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Deformation Analysis of Elastic-Plastic Two Layer Tubes Subject to Pressure: an Analytical Approach

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Abstract

Analytical solutions are obtained for axisymmetric elastic-plastic deformations in tightly fitted concentric tubes with fixed ends subjected to either internal or external pressure. Tresca's yield criterion and its associated flow rule are used to estimate the elastic-plastic response of the assembly. The closed form solutions for all elastic-plastic deformation stages are obtained by assuming linear hardening material behavior. Some numerical examples are handled using the mechanical properties of real engineering materials.

Key words: Composite tubes, Elastoplasticity, Strain hardening, Tresca's criterion.

Introduction

There are many engineering applications of commonly used structures like rods, tubes, annular disks and spherical shells subject to different loading and boundary conditions. The problem of a thick-walled tube with unrestricted ends under pressure is treated comprehensively in the purely elastic state by Timoshenko (1956), Timoshenko and Goodier (1970) and Ugural and Fenster (1995), in the fully plastic stress state by Nadai (1931), and in the elastic-plastic stress state by Parker (2001) and Perry and Aboudi (2003). In this study, the pressurized tube problem is extended to tightly fitted composite tubes with axially constrained ends in the elastic-plastic stress state.

The geometry considered in this work consists of 2 tightly fitted concentric tubes. A long tube of inner radius a and outer radius b is tightly placed in a tube of the same length and of inner radius b and outer radius c. The ends of both tubes are fixed with the help of rigid plates. The composite tube system is then subject to either internal or external pressure. It is the objective of this work to estimate the elastic-plastic stress distribution in this assembly for the aspect ratio: $\bar{a} = a/b = 0.5$. In the framework of infinitesimal deformations, a state of plane strain and Tresca's yield criterion, analytical solutions are obtained for all elastic-plastic deformation stages.

Elastic Solution

A state of plane strain ($\varepsilon_z = 0$) and small deformations are presumed. Using the formal notation for the stresses and strains (Boley and Weiner, 1960; Timoshenko and Goodier, 1970), the equation of equilibrium in the radial direction

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{1}$$

and generalized Hooke's law

$$\varepsilon_r = \varepsilon_r^p + \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] \tag{2}$$

$$\varepsilon_\theta = \varepsilon_\theta^p + \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] \tag{3}$$

$$\varepsilon_z = \varepsilon_z^p + \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)] \tag{4}$$

are valid both in elastic (with plastic strain $\varepsilon_i^p = 0$) and plastic regions. For purely elastic deformations of a tube, Eq. (4) can be solved for σ_z to give

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) \tag{5}$$

The axial stress σ_z may be eliminated from the radial and circumferential strain expressions and results are substituted in the strain-displacement relations $\varepsilon_r = du/dr$ and $\varepsilon_\theta = u/r$ to obtain the following stress-displacement relations:

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{\nu u}{r} + (1-\nu)u'\right] \tag{6}$$

$$\sigma_\theta = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{(1-\nu)u}{r} + \nu u'\right] \tag{7}$$

where a prime denotes differentiation with respect to the radial coordinate r. Substituting the stresses from Eqs. (6) and (7) in the equation of equilibrium (1), and simplifying we get

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = 0 \tag{8}$$

Equation (8) is the governing equation for the radial displacement $u(r)$. The solution of this equation gives the radial displacement in an elastic tube with fixed ends. The general solution is

$$u(r) = \frac{C_1}{r} + C_2 r \tag{9}$$

where C_1 and C_2 are arbitrary integration constants. The stresses are then determined as

$$\sigma_r(r) = \frac{E}{1+\nu}\left[-\frac{C_1}{r^2} + \frac{C_2}{1-2\nu}\right] \tag{10}$$

$$\sigma_\theta(r) = \frac{E}{1+\nu}\left[\frac{C_1}{r^2} + \frac{C_2}{1-2\nu}\right] \tag{11}$$

$$\sigma_z(r) = \frac{2\nu E C_2}{(1+\nu)(1-2\nu)} \tag{12}$$

Note that the axial stress σ_z is constant throughout the tube.

