00000000
-0.5235	-0.00023298	-0.00023298	-0.00023298	-0.00023298
-0.5027	-0.04930398	-0.04930399	-0.04930404	-0.04930424
-0.4817	-0.09881307	-0.09881310	-0.09881320	-0.09881359
-0.4398	-0.19716182	-0.19716186	-0.19716205	-0.19716280
-0.3665	-0.36600625	-0.36600633	-0.36600664	-0.36600791
-0.2617	-0.59330573	-0.59330583	-0.59330625	-0.59330791
-0.1571	-0.79165836	-0.79165846	-0.79165884	-0.79166038
0	-1.00000000	-1.00000000	-1.00000000	-1.00000000
0.1571	-1.03451844	-1.03451823	-1.03451741	-1.03451412
0.2617	-0.91520619	-0.91520584	-0.91520440	-0.91519865
0.3665	-0.65255757	-0.65255716	-0.65255550	-0.65254886
0.4398	-0.38170472	-0.38170441	-0.38170318	-0.38169828
0.4817	-0.19890909	-0.19890891	-0.19890822	-0.19890544
0.5027	-0.10099804	-0.10099795	-0.10099759	-0.10099615
0.5235	-0.00048505	-0.00048505	-0.00048505	-0.00048504
$\pi/6$	-0.00000000	-0.00000000	-0.00000000	-0.00000000

Table 3. Comparison of the numerical solutions with different step sizes for $g(\theta)$ over $[-\pi/6, \pi/6]$ with Re = 3, R = 5, N = 0.015 and Pr = 1.5.

	$g(\theta)$			
θ	$h = \pi/6000$	$h = \pi/3000$	$h = \pi/1500$	$h = \pi/750$
-$\pi/6$	0.00000000	0.00000000	0.00000000	0.00000000
-0.5235	0.00559399	0.00559399	0.00559398	0.00559397
-0.5027	1.18006867	1.18006848	1.18006771	1.18006463
-0.4817	2.35629112	2.35629071	2.35628906	2.35628248
-0.4398	4.65984517	4.65984424	4.65984052	4.65982565
-0.3665	8.47802967	8.47802757	8.47801917	8.47798553
-0.2617	13.23452174	13.23451739	13.23449997	13.23443032
-0.1571	16.87206220	16.87205508	16.87202662	16.87191276
0	19.83688921	19.83687806	19.83683347	19.83665512
0.1571	19.73229713	19.73228463	19.73223461	19.73203456
0.2617	18.02488235	18.02487126	18.02482689	18.02464950
0.3665	14.47627227	14.47626357	14.47622877	14.47608970
0.4398	9.82874201	9.82873569	9.82871044	9.82860941
0.4817	5.71669811	5.71669421	5.71667860	5.71661614
0.5027	3.08579173	3.08578956	3.08578089	3.08574620
0.5235	0.01580050	0.01580049	0.01580043	0.01580017
$\pi/6$	0.00000000	0.00000000	0.00000000	0.00000000

elastic number N as a parameter. Figure 2 shows that the velocity distribution for a Newtonian and a viscoelastic fluid is almost the same as that for low values of both the Reynolds number and the cross-flow Reynolds number. We observe from the other figures illustrating velocity profiles that the main effect of the elastic number on the converging flow is to increase the velocity slightly in the interval $-\pi/6 < \theta < 0$ whereas it is to decrease the velocity more prominently in the interval $0 < \theta < \pi/6$. This reduction in velocity values in the region $(0, \pi/6)$ is more pronounced with an increase in either the cross-flow Reynolds number or Reynolds number. Moreover, increasing N has a tendency to make the velocity profiles more symmetrical.

