Use of Non-Uniform Rational B-Splines for Three-Dimensional Computer Simulation of Warp Knitted Structures

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Abstract

Some specific curves and surfaces have been used by engineers in the aerospace and automotive industries to design parts, such as car-body panels, for the last three decades. In this study, a method is developed for 3D solid computer representation of warp knitted structures using a similar technique. For this purpose, a three-dimensional cylindrical uniform solid yarn model was developed, such that the central axis is a space curve, by using the Non-Uniform Rational B-Spline (NURBS) surfaces. Then this model was applied to warp knitted fabrics using a CAD program written in C++ on a Silicon Graphics (SGI) workstation, which has an Open Inventor graphical library. The computer generated images of some structures are also given as examples in the study.

This paper describes the methodology and mathematics used to create the three-dimensional solid yarn model and apply this model to textile structures.

Key Words: Textile, CAD, 3D Solid Modelling, Warp Knitting, NURBS.

Çözgülü Örme Yapıların Bilgisayarda Üç Boyutlu Simülasyonu için Uniform Olmayan Rasyonel B-Spline Kullanımı

Özet

Bazı özel eğri ve yüzeyler, havacılık ve otomotiv endüstrisindeki mühendisler tarafından otomobil parçalarının tasarımı gibi amaçlar için otuz yıldır kullanılmaktadır. Bu çalışmada benzer bir teknik kullanılarak, çözgülü örme yapıların bilgisayarda üç boyutlu katı simülasyonunu sağlayacak bir metot geliştirilmiştir. Bu amaçla NURBS yüzeyleri kullanılarak merkez ekseni uzayda bir eğri olan üç boyutlu silindirik bir iplik modeli geliştirilmiştir. Daha sonra C++ programlama dili ve Open Inventor grafik kütüphanesi ile Silicon Graphics (SGI) iş istasyonu üzerinde yazılan bir bilgisayar destekli tasarım (CAD) programında, bu katı iplik modeli kullanılarak çözgülü örme kumaşlar elde edilmiştir. Bu şekilde simüle edilen yapılardan bazıları örnek olarak makalede sunulmaktadır.

Bu makalede üç boyutlu katı iplik modelini geliştirmek ve bu modeli kumaşlara uygulamak için kullanılan hesaplamalar ve yöntemler anlatılmaktadır.

Anahtar Sözcükler: Tekstil, Çözgülü Örme, CAD (Bilgisayar Destekli Tasarım), 3 Boyutlu Katı Modelleme, NURBS.

Introduction

In textiles, the general aim of computer aided design (CAD) systems is to provide a fast, easy design and realistic simulation of the structures on the computer screen to enable manufacturers to assess their designs before the real knitting or weaving process. However, even the recent CAD systems and simulation programs in warp knitting can only provide two- dimensional representations of structures (Breitenstein, 1995; Candan, 1994; Latifi, 1990; Stumpf, 1996; Karl Mayer Textilmaschinenfabrik GmbH., 1992, 1995; ALC Computertectechnik GmbH, 1995). This presentation is limited from the perspective of the designer and customer, since the fabrics are not pictures. Furthermore, designers and customers want to predict the comfort, drape and handling characteristics of the fabrics as well as the aesthetic aspects of the design. Therefore most of them still prefer to make their decisions after seeing the physical samples instead of 2D computer images which do not have geometrical definition of the sample displayed. However, if three-dimensional solid models can be created on the computer screen, designers and customers will have much more realistic and adequate representation of structures and they can manipulate the models (e.g., zooming, rotating and scaling) and consider other aspects of the fabrics as well as the aesthetic ones. In fact, there is a general trend towards three-dimensional modelling in all fields, parallel to the advancement in computing in terms of both hardware and software. However, the application of the 3D solid modelling concept in textiles is an even more difficult task than in other fields due to the complexity, flexibility and irregularity of the structures (Göktepe, 1998).

Nurbs Curves and Surfaces

Basic concepts of NURBS curves and surfaces are explained briefly in the next section to aid understanding of the geometric aspects of the computer modelling processes in this work.

A *spline curve* is a continuous curve that is composed of several polynomial segments (Farin, 1993). NURBS curves or surfaces are described by some *parametric* equations that are not explicitly part of the geometry. A curve is described in terms of one parameter, u, as follows:

$$x = f(u) \ \ y = g(u) \ \ z = h(u)$$

This single parameter is mapped into a three-dimensional (x, y, z) space by the three functions given above (Figure 1).
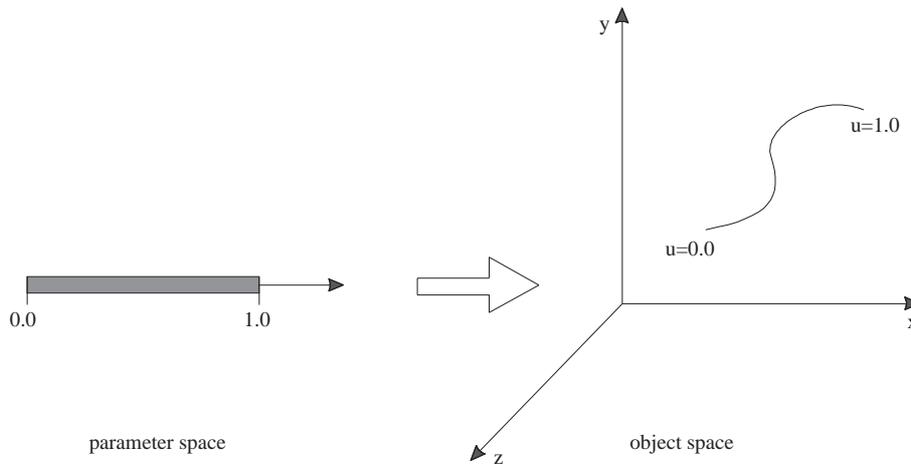

Figure 1. Mapping parametric curve to object space

These equations are provided by the most common graphical libraries such as Open GL and Open Inventor, which was used in this study. Therefore a programmer does not need to specify the equations explicitly. Instead, he/she defines the following components that make up the parametric functions:

- Control points

- Knot sequence

- Order

Control points are points in object space that affect the shape of the curve in some way. The curve may pass near the control points or pass through some of them (Figure 2). The control points can be a set of data through which a curve is fitted or a grid of points used to describe a curved surface.
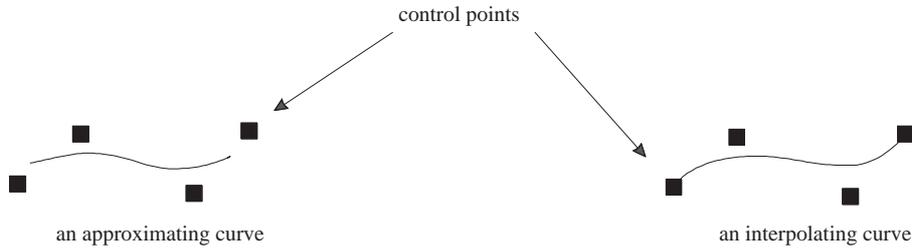

Figure 2. Using control points to shape the curve

The *knot sequence* is simply a list of non-decreasing numbers which defines how the control points affect the curve. The knot sequence determines whether the curve passes through and interpolates between some (usually the first and last) of the control points (an interpolating curve in Figure 2) or passes near the control points (an approximating curve in Figure 2). The curves which are used in this study have a multiplicity of knots, which is four at the beginning and end. There are uniformly spaced single knots between them (for example, 0,0,0,0,1,2,3,4,4,4,4), so that the curve representing the yarn passes through the end points.

The *order* of a curve determines the form of the parametric equations. The order is equal to one plus the maximum exponent (*degree*) of the variables in the parametric equations. For example, the parametric equations of a cubic curve (*degree*=3, *order*=4) has the following form:

$$x(u) = A_x u^3 + B_x u^2 + C_x u + D_x$$

$$y(u) = A_y u^3 + B_y u^2 + C_y u + D_y$$

$$z(u) = A_z u^3 + B_z u^2 + C_z u + D_z$$

In Open Inventor, the order of a curve is not explicitly specified. It is calculated as follows:

order = number of knots - number of control points

The order of the curve determines the minimum number of control points necessary to define the curve. For example, to create a cubic curve at least four points need to be specified. Two or more curve segments can be joined into a *piecewise cubic curve* by specifying more than four control points (Figure 3).
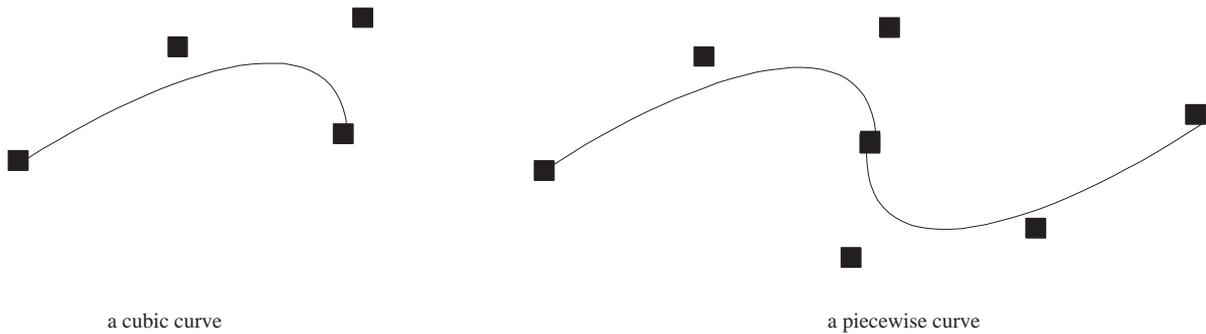

