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The Nonlocal Solution of the Elastic Half-Plane Loaded at the Origin by Shear Force

Reha ARTAN
İTÜ İnşaat Fakültesi, Mekanik Anabilim Dalı,
Maslak 80626, İstanbul-TURKEY

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Abstract

In this paper, the nonlocal continuum theory is applied to the problem of elastic half plane loaded at the origin by a force P directed along x axis. The solution of this problem in the frame of classical elasticity can be found in every reference book of elasticity. First constitutive equations of nonlocal theory is given. The nonlocal stress field is determined. According to the classical elasticity solution of this problem, stresses become infinite at the application point. That is, classical elasticity solution is not valid in the neigbourhood of origin. To remedy this situation nonlocal continuum theory is used. The results are compared with the classical elasticity solution. Interestingly enough none of the classical singularities exist in the nonlocal solution.

Key Words: Nonlocal Elasticity, Nonlocal Elastic Half-plane, Nonlocal Continuum Theory

Orjin Noktasına Etkiyen Kesme Kuvveti ile Yüklü Yarım Düzlem Probleminin Yerel Olmayan Elastisitede Çözümü

Özet

Bu çalışmada x ekseni dogrultusunda tekil yükle yüklü yarım düzlem problemi yerel olmayan elastisite teorisi çerçevesinde çözülmüştür. Bu problem klasik elastisitede daha önce çözülmüş olup çözüm herhangi bir elastisite teorisi kitabında kolayca bulunabilir. Ancak problem yerel olmayan teori kullanılarak henüz incelenmemiştir. Problemin klasik elastisitedeki çözümünde kuvvetin tatbik noktasında gerilme tekillikleri mevcuttur. Yani klasik çözüm kuvetin tatbik noktasının komşuluğunda geçerli değildir. Bu bölgedeki gerilmeleri hesaplamak için yerel olmayan teorinin kullanılması uygundur. Yerel olmayan teorinin temel denklemleri kısaca verildikten sonra, problemin yerel olmayan elastisitede ki çözümü yapılmış ve sonuçlar klasik teorinin sonuçlarıyla karşılaştırılmıştır.

Anahtar Sözcükler: Yerel olmayan elastisite, Yerel olmayan elastik yarım düzlem, Yerel olmayan teori

Introduction

The problem of an elastic half-plane loaded at the origin by a force P directed along the x axis is solved in the frame of nonlocal elasticity. The classical elasticity solution of this problem can be found in every reference book on the mathematical theory of elasticity. But the problem has not been solved in the frame of nonlocal elasticity yet. According to the classical elasticity solution of this problem, stresses at the application point of the force become infinite. In other words, the classical elasticity solution contains artificial infinite stresses at the application point of the force, but this solution does not display the actual situation. In particular, the most important question about the value of maximum pressure remains unanswered. In this paper, the classical elasticity singularities are eliminated and the maximum stresses are calculated.

In the nonlocal theory, the constitutive relations are nonlocal in character and the stress at a given point does not only depend on the strain at the same point, but also on the strains at all points of the body. The governing equations of the nonlocal elasticity are given in Altan (1989), Eringen (1974), Eringen (1976) and Eringen (1987). Some of the early ideas for the nonlocal elastic solids were explored by Eringen, Edelen and Kunin. Eringen and Edelen (1972), Kunin (1968). The program Mathematica, Derive and LaTeX are used throughout.)

The Nonlocal Solution of the Elastic Half-Plane Loaded at the Origin by a Force P Directed Along the x Axis

The classical elasticity solution of the elastic half-plane loaded at the origin by a force P directed along the x axis in cartesian coordinates is Rekach (1979) and (Figure 1)

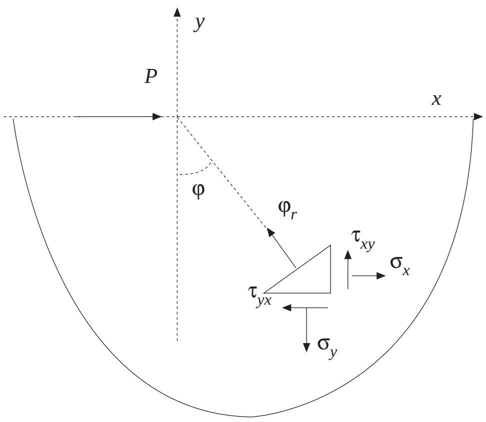

Figure 1. Elastic half plane loaded at the origin by a shear force

$$\sigma_x = -A\,P\frac{x^3}{(x^2+y^2)^2} \tag{1}$$

$$\sigma_y = -A\,P\frac{xy^2}{(x^2+y^2)^2} \tag{2}$$

$$\tau_{xy} = -A\,P\frac{x^2 y}{(x^2+y^2)^2} \tag{3}$$

Where

$$A = \frac{2}{\pi\delta} \tag{4}$$

δ is the thickness of the medium. The nonlocal stress field can be obtained as follows:

$$t_{xx}(x,y,a) = \int\int \alpha(|\mathbf{x}'-\mathbf{x}|)\sigma_{xx}(x',y')\,dx'\,dy' \tag{5}$$

$$t_{yy}(x,y,a) = \int\int \alpha(|\mathbf{x}'-\mathbf{x}|)\sigma_{yy}(x',y')\,dx'\,dy' \tag{6}$$