The expressions for the elastic stresses and displacements for the inner and outer tubes contain 4 unknown integration constants: C_1, C_2, C_3 and C_4. For an assembly subjected to internal pressure P, these constants are determined from $\sigma_r^{eI}(a) = -P$, $\sigma_r^{eI}(b) = \sigma_r^{eO}(b)$, $u^{eI}(b) = u^{eO}(b)$ and $\sigma_r^{eO}(c) = 0$. Here the superscripts eI and eO denote the inner and outer elastic regions (tubes), respectively. For an assembly subjected to external pressure, the integration constants are determined from $\sigma_r^{eI}(a) = 0$, $\sigma_r^{eI}(b) = \sigma_r^{eO}(b)$, $u^{eI}(b) = u^{eO}(b)$ and $\sigma_r^{eO}(c) = -P$.

To present the numerical results, it is convenient to use the following nondimensional variables: radial coordinate $\bar{r} = r/c$, stress $\bar{\sigma}_j = \sigma_j/\sigma_{01}$, displacement $\bar{u} = uE_1/\sigma_{01}c$ and pressure $\bar{P} = P/\sigma_{01}$. Here subscript 1 is used to denote the properties of the inner tube. As an example, a composite tube system with inner radius $\bar{a} = 0.5$ and interface radius $\bar{b} = 0.75$ is considered. The assembly consists of brass inner ($E = 105$ GPa, $\nu = 0.35$, $\sigma_0 = 410$ MPa) and copper outer ($E = 120$ GPa, $\nu = 0.364$, $\sigma_0 = 265$ MPa) tubes.

When the assembly is subjected to internal pressure, the largest difference between the principal stress components occurs at the inner surface $r = a$ and the plastic deformation commences at this surface according to the yield condition $\sigma_y = \sigma_\theta - \sigma_r$. The elastic limit internal pressure is calculated as $\bar{P}_e = 0.387625$. The dimensionless integration constants are obtained as $\bar{C}_1 = C_1/c^2 = 6.58928 \times 10^{-4}$, $\bar{C}_2 = C_2 = 1.77713 \times 10^{-4}$, $\bar{C}_3 = C_3/c^2 = 6.58189 \times 10^{-4}$ and $\bar{C}_4 = C_4 = 1.79027 \times 10^{-4}$. The corresponding stresses and displacement are plotted against the nondimensional radial coordinate in Figure 1. The radial stress and displacement are continuous at the interface satisfying interface conditions, but since the tubes are made of different materials the circumferential and axial stresses are discontinuous.

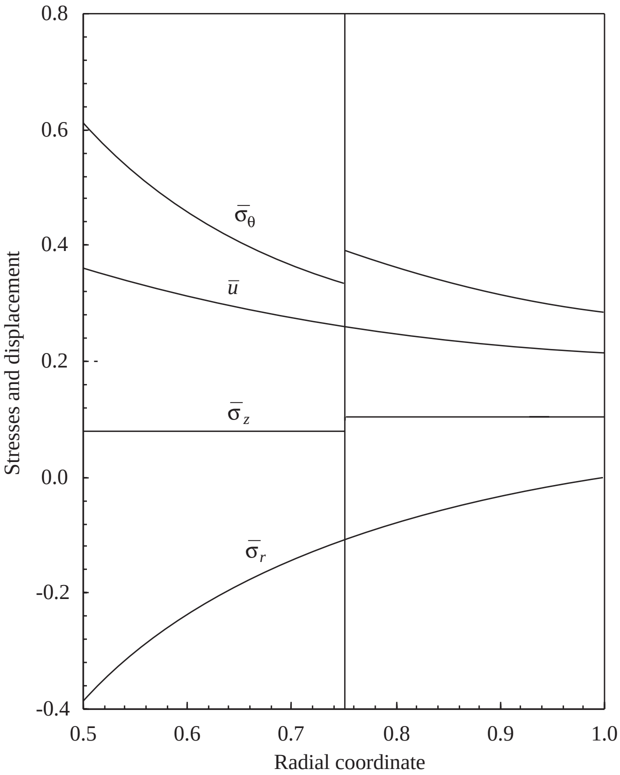

Figure 1. Stresses and displacement at the elastic limit pressure $\bar{P}_e = 0.387625$.