Figure 3. A piecewise cubic curve

The order of the curve also affects the behaviour of the curve when a control point is moved. For example, such a change in the position of a control point affects at most three segments of a quadratic B-spline curve. This is called a *local control* property, a feature which makes these curves very popular. For B-spline cubic curves, this control is not so local since movement of a control point affects four segments of the curve. As the order of the curve is increased, the change in the curve becomes *global* (affects the whole curve) and unpredictable. A NURBS curve can have an order of up to eight in Open Inventor (Wernecke, 1995). The curves with higher order are not preferable because of their unpredictable behaviour. Cubic curves are the most commonly used curves since they provide enough control for most geometric modelling applications without the drawbacks of higher-order curves. They are also found to be suitable for accurate representation of warp knitted stitches in this study, since they provide a smooth *continuity* at breakpoints as well as other advantages (Göktepe, 1998).

The *continuity* of a curve describes the behaviour of the curve segments at breakpoints. The maximum continuity possible is *order*-2. In this study, the highest possible continuity, which is C^2 for the cubic curves, is used to obtain the curves which have identical curvature at the breakpoint.

In brief, the (non-rational) uniform cubic B-spline curves with a continuity of two were used for representing yarns. The knot sequence for these curves has a multiplicity of four at the beginning and end, which causes the curve to pass through the first and last control points. In between, the curve shape is uniform.

Solid Yarn Model

For simplicity, the yarn was modelled as a monofilament yarn which was assumed to be uniform with a circular cross-section. The path followed by the thread in a fabric (i.e., central axis) refers to the first parameter (u) of the NURBS surface and the circle (i.e., cross-section) refers to the second parameter (v) of the same surface (Figure 4). Hence, the curve used for representation in simple form defines the first parameter of the surface. The values for the second parameter (v) are calculated and set so that the curve is wrapped with a cylindrical surface. This also can be thought of as sweeping the central axis with a circle (cross-section) along the yarn to create a 3D solid image of loops and fabrics.
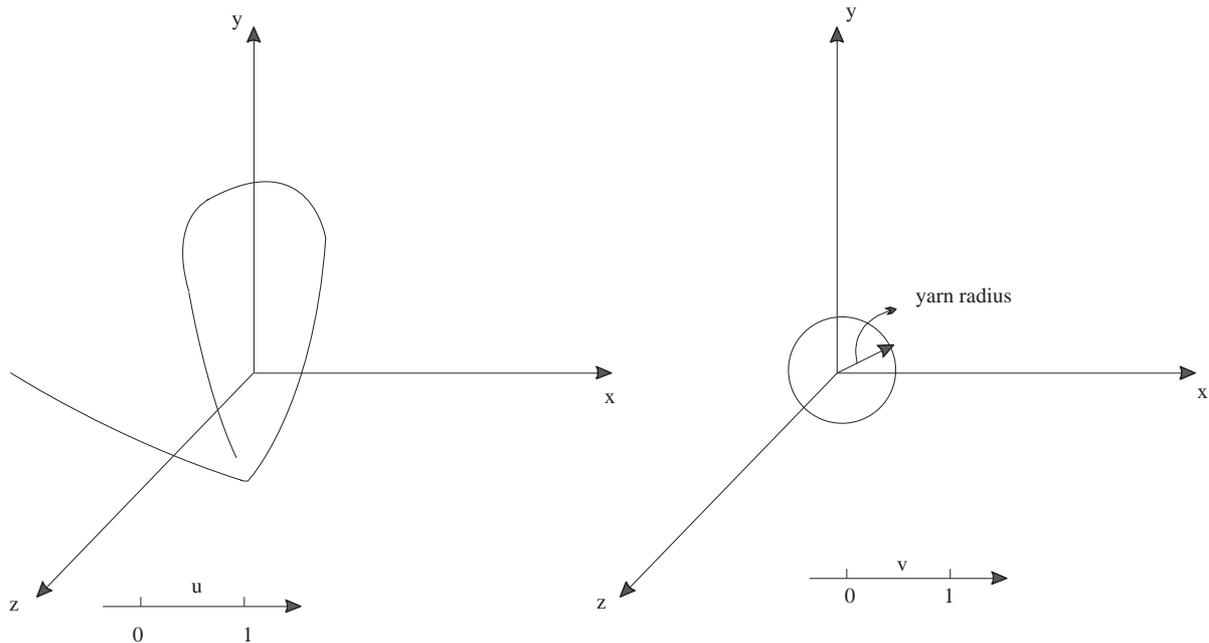

Figure 4. The u and v parameters (mapped to object space) for solid loop model

A new set of control points is needed to define the surface. For calculation of the co-ordinates of the control points, it is necessary to obtain the tangents to the curve of the central axis at the control points. Then the circles defining the yarn cross-section are placed around the control points perpendicular to their tangents.