$$t_{xy}(x,y,a) = \int\int \alpha(|\mathbf{x}'-\mathbf{x}|)\tau_{xy}(x',y')\,dx'\,dy' \tag{7}$$

where $\alpha(|\mathbf{x}'-\mathbf{x}|)$ is called the kernel function and is the measure of the effect of the strain at point $\mathbf{x'}$ on the stress at point $\mathbf{x}$. Artan (1996a), Artan (1996b), Artan (1996c), Artan (1997), Eringen (1976). In this article, the kernel function of the nonlocal medium will be chosen as follows:

$$\alpha(|\mathbf{x}-\mathbf{x}'|) = \begin{cases} B\big(1-\dfrac{|\mathbf{x}-\mathbf{x}'|^2}{a^2}\big) & |\mathbf{x}-\mathbf{x}'| \le a \\ 0 & |\mathbf{x}-\mathbf{x}'| \ge a \end{cases} \tag{8}$$

where a is the atomic distance and B is a constant. In the Cartesian coordinates (8) becomes (Figure 2)

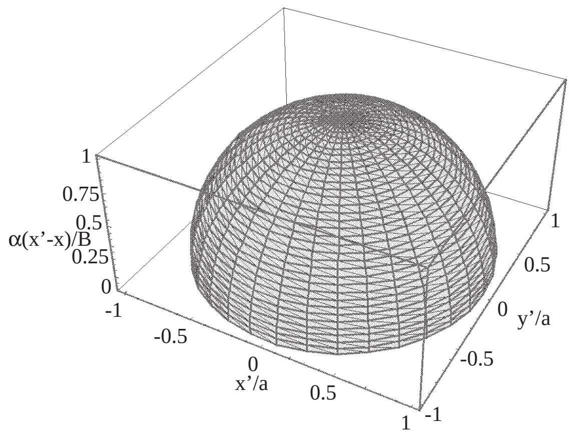

Figure 2. Kernel function

$$\alpha(x,y) = B\left(1-\frac{(x'-x)^2+(y'-y)^2}{a^2}\right); \quad |\mathbf{x}-\mathbf{x}'| \le a \tag{9}$$

The values of a and B are Artan (1996b)

$$a = 4 \times 10^{-8}\, cm, \quad B = \frac{2}{\pi\, a^2} \tag{10}$$

When the distance to the boundary is less than one atomic measure, the nonlocal stress field in the x direction is calculated as (Figure 4)

$$t_{xx}(x,y,a) = \int_{0}^{y-a}\int_{\alpha_1}^{\alpha_2} \alpha(x,y)\sigma_{xx}(x',y')\, dx'dy' \tag{11}$$

where

$$\begin{aligned} \alpha_1 &= x - \sqrt{a^2 - (y'-y)^2} \\ \alpha_2 &= x + \sqrt{a^2 - (y'-y)^2} \end{aligned} \tag{12}$$

When the distance to the boundary is greater than one atomic measure, the nonlocal stress field in the x direction is calculated as (Figure 3)

$$t_{xx}(x,y) = \int_{y-a}^{y+a}\int_{\alpha_1}^{\alpha_2} \alpha(x,y)\sigma_{xx}(x',y')\, dx'dy' \tag{13}$$

In the above equations, the first integral over x' is calculated exactly, and then the second integral over y' is calculated approximately. The nonlocal stress field becomes

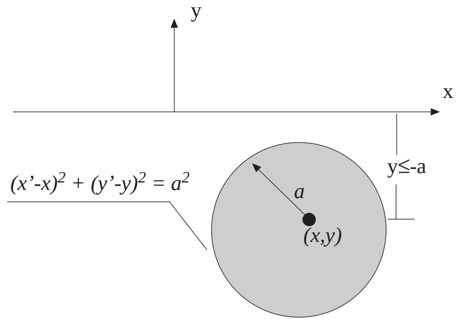

Figure 3. Integration domain for $y \leq -a$

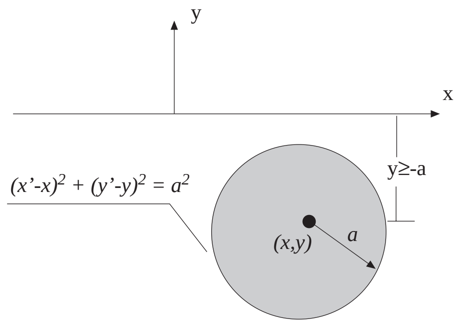

Figure 4. Integration domain for $y \geq -a$

$$\begin{aligned} t_{xx}(x,y,a) &= \frac{2\,P\,A}{\pi a^2}(0.56\, r(x,y,0.5(y-a),a) + 0.22 r(x,y,0.1(y-a),a)) \\ &\times\ (a-y); \quad -a \leq y \leq 0 \end{aligned} \tag{14}$$

$$\begin{aligned} t_{xx}(x,y,a) &= \frac{2PA}{\pi\, a^2}\Big(r(x,y,y-3a/8,a)\frac{3a}{8} + r(x,y,y-a/4,a)\,\frac{3a}{8} \\ &+\ r(x,y,y,a)\,\frac{a}{8} + r(x,y,y+3a/8,a)\frac{3a}{8} + r(x,y,y-a/4,a)\,\frac{a}{8} \\ &+\ r(x,y,y,a)\,\frac{a}{8}\Big); \quad y \leq -a \end{aligned} \tag{15}$$