For the external pressure case, yielding commences at $r = a$ as well but the yield criterion is $\sigma_y = \sigma_r - \sigma_\theta$, which differs from that of internal pressure. The elastic limit external pressure is calculated as $\bar{P}_e = 0.396603$ for the assembly considered above. The dimensionless integration constants are calculated as $\bar{C}_1 = -6.58929 \times 10^{-4}$, $\bar{C}_2 = -7.90715 \times 10^{-4}$, $\bar{C}_3 = -7.11983 \times 10^{-4}$ and $\bar{C}_4 = -6.96397 \times 10^{-4}$. The stresses and displacement at this critical pressure are presented in Figure 2.

The deformation behavior of the composite tube assembly subjected to internal pressure is different from that of the external pressure case. As shown in Figures 1 and 2, the circumferential and axial stresses are tensile when the system is subjected to internal pressure and they are compressive when the pressure is applied externally.

Elastic-Plastic Solution

As stated above, the largest and smallest principal stress components occur at the inner surface of the composite tube system irrespective of whether the pressure is applied internally or externally. For the internal pressure case, the yield condition is $\sigma_y = \sigma_\theta - \sigma_r$ and for the external pressure case it is $\sigma_y = \sigma_r - \sigma_\theta$. The plastic region formed at the inner surface propagates toward the tube interface as the pressure is increased. If the pressure is further increased, another plastic zone is formed at the interface. The formulation and closed form solutions of these 2 prospective plastic regions are given next.

Plastic Region I

In this plastic region, stresses satisfy $\sigma_\theta > \sigma_z > \sigma_r$. Accordingly, Tresca's yield condition reads

$$\sigma_y = \sigma_\theta - \sigma_r \tag{13}$$

For a linearly hardening material, the relation between the yield stress σ_y and the equivalent plastic strain ε_{EQ} is given by

$$\sigma_y = \sigma_0(1 + \eta\varepsilon_{EQ}) \tag{14}$$

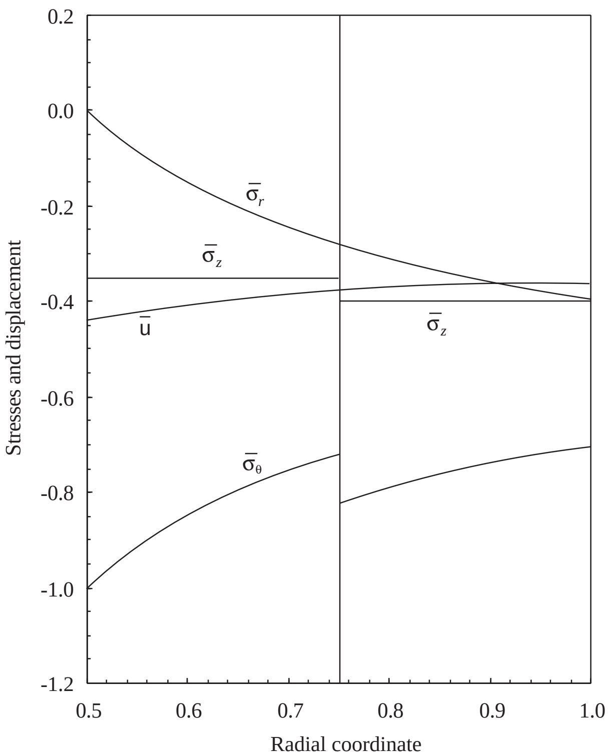

Figure 2. Stresses and displacement at the elastic limit pressure $\bar{P}_e = 0.396603$.

where η represents the hardening parameter. The inverse relation is obtained as

$$\varepsilon_{EQ} = \frac{1}{\eta}\left[\frac{\sigma_y}{\sigma_0} - 1\right] \tag{15}$$

The associated flow rule and the equivalence of increment of plastic work give $\varepsilon_\theta^p = -\varepsilon_r^p$, $\varepsilon_z^p = 0$ and $\varepsilon_\theta^p = \varepsilon_{EQ}$. Since $\varepsilon_z^p = 0$, the axial stress is the same as that given by Eq. (5). It is eliminated from the total strain expressions to obtain

$$\varepsilon_r = -\frac{1}{\eta}\left[\frac{\sigma_\theta - \sigma_r}{\sigma_0} - 1\right] + \frac{1+\nu}{E}[(1-\nu)\sigma_r - \nu\sigma_\theta] \tag{16}$$

$$\varepsilon_\theta = \frac{1}{\eta}\left[\frac{\sigma_\theta - \sigma_r}{\sigma_0} - 1\right] + \frac{1+\nu}{E}[(1-\nu)\sigma_\theta - \nu\sigma_r] \tag{17}$$