A plane in 3D space can be determined by a point on it and a vector which is perpendicular to this plane. Figure 5 shows such a plane (p), which is perpendicular to V. V is the tangent line to a 3D space curve (s) at a point (V_1).
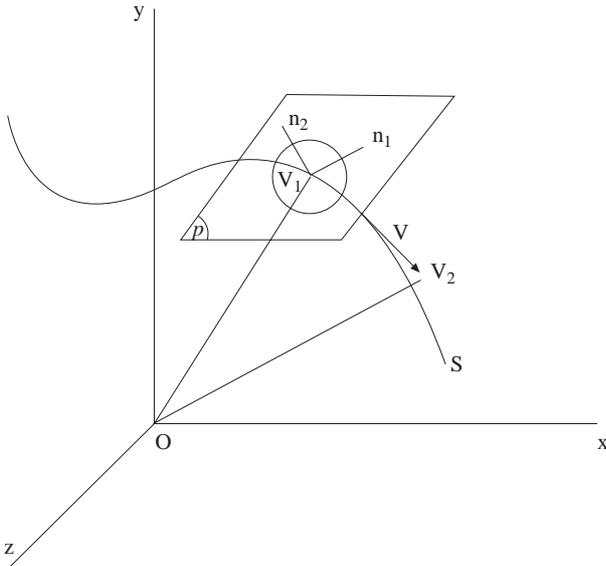

Figure 5. A plane and its normal vectors

Let V_2 be a point on the tangent line to the curve (s); then the vector V is given by

$$V = V_2 - V_1 = (a_1, a_2, a_3)$$

If V_2 and V_1 are two vectors in 3D space:

$$V_2 = (x_2, y_2, z_2)$$

$$V_1 = (x_1, y_1, z_1)$$

Then,

$$a_1 = x_2 - x_1$$
$$a_2 = y_2 - y_1$$

$$a_3 = z_2 - z_1$$

Considering the normal vector V, the equation of the plane p can be written as

$$a_1 x + a_2 y + a_3 z + d_0 = 0$$

Since plane p contains V_1 (as a point) then V_1 satisfies this equation so,

$$d_0 = -(a_1 x_1 + a_2 y_1 + a_3 z_1)$$

Since V is not a zero vector, then at least one of a_1, a_2, a_3 is not zero. Suppose that $a_1 \neq 0$, thus,

$$x = -\frac{d_0}{a_1} - \frac{a_2}{a_1}y - \frac{a_3}{a_1}z$$

Vectoral equation of p can be written as

$$r(y,z) = (x(y,z), y, z)$$

$$r(y,z) = (-\frac{d_0}{a_1} - \frac{a_2}{a_1}y - \frac{a_3}{a_1}z, y, z)$$

$$r_y = (-\frac{a_2}{a_1}, 1, 0) = \frac{1}{a_1}(-a_2, a_1, 0)$$

$$r_z = (-\frac{a_3}{a_1}, 0, 1) = \frac{1}{a_1}(-a_3, 0, a_1)$$

$$\|r_y\|^2 = \frac{1}{a_1^2}(a_1^2 + a_2^2), \ \ \|r_z\|^2 = \frac{1}{a_1^2}(a_1^2 + a_3^2),$$

The Gram-Schmidt method (Grossman, 1991) can be used for orthogonalisation since $a_1 \neq 0$ so r_x and r_y are linearly independent vectors (i.e., they are not parallel). Define N_1 and N_2 to be two orthogonal vectors on plane p by using Gram-Schmidt orthogonalisation for the vectors r_y and r_z:

$$N_1 = r_y, \ \ N_2 = r_z - \frac{\langle r_y, r_z \rangle r_y}{\langle r_y, r_y \rangle}$$

Scalar product of r_y and r_z:

$$\langle r_y, r_z \rangle = \frac{1}{a_1^2}(a_2 a_3) = \frac{a_2 a_3}{a_1^2},$$

$$N_2 = \frac{1}{a_1}(-a_3, 0, a_1) - \frac{\frac{a_2 a_3}{a_1^2}\frac{1}{a_1}(-a_2, a_1, 0)}{\frac{a_1^2 + a_2^2}{a_1^2}},$$

So,

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$$N_2 = \frac{1}{a_1}(-a_3, 0, a_1) - \frac{a_2 a_3}{a_1(a_1^2 + a_2^2)}(-a_2, a_1, 0)$$

