PARADOX OF VON ENGEL-STEENBECK AND CHANNEL MODEL OF THE ELECTRIC ARC

by

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It is offered simple algorithm with which help it is possible to estimate heat flux, average temperature, and temperature profile in the channel of the electroarc plasma generator, and also the shape of the positive arc column depending on the rate of flux scavenged of plasma-forming gas through a plasma generator.

Key words: *temperature, plasma, channel, arc, model*

Introduction

The paradox of von Engel-Steenbeck is well known for the experts in the field of physics of the low-temperature plasma [1-3]. It consists in the fact that: the more we cool the arc discharge in order, to put it out, on the contrary, it becomes hotter, but is more thin. Obviously, the increase of heat removal from an arc leads, in turn, to increasing of efficiency of the mechanism of the transfer of energy by an electric arc from voltage source, and diffused then in space by its cooling by a stream of cold gas. Thus a heat flux:

$$\theta = \int_0^T \lambda(T)\,\mathrm{d}T$$

through boundary of the conducting channel, increases according to a relation [4]:

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}r}\right|_{r_c} = -\frac{W}{2\pi r_c} \tag{1}$$

The minus sign means, that the heat flux is is oriented opposite to the increments of radial coordinate.

Direct numerical modeling of a heating of the gas flow scavenged through an electric arc is labour-consuming problem. Desire to build adequate mathematical model of the mentioned processes leads to the necessity of the numerical solution of the equations of a magnetohydrodynamics. That in turn poses a series of additional questions, bounded with nonlinearity of system, closure, a selection of the finite-difference grid,

boundary conditions, and so on. Unfortunately, there are no yet universal recommendations how to solve mentioned questions. In numerous publications [4-6] only various special cases are reviewed – some methods of the solution of this problem.

The most successful among them should be counted, apparently, those examinations, in which with the least mathematical expenses (simplicity of model and programm embodying) is possible to describe the given process adequately. The urgency of such works is obvious, as they give specialists on energetics and technology the tool for searching optimum operation modes of energy installations.

About channel model of an arc which it is longitudinal blasted

The paradox of von Engel-Steenbeck can be described adequately enough with the help of channel model of an electric arc as follows.

In a case interesting us the gas moving along an axis of a plasma generator, flowing along the arc and consequently it is possible to treat arc column (in our case – the current-conducting channel with conductance σ_0) as the fixed cylinder with evenly distributed heat sources along axis and radius. The radius of the current-conducting channel r_c is determined then from the condition that heat flux and temperatures on the boundary of conductive and non-conducting fields of an electric arc, are equal. The problem in this case can be formulated as follows.

The radial heat flux heats up the gas moving with average velocity U in non-conducting area of the discharge. Gas is chilled both because of a convective removal of heat, and by thermal conduction. Convective removal of heat (in unit of time), carried out along an axis of a tube, in average it is possible to describe as:

$$\rho c_p \frac{U(T - T_{\text{blow}})}{L}$$

Thus, the transport equation of heat for a non-conducting zone in usual notations will become:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\lambda \frac{\mathrm{d}T}{\mathrm{d}r}\right) - \rho c_p \frac{U(T - T_{\text{blow}})}{L} = 0, \quad r_c \le r \le R$$

Radius R of the cylindrical channel restricting the discharge. Channel walls are maintained at some stationary, small enough temperature $T(R) =$ const; because of a linearization on axial coordinate we assume $T = T(r)$. Terms "channel", "boundary of the channel" have the same sense as in paper [4], therefore the relation between temperature on axis T_0, and power W, per unit length of a column, received in mentioned paper is valid:

$$T_0 \le \sqrt{\frac{I_i W}{8\pi k\lambda}}$$

The temperature on the channel wall $T_c \approx T_0$, and heat flux through the wall is determined by the relation (1).

Assuming $\lambda \approx \lambda(T_0) = \text{const}$, considering (according to channel model approach) all transport coefficients approximately equal to their average values along length of the discharge, and also taking into account boundary conditions, that is equality of heat flux and temperatures, we obtain the following boundary-value problem:

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{1}{D^2}(T - T_{\text{blow}}) = 0$$

$$T(R) = \text{const}; \quad T(r_c) = T_0; \quad \lambda\frac{d\theta}{dr}\bigg|_{r=r_c} = \frac{W}{2\pi r_c} \tag{2}$$

where D is introduced instead of $L\sqrt{\text{Pe}}$.