where

$$r(x,y,v,a) = \frac{cc1}{cc2} \tag{16}$$

$cc1$ and $cc2$ are given below

$$\begin{aligned}
cc1 &= 4\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}\,(a^4+2\,a^2\,(-x^2+2\,v\,y-y^2)\\
&+ (x^2+y^2)\,(4\,v^2+x^2-4\,v\,y+y^2))+6\,v\,x\,(a^2+x^2+2\,v\,y-y^2\\
&+ 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\,(-a^2-x^2-2\,v\,y+y^2\\
&+ 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\ \arctan(\frac{-x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v})\\
&+ 6\,v\,x\,(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\\
&\times (-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\\
&\times \arctan(\frac{x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v})+(a^2+v^2-x^2+2\,v\,y-y^2)\\
&\times (a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\\
&\times (-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\\
&\times \log(a^2+x^2+2\,v\,y-y^2-2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\\
&- (a^2+v^2-x^2+2\,v\,y-y^2)\,(a^2+x^2+2\,v\,y-y^2\\
&+ 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\,(-a^2-x^2-2\,v\,y+y^2\\
&+ 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\ \log(a^2+x^2+2\,v\,y-y^2\\
&+ 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})
\end{aligned} \tag{17}$$

$$\begin{aligned}
cc2 &= 2\,a^2\left(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}\right)\\
&\times \left(-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}\right)
\end{aligned} \tag{18}$$

The polynomial $t_{xx}^{OB}(\xi)$ is fitted for $t_{xx}(-y\tan(\pi/6), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xx}^{OB}(\xi)$ is valid on the line OB, Figures 5 and 6):

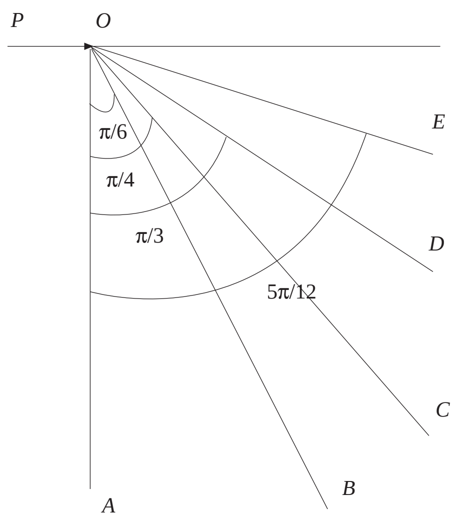

Figure 5. The stress diagrams are given on these lines

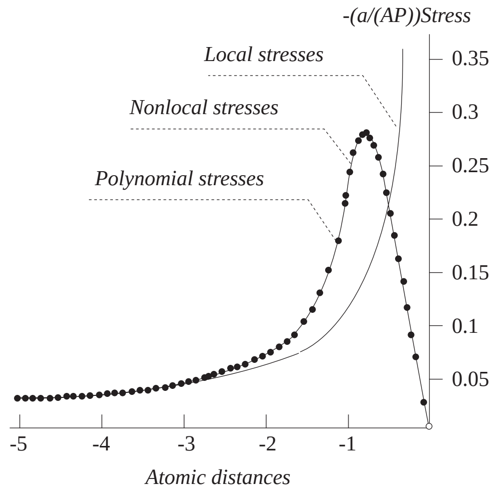

Figure 6. Normal stresses in the direction x axis on the line OB

$$t_{xx}^{OB}(\xi) = \frac{A\,P}{a}(0.00015595 - 0.39657786\xi + 0.40881854\xi^2 + 1.08755198\xi^3 + 1.67047100\xi^4 + 1.88204434\xi^5 + 0.71096324\xi^6); \quad -1 \le \xi \le 0 \tag{19}$$

$$t_{xx}^{OB}(\xi) = \frac{A\,P}{a}(1.49263876 + 2.75486289\xi + 2.23324620\xi^2 + 0.96245702\xi^3 + 0.22920226\xi^4 + 0.02846721\xi^5 + 0.00143934\xi^6); \quad \xi \le -1; \quad \xi = y/a \tag{20}$$

The polynomial $t_{xx}^{OC}(\xi)$ is fitted for $t_{xx}(-y\tan(\pi/4), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xx}^{OC}(\xi)$ is valid on the line OC, Figures 5 and 7)

$$t_{xx}^{OC}(\xi) = \frac{A\,P}{a}(-0.00078165 - 0.754285596\xi - 0.10498005\xi^2 - 1.48323233\xi^3 - 3.90964497\xi^4 - 2.04702453\xi^5 + 0.01528694\xi^6); \quad -1 \le \xi \le 0 \tag{21}$$

$$t_{xx}^{OC}(\xi) = \frac{A\,P}{a}(1.18520003 + 1.83861248\xi + 1.38846323\xi^2 + 0.57617936\xi^3 + 0.13407081\xi^4 + 0.01639597\xi^5 + 0.00081994\xi^6); \quad \xi \le -1; \quad \xi = y/a \tag{22}$$

The polynomial $t_{xx}^{OD}(\xi)$ is fitted for $t_{xx}(-y\tan(\pi/3), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xx}^{OD}(\xi)$ is valid on the line OD, see Figures 5 and 8)