Thus,

$$a_1 N_2 = (-a_3, 0, a_1) - \frac{a_2 a_3}{(a_1^2 + a_2^2)}(-a_2, a_1, 0)$$

$$a_1 N_2 = \frac{1}{a_1^2 + a_2^2}[-a_3(a_1^2 + a_2^2)$$

$$+a_2^2 a_3, -a_1 a_2 a_3, a_1(a_1^2 + a_2^2)]$$

$$a_1 N_2 = \frac{1}{a_1^2 + a_2^2}[-a_1^2 a_3, -a_1 a_2 a_3, a_1(a_1^2 + a_2^2)]$$

So,

$$N_2 = \frac{1}{a_1^2 + a_2^2}[-a_1 a_3, -a_2 a_3, (a_1^2 + a_2^2)]$$

Then define

$$n_1 = \frac{N_1}{\|N_1\|} = \frac{a_1}{a_1\sqrt{a_1^2 + a_2^2}}(-a_2, a_1, 0)$$

$$\tilde{C}(\theta) = \begin{pmatrix} \underbrace{x_1 - r\cos\theta \dfrac{a_2}{\sqrt{a_1^2 + a_2^2}} - r\sin\theta \dfrac{a_1 a_3}{a\sqrt{a_1^2 + a_2^2}}}_{x}, \\[2mm] \underbrace{y_1 + r\cos\theta \dfrac{a_1}{\sqrt{a_1^2 + a_2^2}} - r\sin\theta \dfrac{a_2 a_3}{a\sqrt{a_1^2 + a_2^2}}}_{y}, \\[2mm] \underbrace{z_1 + r\sin\theta \dfrac{\sqrt{a_1^2 + a_2^2}}{a}}_{z} \end{pmatrix} \quad 0 \le \theta \le 2\pi \qquad (2)$$

This formula gives the co-ordinates of a point corresponding to the θ angle on the circle with the radius of r. The radius r is calculated automatically by the program from the linear density of the yarn according to the material (PET, Nylon etc.). The tangent (V) to the curve, which defines the loop at a control point, is obtained by using the previous and next points of the control point. For example, in Figure 6, the tangent at point N_3 is obtained by the points N_2 and N_4 as follows:

$$V = N_4 - N_2 = (a_1, a_2, a_3)$$

$$N_4 = (x_4, y_4, z_4) \quad N_2 = (x_2, y_2, z_2)$$

Let

$$a^2 = a_1^2 + a_2^2 + a_3^2$$

Hence,

$$a = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$n_2 = \frac{N_2}{\|N_2\|} = \frac{1}{a\sqrt{a_1^2 + a_2^2}}(-a_1 a_3, -a_2 a_3, a_1^2 + a_2^2)$$

where n_1 and n_2 is an orthonormal basis for the plane p and satisfies the following condition:

$$\langle n_1, n_2 \rangle = 0$$

Thus, the equation of a circle in the plane p can be written as follows:

$$\tilde{C}(\theta) = V_1 + r(\cos\theta\, n_1 + \sin\theta\, n_2),$$
$$0 \le \theta \le 2\pi \qquad (1)$$

where r is the radius of the circle. Hence, the open (x, y, z) form of the equation can be given as follows:

$$a_1 = x_4 - x_2 \quad a_2 = y_4 - y_2 \quad a_3 = z_4 - z_2$$

In the program, the circle (cross-section) is defined by a uniform cubic B-spline curve that does not pass through the end points. The program generates the co-ordinates of the first control point by putting zero as the value of θ in Equation 2. Then it calculates the co-ordinates of the subsequent points by increasing the value of θ by $\pi/2$ for each point until θ becomes equal to 3π. This process is repeated for each control point on the central axis.

It is noted that θ is defined as $0 \le \theta \le 3\pi$ in the program instead of $0 \le \theta \le 2\pi$ given in the equation. The condition of $0 \le \theta \le 2\pi$ generates five

control points to describe a circle. However, in this case, the ends of the curve do not meet to generate a closed circle due to the behaviour of the approximation curve used. Therefore the condition for θ had to be modified as $0 \leq \theta \leq 3\pi$. Thus, for the second parameter v, the number of control points is set as seven and the knot sequence is defined as