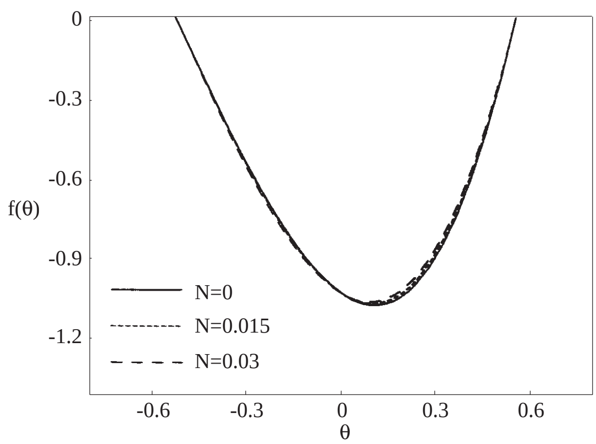

Figure 2. Velocity profiles in a 60° converging channel for Re = 0.5 and R = 2.

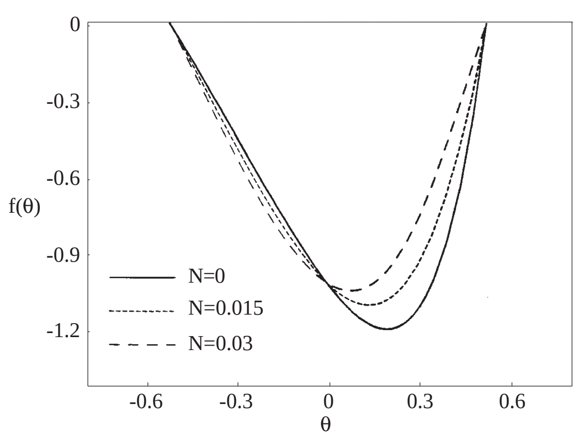

Figure 3. Velocity profiles in a 60° converging channel for Re = 0.5 and R = 5.

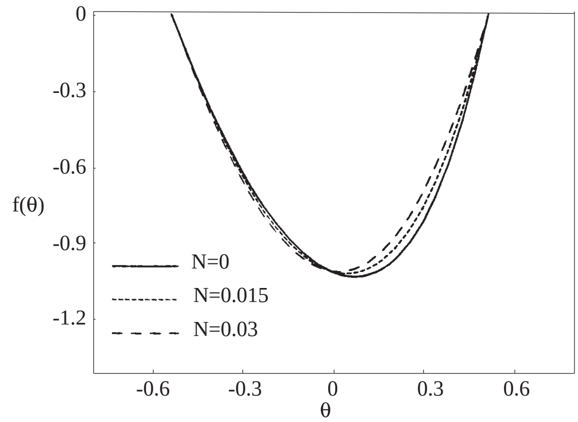

Figure 4. Velocity profiles in a 60° converging channel for Re = 5 and R = 2.

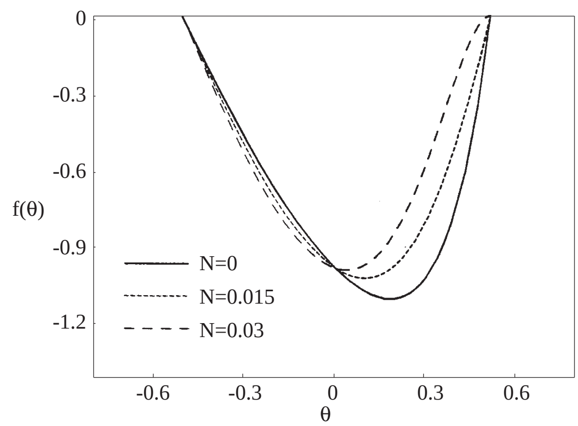

Figure 5. Velocity profiles in a 60° converging channel for Re = 5 and R = 5.