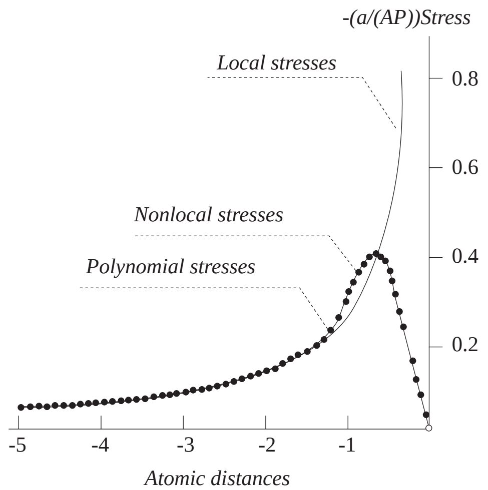

Figure 7. Normal stresses in the direction x axis on the line OC

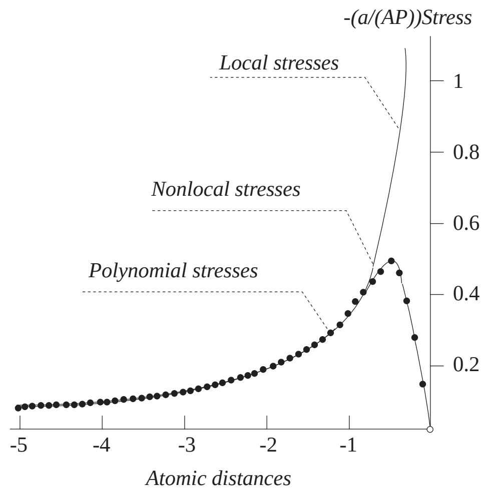

Figure 8. Normal stresses in the direction x axis on the line OD

$$t_{xx}^{OD}(\xi) = \frac{AP}{a}(0.00198456 - 1.15248740\xi + 0.99820896\xi^2 - 0.13497878\xi^3 - 14.7541391\xi^4 - 22.0957259\xi^5 - 9.30953289\xi^6); \quad -1 \le \xi \le 0 \tag{23}$$

$$t_{xx}^{OD}(\xi) = \frac{AP}{a}(1.01589065 + 1.24964382\xi + 0.78512136\xi^2 + 0.27699469\xi^3 + +0.05554837\xi^4 + 0.00591481\xi^5 + 0.00025970\xi^6); \quad \xi \le -1; \quad \xi = y/a \tag{24}$$

The polynomial $t_{xx}^{OE}(\xi)$ is fitted for $t_{xx}(-y\tan(5\pi/12), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xx}^{OE}(\xi)$ is valid on the line OE, Figures 5 and 9)

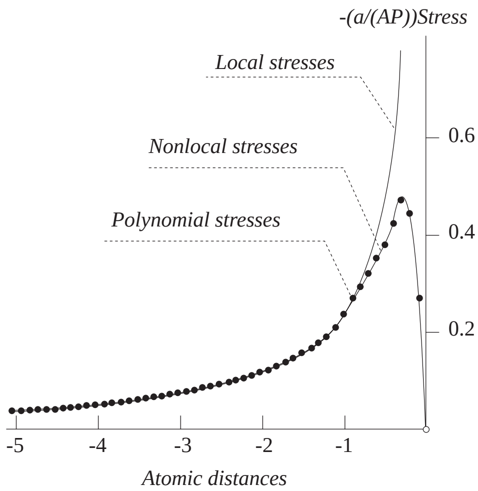

Figure 9. Normal stresses in the direction x axis on the line OE

$$t_{xx}^{OE}(\xi) = \frac{AP}{a}(0.00299421 - 2.04372120\xi + 15.9852596\xi^2 + 132.638002\xi^3 + 366.199774\xi^4 + 492.730500\xi^5 + 327.614427\xi^6 + 86.2433531\xi^7); \ -1 \le \xi \le 0 \tag{25}$$

$$t_{xx}^{OE}(\xi) = \frac{AP}{a}(0.76009320 + 0.941979749\xi + 0.58651882\xi^2 + 0.20296328\xi^3 + 0.03961192\xi^4 + 0.00407712\xi^5 + 0.00017191\xi^6); \quad \xi \le -1; \quad \xi = y/a \tag{26}$$

When the distance to the boundary is less than one atomic measure, the nonlocal stress field in the y direction is calculated as (Figure 4)

$$t_{yy}(x, y, a) = \int_0^{y-a} \int_{\alpha_1}^{\alpha_2} \alpha(x, y)\sigma_{yy}(x', y')\, dx'dy' \tag{27}$$

When the distance to the boundary is greater than one atomic measure the nonlocal stress field in the y direction is calculated as (Figure 3)

$$t_{yy}(x,y) = \int_{y-a}^{y+a}\int_{\alpha_1}^{\alpha_2} \alpha(x,y)\sigma_{yy}(x',y')\,dx'dy' \quad (28)$$