Equations (16) and (17) are substituted in the strain-displacement relations to obtain stress-displacement relations

$$\sigma_r = -\frac{\sigma_0}{2+H(1+\nu)} + \frac{E}{[2+H(1+\nu)](1+\nu)(1-2\nu)}\left\{\frac{1+H\nu(1+\nu)}{r}u + \left[1+H(1-\nu^2)\right]u'\right\} \tag{18}$$

$$\sigma_\theta = \frac{\sigma_0}{2+H(1+\nu)} + \frac{E}{[2+H(1+\nu)](1+\nu)(1-2\nu)}\left\{\frac{1+H(1-\nu^2)}{r}u + [1+H\nu(1+\nu)]u'\right\} \tag{19}$$

where $H = \eta\sigma_0/E$ is the normalized hardening parameter. The governing equation for the radial displacement is obtained by combining Eqs. (18) and (19) and the equation of equilibrium (1)

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = \frac{2\sigma_0(1+\nu)(1-2\nu)}{E[1+H(1-\nu^2)]}r \tag{20}$$

The general solution is

$$u(r) = \frac{C_5}{r} + C_6 r - \frac{\sigma_0(1+\nu)(1-2\nu)(2\ln r - 1)r}{2E[1+H(1-\nu^2)]} \tag{21}$$

By virtue of Eqs. (18), (19) and (5), the stresses are determined as

$$\sigma_r(r) = -\frac{EHC_5}{[2+H(1+\nu)]r^2} + \frac{EC_6}{(1+\nu)(1-2\nu)} + \frac{(2\ln r - 1)\sigma_0}{2[1+H(1-\nu^2)]} \tag{22}$$

$$\sigma_\theta(r) = \frac{EHC_5}{[2+H(1+\nu)]r^2} + \frac{EC_6}{(1+\nu)(1-2\nu)} + \frac{(2\ln r + 1)\sigma_0}{2[1+H(1-\nu^2)]} \tag{23}$$

$$\sigma_z(r) = \frac{2E\nu C_6}{(1+\nu)(1-2\nu)} + \frac{2\nu\ln r\sigma_0}{1+H(1-\nu^2)} \tag{24}$$

The nonzero plastic strain components are obtained from $\varepsilon_\theta^p = -\varepsilon_r^p = \varepsilon_{EQ}$ as

$$\varepsilon_\theta^p = -\varepsilon_r^p = \frac{2C_5}{[2+H(1+\nu)]r^2} - \frac{(1-\nu^2)\sigma_0}{E[1+H(1-\nu^2)]} \tag{25}$$

Plastic Region II

The stress state is such that $\sigma_r > \sigma_z > \sigma_\theta$ and Tresca's yield condition takes the form

$$\sigma_y = \sigma_r - \sigma_\theta \tag{26}$$

The flow rule associated with this yield condition and the equivalence of increment of plastic work give $\varepsilon_r^p = -\varepsilon_\theta^p$, $\varepsilon_z^p = 0$ and $\varepsilon_r^p = \varepsilon_{EQ}$. Hence, the total strains become

$$\varepsilon_r = \frac{1}{\eta}\left[\frac{\sigma_r - \sigma_\theta}{\sigma_0} - 1\right] + \frac{1+\nu}{E}[(1-\nu)\sigma_r - \nu\sigma_\theta] \tag{27}$$

$$\varepsilon_\theta = -\frac{1}{\eta}\left[\frac{\sigma_r - \sigma_\theta}{\sigma_0} - 1\right] + \frac{1+\nu}{E}[(1-\nu)\sigma_\theta - \nu\sigma_r] \tag{28}$$

Using strain-displacement relations, one obtains

$$\sigma_r = \frac{\sigma_0}{2+H(1+\nu)} + \frac{E}{[2+H(1+\nu)](1+\nu)(1-2\nu)}\left\{\frac{1+H\nu(1+\nu)}{r}u + \left[1+H(1-\nu^2)\right]u'\right\} \tag{29}$$

$$\sigma_\theta = -\frac{\sigma_0}{2+H(1+\nu)} + \frac{E}{[2+H(1+\nu)](1+\nu)(1-2\nu)}\left\{\frac{1+H(1-\nu^2)}{r}u + [1+H\nu(1+\nu)]u'\right\} \tag{30}$$