In order to investigate the effect of fluid elasticity on the temperature distribution we have plotted the function $g(\theta)$ against θ in Figures 6 to 9 for two different values of the Reynolds number and the cross-flow Reynolds number, with parametric values of the elastic number and the Prandtl number. For low values of both the Reynolds number and the cross-flow Reynolds number, Figure 6 shows that the effect of the elastic number is insignificant on the temperature profiles, as it is on the velocity profiles. On the other hand, it is evident from Figures 7 to 9 that the elastic number N affects the temperature profiles in different ways, depending on the chosen values of the other parameters. For instance, when Re = 0.5 and R = 5, we notice that for a viscoelastic fluid the temperature is less than the corresponding temperature for a Newtonian fluid (see Figure 7). However, for Re = 5 and R = 2 an opposite effect is observed from that of Figure 8, that is, the presence of viscoelasticity leads to an increase in temperature in comparison to that for a Newtonian fluid. In addition, as the Reynolds number is increased from 0.5 to 5, while keeping the cross-flow Reynolds number fixed at 5, we conclude that the temperature profiles become more symmetrical and the difference between magnitudes of Newtonian and viscoelastic temperature profiles goes on increasing (see Figures 7 and 9). Finally, it is apparent from the figures relevant to the temperature distributions that an increase in the Prandtl number results in an appreciable increase in the temperature, which increases further when the Reynolds number (and/or cross-flow Reynolds number) takes higher values.

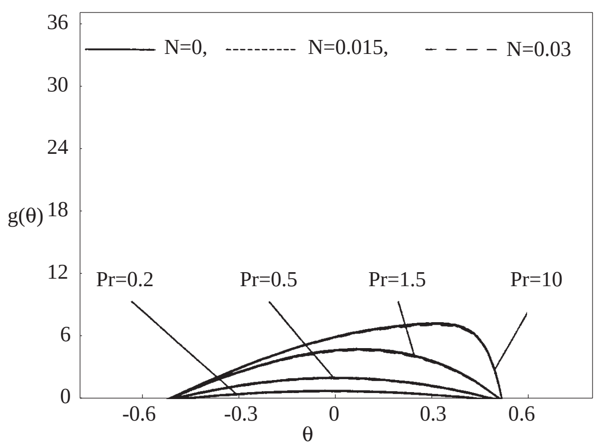

Figure 6. Temperature distributions in a 60° converging channel for Re = 0. 5 and R = 2.

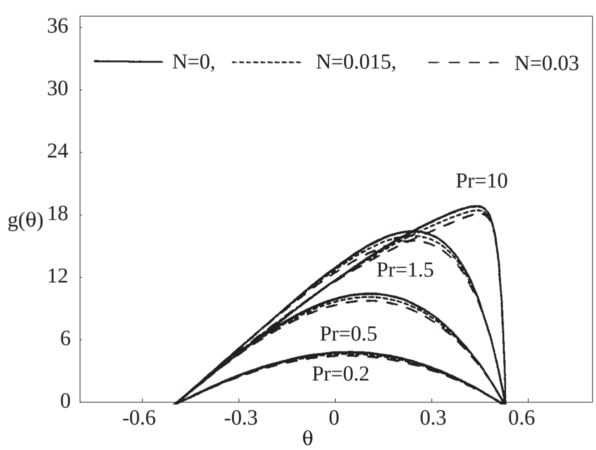

Figure 7. Temperature distributions in a 60° converging channel for Re = 0. 5 and R = 5.

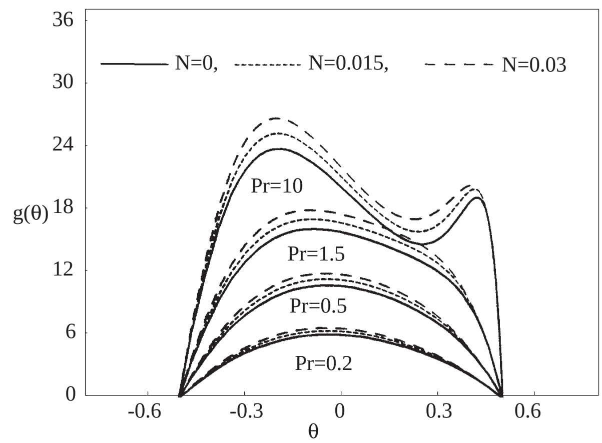

Figure 8. Temperature distributions in a 60° converging channel for Re = 5 and R = 2.