In the above equations the first integral over x' is calculated exactly, and then the second integral over y' is calculated approximately. The nonlocal stress field becomes

$$\begin{aligned} t_{yy}(x,y,a) &= \frac{2\,P\,A}{\pi a^2}(0.55\,s(x,y,0.5(y-a),a) + 0.22 s(x,y,0.1(y-a),a)) \\ &\times \ (a-y); \quad -a \le y \le 0 \end{aligned} \tag{29}$$

$$\begin{aligned} t_{yy}(x,y,a) &= \frac{2PA}{\pi\,a^2}\Big(s(x,y,y-3a/8,a)\frac{3a}{8} + s(x,y,y-a/4,a)\,\frac{3a}{8} \\ &+ \ s(x,y,y,a)\,\frac{a}{8} + s(x,y,y+3a/8,a)\frac{3a}{8} + s(x,y,y-a/4,a)\,\frac{a}{8} \\ &+ \ s(x,y,y,a)\,\frac{a}{8}\Big); \quad y \le -a \end{aligned} \tag{30}$$

where

$$\begin{aligned} s(x,y,v,a) &= -\frac{v\,x\,\arctan(\frac{-x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v})}{a^2} - \frac{v\,x\,\arctan(\frac{x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v})}{a^2} \\ &- \ \frac{v^2\,\log(\sqrt{a^2+x^2+2\,v\,y-y^2-2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}})}{a^2} \\ &+ \ \frac{v^2\,\log(\sqrt{a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}})}{a^2} \end{aligned} \tag{31}$$

The polynomial $t_{yy}^{OB}(\xi)$ is fitted for $t_{yy}(-y\tan(\pi/6), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{yy}^{OB}(\xi)$ is valid on the line OB, Figures 5 and 10)

$$\begin{aligned} t_{yy}^{OB}(\xi) &= \frac{A\,P}{a}(-0.00020307 - 0.16016250\xi + 0.02111223\xi^2 \\ &- \ 0.87036674\xi^3 - 2.17658376\xi^4 - 2.29169096\xi^5 \\ &- \ 0.95039343\xi^6); \quad -1 \le \xi \le 0 \end{aligned} \tag{32}$$

$$\begin{aligned} t_{yy}^{OB}(\xi) &= \frac{A\,P}{a}(0.18372122 - 0.21338880\xi - 0.29804238\xi^2 \\ &- \ 0.14811556\xi^3 - 0.03710521\xi^4 - 0.00469058\xi^5 \\ &- \ 0.00023803\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{33}$$

The polynomial $t_{yy}^{OC}(\xi)$ is fitted for $t_{yy}(-y\tan(\pi/4), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{yy}^{OC}(\xi)$ is valid on the line OB, Figures 5 and 11):

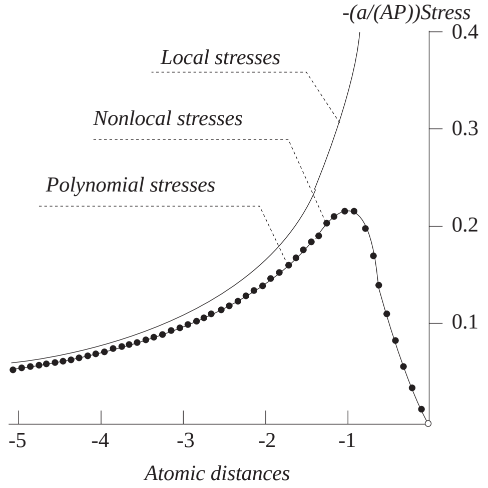

Figure 10. Normal stresses in the direction y axis on the line OB

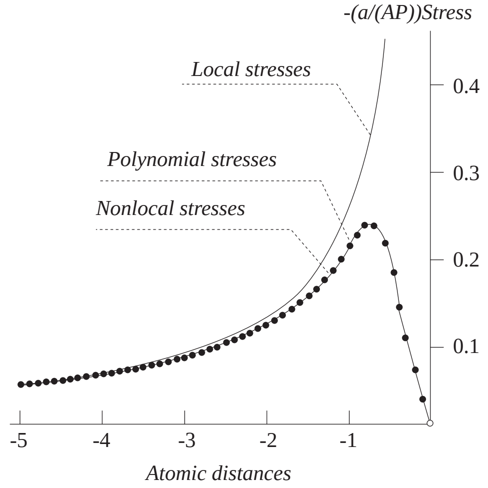

Figure 11. Normal stresses in the direction y axis on the line OC

$$\begin{aligned} t_{yy}^{OC}(\xi) &= \frac{AP}{a}(0.00001860 - 0.23945022\xi + 0.66939554\xi^2 \\ &+ 2.37593136\xi^3 + 6.04416442\xi^4 + 7.64164157\xi^5 \\ &+ 3.27812680\xi^6); \quad -1 \le \xi \le 0 \end{aligned} \tag{34}$$

$$\begin{aligned} t_{yy}^{OC}(\xi) &= \frac{AP}{a}(0.46726459 + 0.40394993\xi + 0.19014221\xi^2 \\ &+ 0.05065045\xi^3 + 0.00737001\xi^4 + 0.00050934\xi^5 \\ &+ 0.00001050\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{35}$$

The polynomial $t_{yy}^{OD}(\xi)$ is fitted for $t_{yy}(-y\tan(\pi/3), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{yy}^{OD}(\xi)$ is valid on the line OD, Figures 5 and 12):

$$\begin{aligned} t_{yy}^{OD}(\xi) &= \frac{AP}{a}(-0.00326344 - 0.75627802\xi - 4.78694975\xi^2 \\ &- 33.4209775\xi^3 - 101.618998\xi^4 - 145.439415\xi^5 \\ &- 99.4157851\xi^6 - 26.3215407\xi^7); \quad -1 \le \xi \le 0 \end{aligned}$$

$$\begin{aligned} t_{yy}^{OD}(\xi) &= \frac{AP}{a}(0.34328747 + 0.40974695\xi + 0.24615127\xi^2 \\ &+ 0.08239331\xi^3 + 0.01558087\xi^4 + 0.00155475\xi^5 \\ &+ 0.00006352\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{36}$$