A governing differential equation for this region is obtained by substitution of the stresses in the equation of equilibrium (1). The result is

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = -\frac{2\sigma_0(1+\nu)(1-2\nu)}{E[1+H(1-\nu^2)]}r \tag{31}$$

The general solution is derived as

$$u(r) = \frac{C_7}{r} + C_8 r - \frac{\sigma_0(1+\nu)(1-2\nu)(2\ln r - 1)r}{2E[1+H(1-\nu^2)]} \tag{32}$$

and as a result

$$\sigma_r(r) = -\frac{EHC_7}{[2+H(1+\nu)]r^2} + \frac{EC_8}{(1+\nu)(1-2\nu)} - \frac{(2\ln r - 1)\sigma_0}{2[1+H(1-\nu^2)]} \tag{33}$$

$$\sigma_\theta(r) = \frac{EHC_7}{[2+H(1+\nu)]r^2} + \frac{EC_8}{(1+\nu)(1-2\nu)} - \frac{(2\ln r + 1)\sigma_0}{2[1+H(1-\nu^2)]} \tag{34}$$

$$\sigma_z(r) = \frac{2E\nu C_8}{(1+\nu)(1-2\nu)} - \frac{2\nu\ln r\sigma_0}{1+H(1-\nu^2)} \tag{35}$$

$$\varepsilon_r^p = -\varepsilon_\theta^p = -\frac{2C_7}{[2+H(1+\nu)]r^2} - \frac{(1-\nu^2)\sigma_0}{E[1+H(1-\nu^2)]} \tag{36}$$

Numerical Results

The composite tube (brass-copper assembly with $\bar{a} = 0.5$ and $\bar{b} = 0.75$) for which elastic solutions were obtained is taken into consideration and the elastic-plastic deformations are investigated. Here we define the following dimensionless integration constants: $\bar{C}_5 = C_5/c^2$, $\bar{C}_6 = C_6$, $\bar{C}_7 = C_7/c^2$, $\bar{C}_8 = C_8$ and use hardening parameters $H_1 = H_2 = 0.25$.

In the first stage of elastic-plastic deformation of the composite tube subjected to internal pressure, the assembly consists of an inner plastic (plastic region I), an inner elastic and an outer elastic region for the values of pressure $P > P_e$. The expressions for the plastic and elastic regions contain 6 unknown integration constants: C_1, C_2 (plastic region I), C_3, C_4 (inner elastic region), C_5, C_6 (outer elastic region) and an unknown plastic-elastic border radius r_1. For the determination of these unknowns, the following boundary and interface conditions are imposed: $\sigma_r^{pI}(a) = -P$, $\sigma_r^{pI}(r_1) = \sigma_r^{eI}(r_1)$, $\sigma_\theta^{pI}(r_1) = \sigma_\theta^{eI}(r_1)$, $u^{pI}(r_1) = u^{eI}(r_1)$, $\sigma_r^{eI}(b) = \sigma_r^{eO}(b)$, $u^{eI}(b) = u^{eO}(b)$ and $\sigma_r^{eO}(c) = 0$, where the superscripts pI, eI and eO are used to indicate plastic, inner elastic and outer elastic regions. At the critical pressure $\bar{P}_1 = 0.484690$, a new plastic region appears at the interface according to the yield criterion $\sigma_y = \sigma_\theta - \sigma_r$. Performing Newton iterations the unknowns are determined as $\bar{C}_1 = 1.05679 \times 10^{-3}$, $\bar{C}_2 = 9.64013 \times 10^{-4}$, $\bar{C}_3 = 8.48129 \times 10^{-4}$, $\bar{C}_4 = 2.28740 \times 10^{-4}$, $\bar{C}_5 = 8.47177 \times 10^{-4}$, $\bar{C}_6 = 2.30432 \times 10^{-4}$ and $\bar{r}_1 = 0.567259$. With these, the stresses, displacement and normalized plastic strains ($\bar{\varepsilon}_j^p = \varepsilon_j^p E_1/\sigma_{01}$) are calculated and plotted in Figure 3.