The general solution of the given differential equation looks like [8]:

$$T(r) = T_{\text{blow}} + c_1 I_0(r/D) + c_2 K_0(r/D) \tag{3}$$

Substitution of boundary conditions (2) in the general solution (3) gives the transcendental recurrent set of equations relative to c_1, c_2, and r_c:

$$c_1 = \frac{T(R) - T_{\text{blow}} - c_2 K_0(R/D)}{I_0(R/D)}$$

$$c_2 \frac{K_0(R/D)I_0(r_c/D) - K_0(r_c/D)I_0(R/D)}{I_0(R/D)}$$
$$= (T_0 - T_{\text{blow}}) - [T(R) - T_{\text{blow}}]\frac{I_0(r_c/D)}{I_0(R/D)}$$

$$c_2 \frac{K_0(R/D)I_1(r_c/D) - K_1(r_c/D)I_0(r/D)}{I_0(R/D)}$$
$$= \frac{4kT_0^2}{I_i(D/r_c)} - [T(R) - T_{\text{blow}}]\frac{I_1(r_c/D)}{I_0(R/D)}$$

whence the transcendental equation is as fellows for r_c:

$$\frac{D}{r_c}\frac{K_0(R/D)I_0(r_c/D) - K_0(r_c/D)I_0(R/D)}{K_0(R/D)I_1(r_c/D) - K_1(r_c/D)I_0(R/D)}$$

$$I_i = \frac{(T_0 - T_{\text{blow}})I_0(R/D) - [T(R) - T_{\text{blow}}]I_0(r_c/D)}{4kT_0^2 I_0(R/D) - \frac{r_c}{D}I_i[T(R) - T_{\text{blow}}]I_1(r_c/D)} \tag{4}$$

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and relations for c_1 and c_2:

$$c_1 = \frac{1}{I_0(R/D)}\frac{(T_0 - T_{\text{blow}})K_0(R/D)I_0(r_c/D) - [T(R) - T_{\text{blow}}]K_0(r_c/D)I_0(R/D)}{K_0(r_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)} \quad (5)$$

$$c_2 = \frac{(T_0 - T_{\text{blow}})I_0(R/D) - [T(R) - T_{\text{blow}}]I_0(r_c/D)}{K_0(r_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)} \quad (6)$$

In real conditions, as a rule, temperature of the plasma-forming gas fed to the discharge camera is insignificant and is approximately equal to the temperature of a wall, so it is possible to assume $T(R) \approx T_{\text{blow}}$. Relationships (4)-(6) can be simplified, and then we shall receive:

$$c_1 = [T_0 - T(R)]\frac{K_0(R/D)}{K_0(R_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)}$$

$$c_2 = [T_0 - T(R)]\frac{I_0(R/D)}{K_0(r_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)}$$

and

$$\frac{D}{r_c}\frac{K_0(R/D)I_0(r_c/D) - K_0(r_c/D)I_0(r/D)}{K_0(R/D)I_1(r_c/D) - K_1(r_c/D)I_0(r/D)}$$

$$I_i = [T_0 - T(R)]\frac{I_0(R/D)}{4kT_0^2 I_0(R/D)}$$

If now we neglect also temperature on the wall $T(R)$ in comparison with temperature in interior of discharge $T(r_c) \approx T_0$, relations (4)-(6) will be even more simplified:

$$c_1 = T_0\frac{K_0(R/D)}{K_0(r_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)} \quad (7)$$

$$c_2 = T_0\frac{I_0(R/D)}{K_0(r_c/D)I_0(R/D) - K_0(R/D)I_0(r_c/D)} \quad (8)$$

$$\frac{D}{r_c}\frac{K_0(R/D)I_0(r_c/D) - K_0(r_c/D)I_0(r/D)}{K_0(R/D)I_1(r_c/D) - K_1(r_c/D)I_0(R/D)} = \frac{I_i}{4kT_0} \quad (9)$$

Formulas (7)-(9), within the frame of an approximative channel model, give explicite solution of the problem posed above.

As one would expect, if $U \to 0$ (that is $D \to \infty$) relation (3) becames the known Steenbeck solution for an non-flowing electric arc:

$$T(r) \quad T(R) \quad \frac{W}{2\pi\lambda} \ln \frac{r}{r_c}$$

Results

Relationship (7)-(9) have the physical transparence and are convenient for research and estimation. So, for example, with the help of asymptotics [9, 10] for functions I_0, I_1, K_0, and K_1, it is easy to show validity of the formula in the case R/D $\ll$ 1, and $U \sim 0$ (when the basic mechanism of a heat removal is the conduction):

$$r_c \quad R\mathrm{e}^{\frac{I_i}{4\mathrm{k}T_0}} \tag{10}$$

which, predictably, coincides with classical result of model of channel [4].