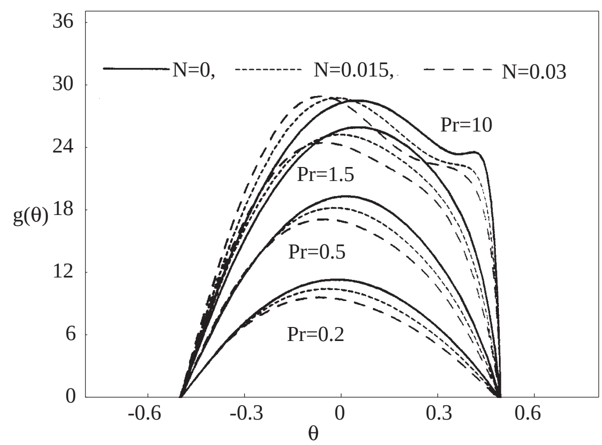

Figure 9. Temperature distributions in a 60° converging channel for Re = 5 and R = 5.

Conclusions

In the present paper we considered the laminar flow of a second-grade-visco-elastic fluid in a porous converging channel of total opening of 60^0. By means of similarity transformations, the governing equations are reduced to a set of ordinary differential equations. Numerical calculations have been carried out for various values given to the non-dimensional parameters Re, R, N and Pr, and the qualitatively significant contributions of the elastic parameter N to the velocity and temperature distributions have been pointed out. From the present investigations, we may conclude the following:

1. The commonly used perturbation technique in the literature, in which perturbation solutions are sought for small values of the elastic pa-

rameter N, does not give satisfactory results for $N > 0.03, Re > 5$ and $R > 5$.

2. For the small values of both the Reynolds number and the cross-flow Reynolds number, the effect of an elastic number is insignificant on velocity and temperature profiles.

3. Elasticity of the fluid increases the velocity slightly in the interval $-\pi/6 < \theta < 0$, whereas it decreases more prominently in the interval $0 < \theta < \pi/6$ and the change in the values of velocity in the region $(0, \pi/6)$ is more pronounced with an increase in either cross-flow Reynolds number or Reynolds number.

4. Elastic elements in the viscous fluid affect temperature profiles in different ways, depending on the values given to the other parameters.

5. The Prandtl number leads to an increase in the temperature at any point and the cross-flow Reynolds number and / or Reynolds number increase it further.

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Nomenclature

$\mathbf{A}_n$	Rivlin-Ericksen tensor of rank n, T^{-1}
c_p	specific heat at constant pressure, $L^2T^{-2}\vartheta^{-1}$
$\mathbf{I}$	identity tensor, dimensionless
k	thermal conductivity, M L T^{-3} ϑ^{-1}
N	elastic number, dimensionless
Pr	Prandtl number, dimensionless
p	pressure, ML^{-1} T^{-2}
R	cross-flow Reynolds number, dimensionless
Re	Reynolds number, dimensionless
r, θ	polar coordinates
r_0	typical length, L
$\mathbf{S}$	extra stress tensor, ML^{-1} T^{-2}
T	temperature, ϑ
$\mathbf{T}$	Cauchy stress tensor, ML^{-1} T^{-2}
T_w	constant temperature of the walls, ϑ
t	time, T
U_0	radial velocity component at the center line, LT^{-1}
u , v	components of the velocity vector, LT^{-1}
V_0	suction / injection velocity at $r = r_0$, LT^{-1}
$\mathbf{v}$	velocity vector, LT^{-1}
α	half angle of corner, dimensionless
α_1, α_2	normal stress moduli, ML^{-1}
β	normal stress modulus, ML^{-1}
μ	coefficient of viscosity, ML^{-1} T^{-1}
ν	kinematic viscosity, L^2 T^{-1}
ρ	density, ML^{-3}

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