The polynomial $t_{yy}^{OE}(\xi)$ is fitted for $t_{yy}(-y\tan(5\pi/12), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{yy}^{OE}(\xi)$ is valid on the line OE, Figures 5 and 13):

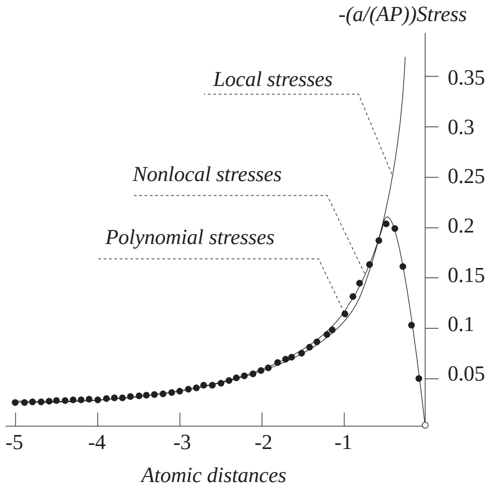

Figure 12. Normal stresses in the direction y axis on the line OD

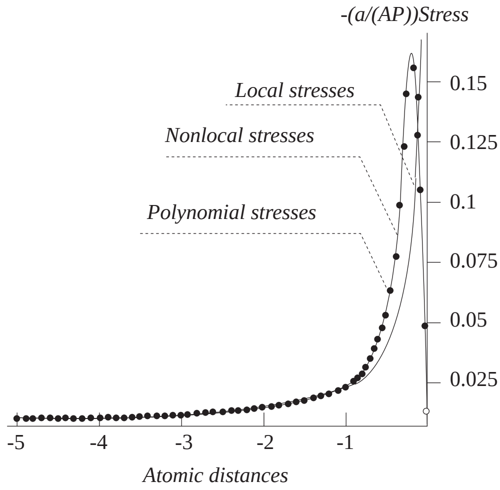

Figure 13. Normal stresses in the direction y axis on the line OE

$$\begin{aligned} t_{yy}^{OE}(\xi) &= \frac{A\,P}{a}(0.00875337 + 0.04121388\xi + 22.4609378\xi^2 \\ &+ 174.808977\xi^3 + 575.792210\xi^4 + 1012.75344\xi^5 \\ &+ 997.239948\xi^6 + 519.762780\xi^7 \\ &+ 111.881302\xi^8); \quad -1 \le \xi \le 0 \quad \xi = y/a \end{aligned} \tag{37}$$

$$\begin{aligned} t_{yy}^{OE}(\xi) &= \frac{A\,P}{a}(0.06890654 + 0.09932850\xi + 0.07140735\xi^2 \\ &+ 0.02858807\xi^3 + +0.00647292\xi^4 + 0.00077484\xi^5 \\ &+ 0.00003809\xi^6); \quad \xi \le -1; \xi = y/a \end{aligned} \tag{38}$$

When the distance to the boundary is less than one atomic measure, the nonlocal shear stress field is calculated as (Figure 4)

$$t_{xy}(x,y,a) = \int_0^{y-a}\int_{\alpha_1}^{\alpha_2} \alpha(x,y)\tau_{xy}(x',y')\,dx'dy' \tag{39}$$

When the distance to the boundary is greater than one atomic measure the nonlocal shear stress field is calculated as (Figure 3)

$$t_{xy}(x,y) = \int_{y-a}^{y+a}\int_{\alpha_1}^{\alpha_2} \alpha(x,y)\tau_{xy}(x',y')\,dx'dy' \tag{40}$$

In the above equations the first integral over x' is calculated exactly, and then the second integral over y' is calculated approximately. The nonlocal stress field becomes

$$\begin{aligned} t_{xy}(x,y,a) &= \frac{2\,P\,A}{\pi a^2}(0.53\,w(x,y,0.5(y-a),a) + 0.22w(x,y,0.1(y-a),a)) \\ &\times \; (a-y); \quad -a \le y \le 0 \end{aligned} \tag{41}$$

$$\begin{aligned} t_{xy}(x,y,a) &= \frac{2PA}{\pi\,a^2}\Big(w(x,y,y-3a/8,a)\frac{3a}{8} + w(x,y,y-a/4,a)\,\frac{3a}{8} \\ &+ w(x,y,y,a)\,\frac{a}{8} + w(x,y,y+3a/8,a)\frac{3a}{8} + w(x,y,y-a/4,a)\,\frac{a}{8} \\ &+ w(x,y,y,a)\,\frac{a}{8}\Big); \quad y \le -a \end{aligned} \tag{42}$$