For $P > P_1$ the assembly consists of inner plastic, inner elastic, outer plastic and outer elastic regions. The stress and displacement expressions for these regions contain 8 unknown integration constants: C_1, C_2 (inner plastic), C_3, C_4 (inner elastic), C_5, C_6 (outer plastic), C_7, C_8 (outer elastic), and 2 unknown plastic-elastic border radii r_1 and r_2. For the evaluation of these unknowns, the conditions stated above (first stage) are still valid and the remaining 3 conditions are $\sigma_r^{pO}(r_2) = \sigma_r^{eO}(r_2)$, $\sigma_\theta^{pO}(r_2) = \sigma_\theta^{eO}(r_2)$ and $u^{pO}(r_2) = u^{eO}(r_2)$, where the subscript pO denotes the outer plastic region. Taking $\bar{P} = 0.6 > \bar{P}_1$ 10 unknowns are determined as $\bar{C}_1 = 1.51165 \times 10^{-3}$, $\bar{C}_2 = 8.60468 \times 10^{-4}$, $\bar{C}_3 = 1.21318 \times 10^{-3}$, $\bar{C}_4 = 3.57320 \times 10^{-4}$, $\bar{C}_5 = 1.48592 \times 10^{-3}$, $\bar{C}_6 = 4.02783 \times 10^{-4}$, $\bar{C}_7 = 1.21446 \times 10^{-3}$, $\bar{C}_8 = 3.30332 \times 10^{-4}$, $\bar{r}_1 = 0.678442$ and $\bar{r}_2 = 0.897980$. Figure 4 shows the consequent stresses, displacement and plastic strains.

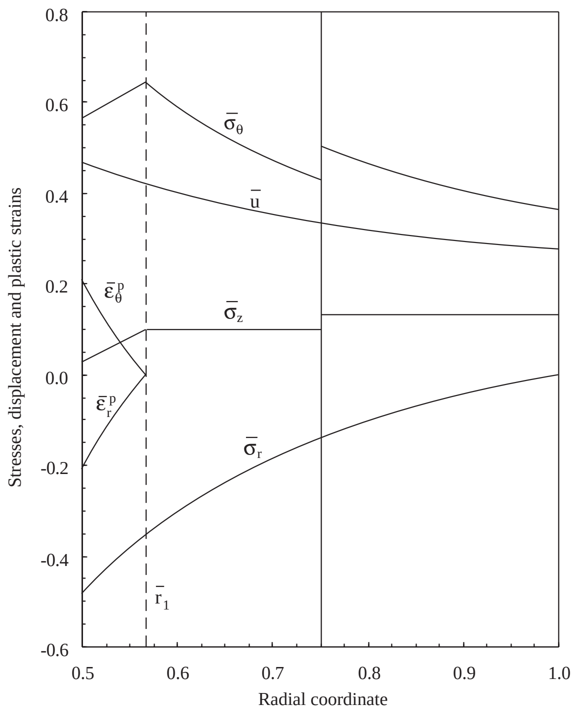

Figure 3. Stresses, displacement and plastic strains at $\bar{P}_1 = 0.484690$.

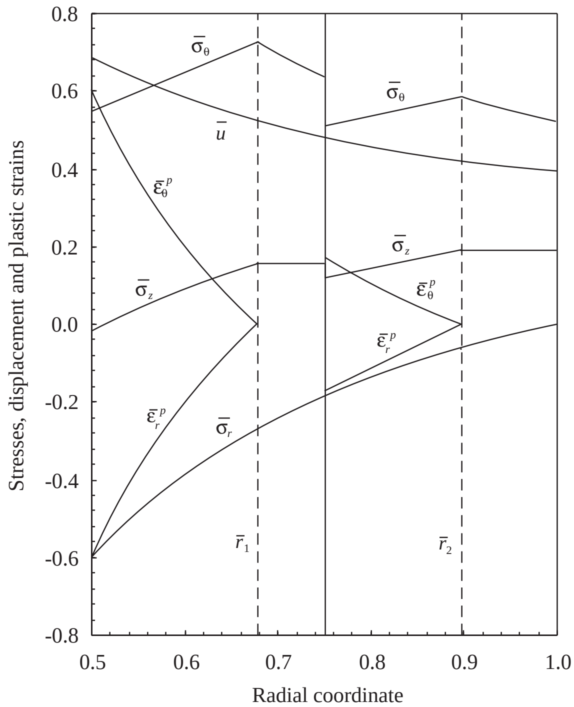

Figure 4. Stresses, displacement and plastic strains at $\bar{P} = 0.6$.