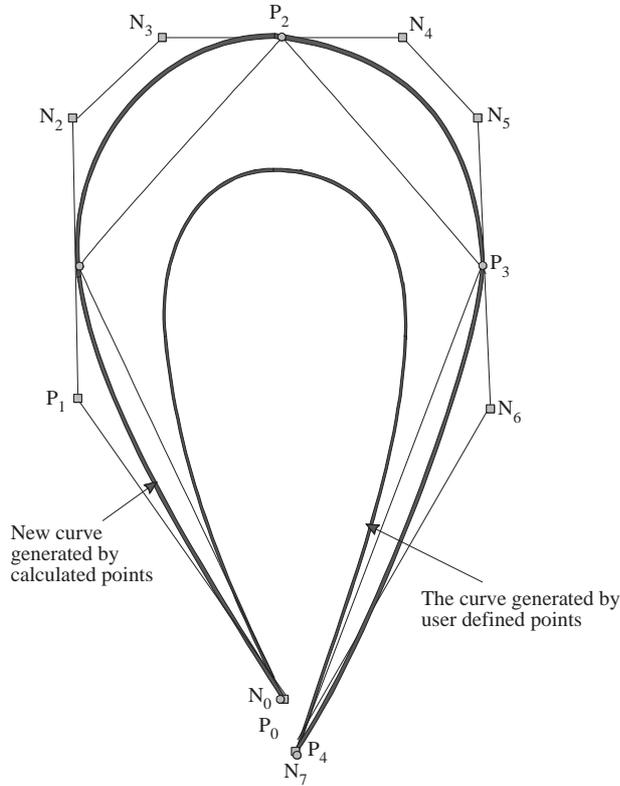$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Figure 6. Redesign of a curve

to generate a closed circular cross-section. For example, a loop defined by a curve (central axis) with 10 control points is defined by a surface with (10x7=) 70 control points in solid form. Figure 7 shows a 3D solid loop model of a warp knit stitch while a point and wire-frame representations of such a warp-knit stitch obtained using the solid yarn model are shown in Figures 8 and 9, respectively.

Application of the Yarn Model to Warp Knitted Structures

It is necessary to have either a theoretical or an empirical loop model to generate knitted structures on the computer screen. This model describes the geometrical configuration of the yarn in knitted fabrics. Then the number of control points needs to be determined in order to define a single loop shape. Once the loop is created, the fabric can be generated by connecting loops to each other according to the design. In this study, experimental work was conducted to obtain the necessary empirical loop models and a CAD program was also developed to enable design preparation (Göktepe, 1998).
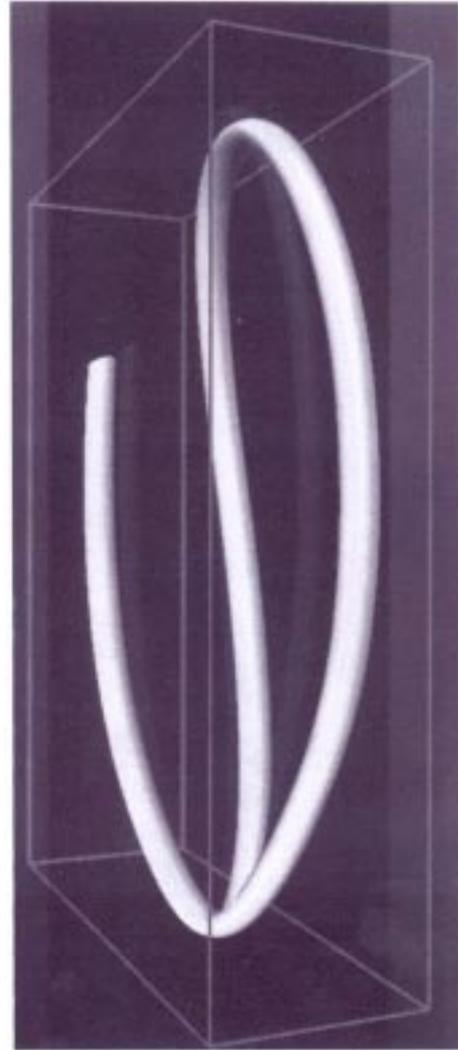
Figure 7. 3D computer generated solid image of a typical warp knit stitch

Determination of the control points number to define a loop

It is obvious that the number of control points depends on the shape of the loop to be created. The minimum possible number of control points must be

used to describe the loop types to allow faster rendering. At least four control points are necessary to create a cubic uniform B-spline curve as mentioned earlier. Hence, *number of control points ≥ 4*
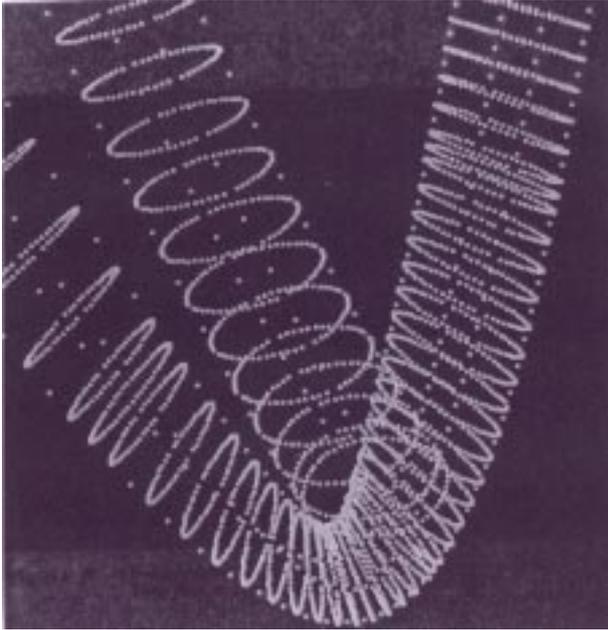