where

$$w(x,y,v,a) = \frac{n1}{n2} \tag{43}$$

$n1$ and $n2$ are given below

$$\begin{aligned}
n1 \;=\;& -((a^2+2\,v^2-x^2+2\,v\,y-y^2)\,(a^2+x^2+2\,v\,y-y^2 \\
+\;& 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\,(-a^2-x^2-2\,v\,y+y^2 \\
+\;& 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\ \arctan(\frac{-x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v})) \\
-\;& (a^2+2\,v^2-x^2+2\,v\,y-y^2)\,(a^2+x^2+2\,v\,y-y^2 \\
+\;& 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\,(-a^2-x^2-2\,v\,y+y^2 \\
+\;& 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\ \arctan(\frac{x+\sqrt{a^2-v^2+2\,v\,y-y^2}}{v}) \\
+\;& v\,(-((a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& (-6\,a^2\,x+6\,x^2\,\sqrt{a^2-v^2+2\,v\,y-y^2}+2\,(a^2-v^2+2\,v\,y-y^2)^{\frac{3}{2}} \\
+\;& 2\,x\,(4\,v^2-x^2-6\,v\,y+3\,y^2)+(a^2+2\,v^2-x^2+2\,v\,y-y^2) \\
\times\;& (-x+\sqrt{a^2-v^2+2\,v\,y-y^2}))))+(-a^2-x^2-2\,v\,y+y^2 \\
+\;& 2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})\,(6\,a^2\,x+6\,x^2\,\sqrt{a^2-v^2+2\,v\,y-y^2} \\
+\;& 2\,(a^2-v^2+2\,v\,y-y^2)^{\frac{3}{2}}-2\,x\,(4\,v^2-x^2-6\,v\,y+3\,y^2) \\
+\;& (a^2+2\,v^2-x^2+2\,v\,y-y^2)\,(x+\sqrt{a^2-v^2+2\,v\,y-y^2})) \\
+\;& 2\,x\,(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& (-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& \log(a^2+x^2+2\,v\,y-y^2-2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
-\;& 2\,x\,(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& (-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& \log(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}))
\end{aligned} \tag{44}$$

$$\begin{aligned}
n2 \;=\;& 2\,a^2\,(a^2+x^2+2\,v\,y-y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2}) \\
\times\;& (-a^2-x^2-2\,v\,y+y^2+2\,x\,\sqrt{a^2-v^2+2\,v\,y-y^2})
\end{aligned} \tag{45}$$

The polynomial $t_{xy}^{OB}(\xi)$ is fitted for $t_{xy}(-y\tan(\pi/6), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xy}^{OB}(\xi)$ is valid on the line OB, Figures 5 and 14):

$$\begin{aligned}
t_{xy}^{OB}(\xi) \;=\;& \frac{A\,P}{a}(0.25985230-0.23481033\xi-0.37333829\xi^2 \\
-\;& 0.27716391\xi^3-0.71705619\xi^4-0.63648440\xi^5 \\
-\;& 0.13470999\xi^6); \quad -1\le\xi\le 0
\end{aligned} \tag{46}$$

$$\begin{aligned}
t_{xy}^{OB}(\xi) \;=\;& \frac{A\,P}{a}(0.59436457+0.78791738\xi+0.54534940\xi^2 \\
+\;& 0.21381195\xi^3+0.04778151\xi^4+0.00566876\xi^5 \\
+\;& 0.000276870\xi^6); \quad \xi\le -1 \quad \xi=y/a
\end{aligned} \tag{47}$$

The polynomial $t_{xy}^{OC}(\xi)$ is fitted for $t_{xy}(-y\tan(\pi/4), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xy}^{OC}(\xi)$ is valid on the line OC, Figures 5 and 15):

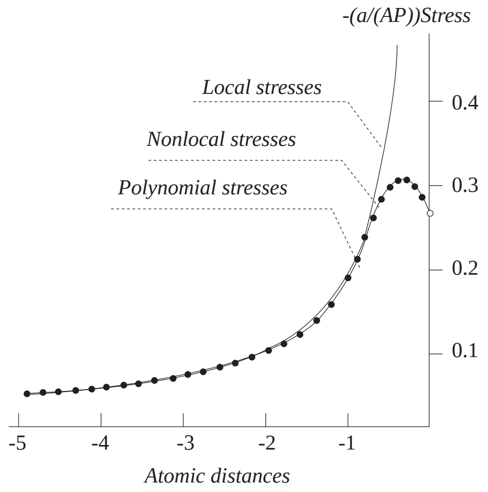

Figure 14. Shear stresses on the line OB

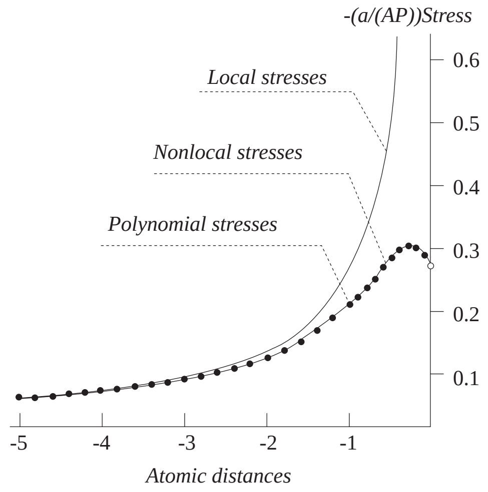

Figure 15. Shear stresses on the line OC

$$\begin{aligned} t_{xy}^{OC}(\xi) &= \frac{AP}{a}(0.26083403 - 0.18340326\xi + 0.07662246\xi^2 \\ &+ 1.85895975\xi^3 + 2.96445409\xi^4 + 1.80965063\xi^5 \\ &+ 0.38235180\xi^6); \quad -1 \le \xi \le 0 \end{aligned} \tag{48}$$

$$\begin{aligned} t_{xy}^{OC}(\xi) &= \frac{AP}{a}(0.34996374 + 0.17731323\xi + 0.01088870\xi^2 \\ &- 0.02316578\xi^3 - 0.00930792\xi^4 - 0.00145299\xi^5 \\ &- 0.00008360\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{49}$$