If the same assembly is subjected to external pressure, yielding begins at the inner surface with the yield condition (26) for the values of the pressure $P > P_e$. Similar to the first stage of the internal pressure case, expressions for the plastic and elastic regions contain 6 unknown integration constants: C_1, C_2 (plastic region II), C_3, C_4 (inner elastic region), C_5, C_6 (outer elastic region) and an unknown plastic-elastic border radius r_1. The following boundary and interface conditions are imposed for the determination of unknowns: $\sigma_r^{pI}(a) = 0$, $\sigma_r^{pI}(r_1) = \sigma_r^{eI}(r_1)$, $\sigma_\theta^{pI}(r_1) = \sigma_\theta^{eI}(r_1)$, $u^{pI}(r_1) = u^{eI}(r_1)$, $\sigma_r^{eI}(b) = \sigma_r^{eO}(b)$, $u^{eI}(b) = u^{eO}(b)$ and $\sigma_r^{eO}(c) = -P$. At the critical pressure $\bar{P}_1 = 0.465615$, a new plastic region appears at the interface according to the same yield criterion (26) and unknowns are determined as $\bar{C}_1 = -9.78014 \times 10^{-4}$, $\bar{C}_2 = -1.71687 \times 10^{-3}$, $\bar{C}_3 = -7.84905 \times 10^{-4}$, $\bar{C}_4 = -9.31358 \times 10^{-4}$, $\bar{C}_5 = -8.47178 \times 10^{-4}$, $\bar{C}_6 = -8.20651 \times 10^{-4}$ and $\bar{r}_1 = 0.545707$. The stresses, displacement and plastic strains for this stage are calculated and plotted in Figure 5.

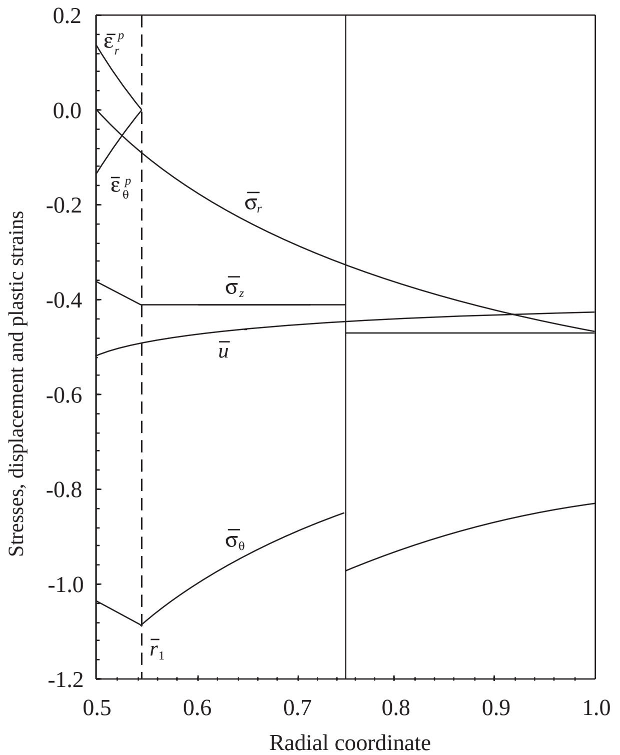

Figure 5. Stresses, displacement and plastic strains at $\bar{P}_1 = 0.465615$.

For the values of pressure $P > P_1$, the composite tube consists of 4 regions, namely, inner plastic, inner elastic, outer plastic and outer elastic. The solution of this elastic-plastic deformation problem involves the determination of 8 integration constants: C_1, C_2 (inner plastic), C_3, C_4 (inner elastic), C_5, C_6 (outer plastic), C_7, C_8 (outer elastic), and 2 unknown plastic-elastic border radii r_1 and r_2. In addition to the conditions listed above, the continuity of 2 stress components and displacement at the plastic elastic border r_2 are imposed to determine the integration constants and border radii. Taking $\bar{P} = 0.6 > \bar{P}_1$, the unknown parameters are determined as $\bar{C}_1 = -1.47860 \times 10^{-3}$, $\bar{C}_2 = -1.80360 \times 10^{-3}$, $\bar{C}_3 = -1.18665 \times 10^{-3}$, $\bar{C}_4 = -1.28612 \times 10^{-3}$, $\bar{C}_5 = -1.55399 \times 10^{-3}$, $\bar{C}_6 = -1.16340 \times 10^{-3}$, $\bar{C}_7 = -1.27010 \times 10^{-3}$, $\bar{C}_8 = -1.10603 \times 10^{-3}$, $\bar{r}_1 = 0.670984$ and $\bar{r}_2 = 0.910319$. The corresponding stresses, displacement and plastic strains are plotted in Figure 6.