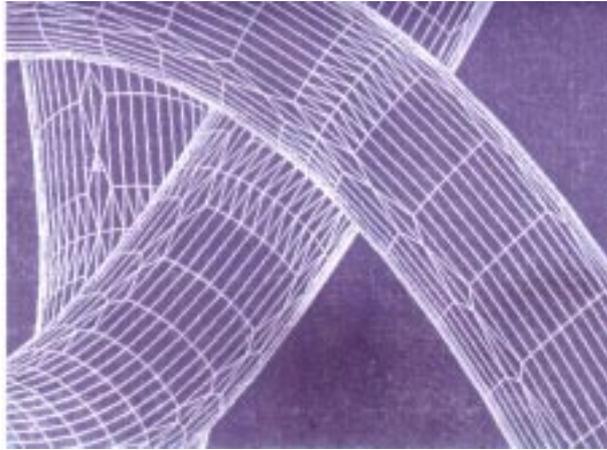**Figure 8.** Point representation of a warp knit loop

Figure 9. Wire frame representation of warp knit loops in a fabric

Basic two-dimensional loop shapes (Figure 10), which were classified by Candan (1994), can be used to determine the minimum number of points to describe the loop shape.

Generally, to define a shape, two points for the beginning and end points and one point for each corner (or turning points for curves) are adequate.

Thus, a closed or open loop and fall-plate loop can be represented by five points while a laid-in loop can be defined using four points. This assumption excludes the cross-over part where the loop connects to the underlap. By including this point, a user needs to enter six control points to define a basic warp knit loop.
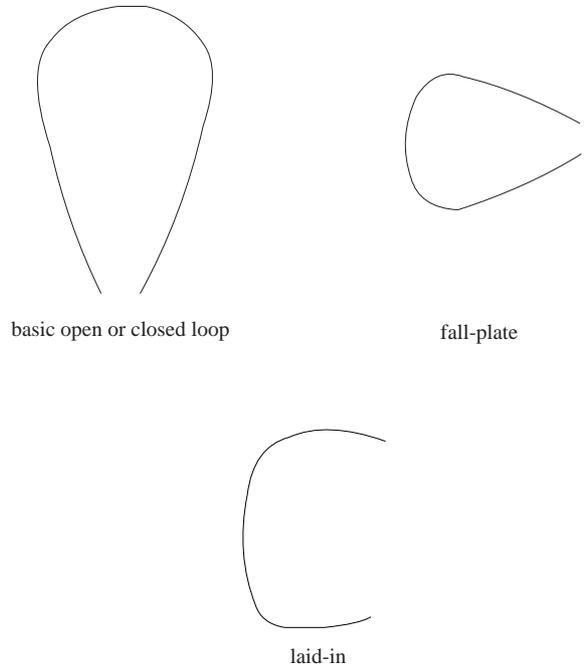

basic open or closed loop   fall-plate

laid-in

Figure 10. Basic 2D loop types

For modelling purposes, the uniform cubic B-spline curves have a drawback since the curves do not pass through the points, except the first and last points. However, it is desirable to have a curve which passes through all the control points for more accuracy and a better control of the curve. To achieve this, in the program, a new set of control points (polygon) is created using the co-ordinates entered by the user (Figure 6). As a result, the program creates ten control points using the six user defined points to represent the loop. For example, the calculation of the co-ordinates for new control points N_1 and N_2 (Figure 6) is given below:

$$r_{01} = \frac{\mid P_1 P_0 \mid}{\mid P_0 P_2 \mid}, r_{12} = \frac{\mid P_2 P_1 \mid}{\mid P_0 P_2 \mid}$$

$$\overrightarrow{P_0 N_1} = \overrightarrow{P_0 P_1} - \overrightarrow{P_0 P_2}.r_{01}/4$$

$$\overrightarrow{P_0N_2} = \overrightarrow{P_0P_1} + \overrightarrow{P_0P_2}.r_{12}/4$$

This vectoral calculation is repeated for all user defined points (except the first and last points). Consequently, the user needs to specify at least six control points to describe the configuration of a basic warp knit loop. Then the program calculates a new set of (ten) control points so that the curve can pass through the six points specified by the user.

Once a loop (or fabric) is created in simple form using the curves, it can be displayed in a solid form using surfaces by the 3D CAD system developed by Göktepe (1998). To define a surface, the control points and knot sequence must be specified for both parameters. The u and v parameters can have a different order and a different knot sequence.