Let's consider now a case when the convective heat removal is essential, that is the case, when $R/D \gg 1$. As it is known [4], in practice almost always is $I_i/4kT_0 \ll 1$. After examination of the left part of a transcendental eq. (9) as functions r_c/D, it is concluded, that to sutisfy this inequality, it must also be $r_c/D \ll 1$ and then, using asymptotics for functions I_0, I_1, K_0, and K_1 for $r_c/D \ll 1$ and $R/D \gg 1$ in the mentioned equations, we shall receive:

$$r_c \quad D\mathrm{e}^{\frac{I_i}{4\mathrm{k}T_0}} \tag{11}$$

It is obvious from formulas for r_c, that the radius of the current-conducting channel decreases with flow in R/D times ($R/D \gg 1$), compared with the value for $U = 0$ (that is without flow). This result is in agreement with effect of decreasing of radius of the current-conducting channel observed in experiments for increasing flow velocity [1, 2].

In fig. 1 the results of calculations of temperature profiles using formulas (3)-(6) are presented. Calculations are carried out for the typical values $T(R)$ = 400 K, T_{blow} = 500 K, and T_{channel} = 12000 K, and various values of a Peclet number. Radius of the tube was R = 1,5 cm. Also, the results of calculations using Steenbeck formula are presented. Calculations are carried out for argon plasma at atmospheric pressure for various flow velocities. Values of quantities ρ, c_{p}, and λ are taken from paper [11, 12].

It is obvins that the temperature profiles in a non-conducting band of an electric arc, for small flow velocities are moving toward the Steenbeck solution, which, thus, is an asymptotic limit for a family of curves, described by formulas (3)-(9).
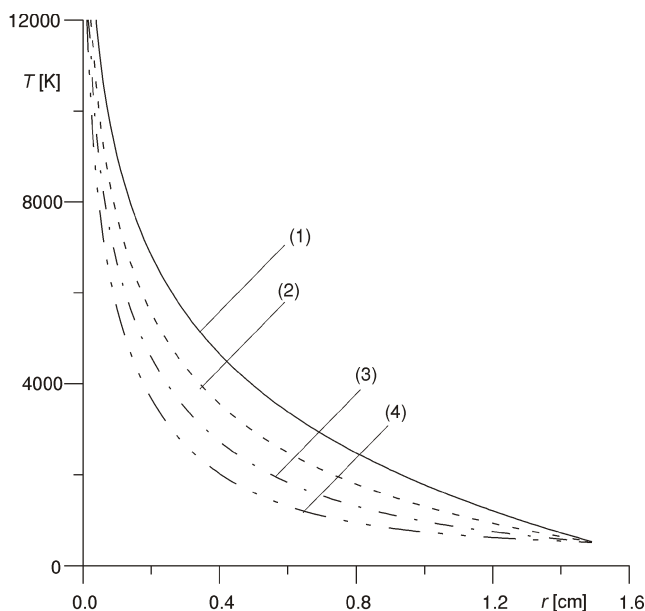
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**Figure 1. Temperature distributions in a non-conducting zone of an electric arc for various flow velocities of blower:
(1) $U = 0$ m/s (profile of Steenbeck); (2) $U = 1$ m/s;
(3) $U = 2.5$ m/s; (4) $U = 5$ m/s**

By averaging temperature profiles along radial coordinate, it is possible to receive as well average temperature of a jet T_a on the outlet from the arc plasma generator. Results of the numerical solution of transcendental eqs. (4) and (9) relative r_c, confirm validity of estimations (10) and (11) for radius of the current-conducting channel .

Finaly, the analysis of the given graphs allows to conclud that exists good agreement between analytical estimation and precise numerical solutions, and that it is possible to recommend the given procedure for use both in model, and, probably, also in engineering calculations.

Nomenclature

c_p – heat capacity, [J/kgK]
I_i – potential of ionizations, [J]
I_0 – modified Bessel function of order zero
I_1 – modified Bessel function of order one
K_0 – McDonald function of order zero
K_1 – McDonald function of order one
k   – Boltzmann constant, [J/K]
L – length of an arc, [m]

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Pe   – number of Peclet ($= \rho c_p U L / \lambda$)
R – radius of the cylindrical channel, [m]
r_c – radius of the current-conducting channel, [m]
T – temperature in the discharge, [K]
T_{blow} – temperature of "cold" plasma-forming gas, which is blown in through a
            front part of the discharge, [K]
U – velocity of blower, [m/s]
W – power put in the discharge per unit his lengths, [W/m]

Greek symbols

λ – thermal conductivity, [W/mK]
ρ – density, [kg/m^3]
θ – heat flux, [W/m]

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