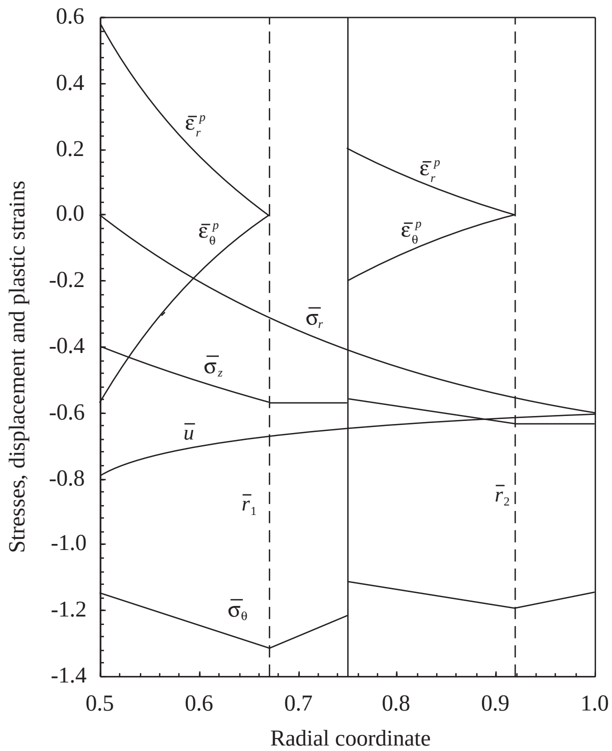

Figure 6. Stresses, displacement and plastic strains at $\bar{P} = 0.6$.

Concluding Remarks

This work represents an extension of previous studies to include composite tubes, strain hardening and axially constrained ends. For unrestricted ends the axial stress σ_z vanishes throughout (Ugural and Fenster, 1995) while it is constant for fixed ends. The tube assembly with fixed ends first yields at the inner surface $r = a$ irrespective of whether internal or external pressure is applied. In the case of internal pressure, the yield condition is $\sigma_y = \sigma_\theta - \sigma_r$ (see Figure 1), while it is $\sigma_y = \sigma_r - \sigma_\theta$ for the external pressure case (see Figure 2). For the aspect ratio ($\bar{a} = 0.5$) considered, elastic-plastic deformation of the assembly consists of 2 stages. In the first stage, plastic deformation begins at the inner surface at elastic limit $P = P_e$ and the plastic region formed here propagates toward the interface with increasing pressure. While this plastic region propagates, another plastic region forms at the interface in the outer tube at a critical value of pressure $P = P_1$. For the values of pressure greater than P_1 the assembly consists of 4 different regions and plastic regions continue propagating as the pressure is increased. The critical values of pressure for both loading cases are determined here using the properties of real engineering materials and the corresponding stress states are presented graphically.

Nomenclature

a, b, c	inner, interface and outer radii of the tube assembly, respectively
C_i	integration constants
E	modulus of elasticity
r, θ, z	cylindrical coordinates
u	radial displacement (dimensionless form $\bar{u} = uE_1/\sigma_{01}c$)
ε_i	strain components
ε_i^p	plastic strain components (normalized form $\bar{\varepsilon}_i^p = \varepsilon_i^p E_1/\sigma_{01}$)
ε_{EQ}	equivalent plastic strain
η	hardening parameter (normalized form $H = \eta\sigma_0/E$)
ν	Poisson's ratio
σ_i	stress components (dimensionless form $\bar{\sigma}_i = \sigma_i/\sigma_{01}$)
σ_0, σ_y	initial and subsequent yield stress
P	pressure (dimensionless form $\bar{P} = P/\sigma_{01}$)
r_i	elastic-plastic border radius

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