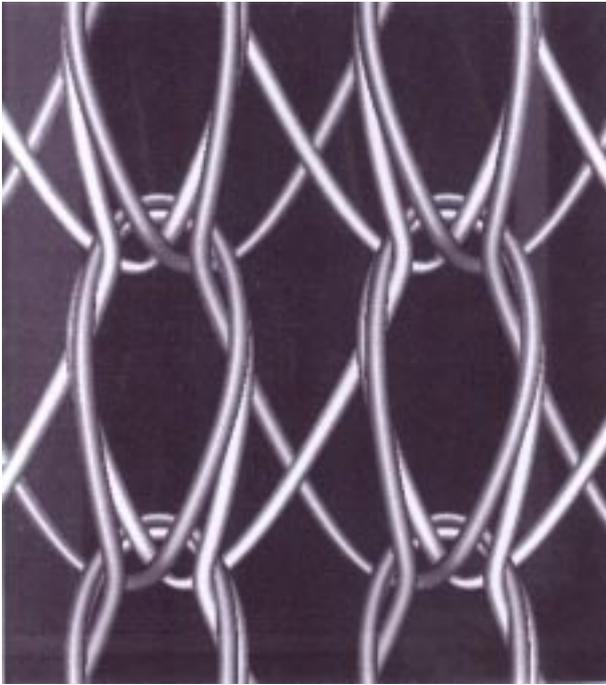Some computer generated 3D solid images of fabrics are given in Figures 11-13.

Figure 11. Computer generated image of Tricot structure

Figure 12. Computer generated and rotated image of Satin structure
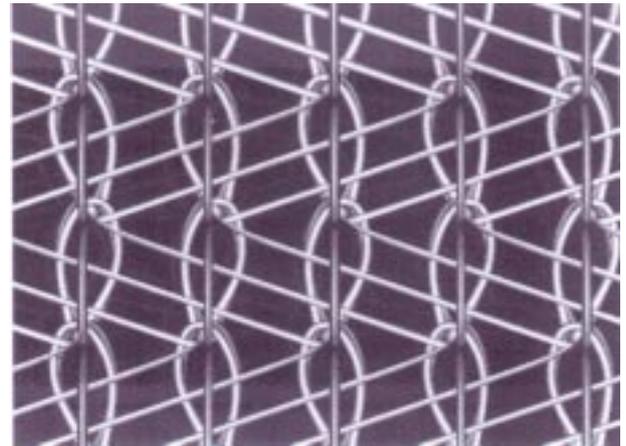
Figure 13. Computer generated image of Queen's Cord structure

Conclusion

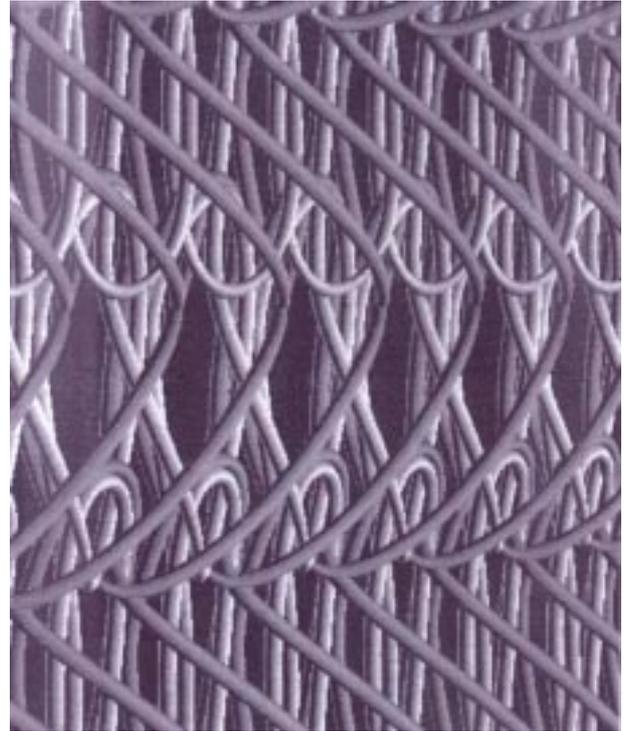In this study, a three-dimensional solid yarn model was developed using NURBS curves and surfaces. This 3D model was successfully used to represent some basic warp knitted fabrics on the computer. Computer generated images of knitted structures show that the cubic NURBS curves and surfaces can be used for modelling and realistic simulation of textile structures.

Multifilament yarn models are recommended for future studies to obtain more realistic representations of textile structures. This will require a much more powerful computer than the Silicon Graphics (SGI) workstation used for this study. Alternatively, pictures of real yarns can be mapped to the surface of the 3D yarn model using the texture mapping method.

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