The polynomial $t_{xy}^{OD}(\xi)$ is fitted for $t_{xy}(-y\tan(\pi/3), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xy}^{OD}(\xi)$ is valid on the line OD, Figures 5 and 16):

$$\begin{aligned} t_{xy}^{OD}(\xi) &= \frac{AP}{a}(0.26000015 - 0.21858457\xi - 0.48035308\xi^2 \\ &+ 1.47129866\xi^3 + 4.45768040\xi^4 + 4.08903565\xi^5 \\ &+ 1.27404293\xi^6); \quad -1 \le \xi \le 0 \end{aligned} \tag{50}$$

$$\begin{aligned} t_{xy}^{OD}(\xi) &= \frac{AP}{a}(0.46652603 + 0.52637802\xi + 0.32272794\xi^2 \\ &+ 0.11458830\xi^3 + 0.02352355\xi^4 + 0.00259044\xi^5 \\ &+ 0.00011840\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{51}$$

The polynomial $t_{xy}^{OE}(\xi)$ is fitted for $t_{xy}(-y\tan(5\pi/12), y, 0.00000004)$ as follows (in other words the approximate polynomial $t_{xy}^{OE}(\xi)$ is valid on the line OE, Figures 5 and 17):

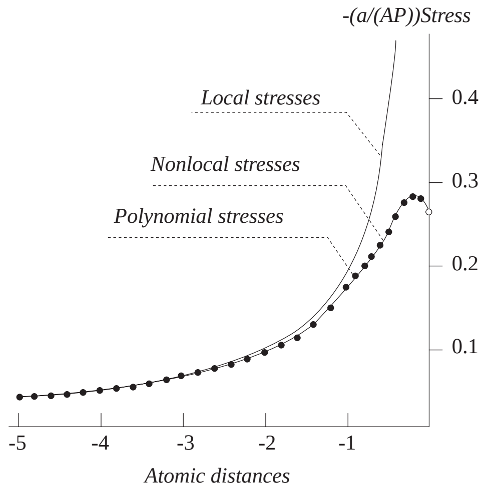

Figure 16. Shear stresses on the line OD

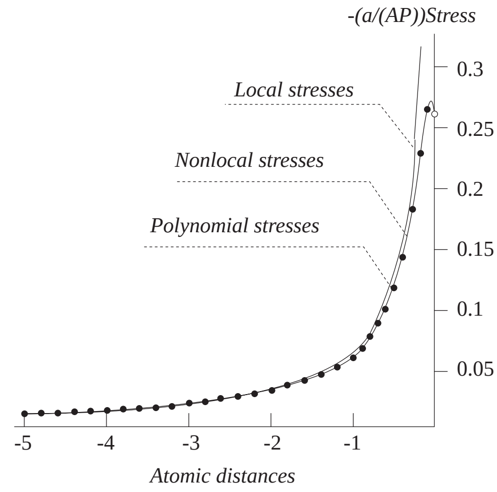

Figure 17. Shear stresses on the line OE

$$\begin{aligned} t_{xy}^{OE}(\xi) &= \frac{AP}{a}(0.25676691 - 0.43454903\xi - 5.04966107\xi^2 \\ &- 13.0728768\xi^3 - 15.78706540\xi^4 - 9.23202576\xi^5 \\ &- 2.09963882\xi^6); \quad -1 \le \xi \le 0 \end{aligned} \tag{52}$$

$$\begin{aligned} t_{xy}^{OE}(\xi) &= \frac{AP}{a}(0.18080841 + 0.22475544\xi + 0.14903892\xi^2 \\ &+ 0.05675152\xi^3 + 0.01242417\xi^4 + 0.00145206\xi^5 \\ &+ 0.00007013\xi^6); \quad \xi \le -1; \quad \xi = y/a \end{aligned} \tag{53}$$

For $a = 0$ the nonlocal stress field reverts to the classical stress field. That is

$$\begin{aligned} &t_{xx}(x, y, 0) = 0.975\sigma_{xx};\ t_{yy}(x, y, 0) = 0.975\sigma_{yy}; \\ &t_{xy}(x, y, 0) = 0.975\tau_{xy} \end{aligned} \tag{54}$$

Conclusion

The most important difference between the stress distributions of the local and nonlocal theories is of course the disappearence of the infinite stress at the tip of the singular force. This property can be seen in Figures 6-17. The unbounded stress obtained in the classical theory was a great obstacle in the interpretation of the actual situation and the efforts of finding the limits of the safe loading remained unanswered, despite the known physical characteristics of the medium. The most striking property in the nonlocal stress distribution is that the maximum stress does not occur at the application point of the force but further down. Similar results have previously been obtained in some other problems Artan (1996b), Artan (1996c), Artan (1997), Eringen and Balta (1979), Kunin (1968). The following significant results are observed:

a) The nonlocal stresses are finite even at the points where local stresses are infinite

b) The maximum stress does not occur at the boundary but further down. Similar results have previously been obtained in some other problems (see Artan (1996a), Artan (1996b), Artan (1996c), Artan (1997), Artan (2000), Eringen and Balta (1979).

c) For $a = 0$ the nonlocal solution reverts to the classical solution.

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