

## Robust Control of Active Suspensions Using Sliding Modes

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### Abstract

In this study, a four degrees of freedom vehicle model is used in order to design and check the performance of sliding mode controlled active suspensions. Force actuators are mounted parallel to the suspensions and a non-chattering control is realized. Sliding mode control is preferred because of its robust character since any change in vehicle parameters should not affect the performance of the active suspensions. The aim is to improve ride comfort by decreasing the amplitudes of motions of the vehicle body. Body bounce and pitch motions of the vehicle are simulated both in the time domain, in the case of travelling over a single rectangular bump type road profile, and the frequency domain. Also the phase plane plots of them are checked. Simulation results are compared with the ones of the passive suspensions. The robust character of the controller has been demonstrated using different system parameters which it is possible to change depending on the conditions of the journey. At the end of the study, the performance of the controller and the improvement in the system performance have been discussed also by considering the ride comfort.

**Key Words:** Sliding mode, Active suspension, Ride comfort, Robust control.

## Aktif Süspansiyonların Kayan Modlar Yönetimi ile Robust Denetimi

### Özet

Bu çalışmada, kayan modlar denetimli aktif süspansiyonların tasarımını gerçekleştirmek ve performansını kontrol etmek amacı ile dört serbestlik dereceli bir taşıt modeli kullanılmıştır. Kuvvet oluşturuçular süspansiyonlara paralel yerleştirilmiş ve atlamasız kontrol gerçekleştirilmiştir. Taşıt parametrelerindeki herhangi bir değişimin aktif süspansiyonların performansını etkilememesi gerektiği için robust niteliğe sahip olan Kayan Modlu Denetim tercih edilmiştir. Taşıtın gövdesinin sıçrama ve kafa vurma hareketlerinin tek bir dikdörtgen engele sahip bir yol profilinden geçmesi halinde zaman boyutunda ve frekans boyutunda benzetimi gerçekleştirilmiştir. Faz planı çizimleri de kontrol edilmiştir. Benzetim sonuçları pasif süspansiyonunkilerle kıyaslanmıştır. Seyir Şartlarına bağlı olarak değişmesi mümkün olduğundan, farklı sistem parametreleri için denetiminin robust karakteri gösterilmiştir. Çalışmanın sonunda denetiminin etkinliği ve sistem performansındaki iyileşme, seyir konforu da dikkate alınarak irdelenmiştir.

**Anahtar Sözcükler:** Kayan modlar, Aktif süspansiyon, Seyir konforu, Robust denetim.

## 1. Introduction

The main functions of a vehicle suspension system are to provide effective isolation from road surface unevenness, to provide stability and directional control during handling manoeuvres with ride comfort and to provide vehicle support. Traditional vehicle suspension systems are composed of two parallel components, that is springs and viscous dampers. Designers of passive suspension systems are faced with the problem of determining the suspension spring and damper coefficients. They have to compromise between two important conflicting factors, ride comfort and road holding. Good ride comfort needs soft springs, but this means poor road holding. Furthermore, when talking about passive suspensions, there is no way to get rid of the resonance frequencies such as the most important one around 1 Hz, which is the result of vehicle body dynamics. Therefore the improvement of vehicle suspension systems has been of growing interest and been the subject of research and development in recent years, for commercial and scientific reasons. The main reason for the commercial activity is the desire of automotive manufacturers to improve the performance and quality of their products. On the other hand, researchers and control system designers have claimed that the automatic control of the vehicle suspension system is possible when the developments in actuators, sensors and electronics have been considered. If the performance characteristics of a desired suspension system have been taken into consideration, the active suspension control becomes more attractive.

In the last fifteen years, many studies have been published on active and semi-active suspension systems. Most of the investigators used a quarter-car model. Prokop and Sharp studied active automotive suspensions by road preview on a quarter model (Prokop and Sharp, 1995). Hrovat (1993) surveyed optimal control techniques in the design of active suspensions in one of his studies starting with a quarter model. Non-linear control of a quarter-car active suspension was reviewed by Alleyne and Hedrick (1995). Burton and Truscott (1995) brought together analysis of active and passive quarter-car systems and a full-scale test rig in their paper. Redfield and Karnopp (1988) examined the optimal performance comparisons of variable component suspensions on a quarter-car model. Yu and Crolla (1998) presented an optimal self-tuning control algorithm using a quarter model considering both external and inter-

nal disturbances. Although the quarter-car model has been proved to be useful for designing control strategies, it does not reflect terms such as pitch motion. However some investigations based on a three-dimensional vehicle model have been published. In one of them Yuksek and Kaya (1995) discuss vibration optimization of a vehicle by calculating optimum values of control forces such that overall maximum vehicle response amplitude is minimized on a full car model.

The aim of this study is to apply the non-chattering sliding mode control to automotive suspension systems. If not prevented, the chattering causes damage to the mechanical components. Sabanovic proposed an effective method for chattering free sliding modes applications (Sabanovic, 1994). The improvements in electromagnetic force sources and sensors make it possible. Dan Cho presented the application of sliding mode control to stabilize an electromagnetic suspension system with experimental results (Dan Cho, 1993). Sliding mode control was proposed first in the Soviet Union by Emelyanov and Utkin. A survey paper by Utkin references many of the early contributions (Utkin, 1977). Because of the language and reference problems, it has taken a long time to enter western literature (Hung, 1993). Utkin (1981) published an excellent book on sliding mode control. Young (1978) showed that the method is successfully applicable to robot manipulators. Yagiz (1997) proposed the application of sliding mode control on a quarter-vehicle model. The advantages of this method are the applicability to non-linear systems, simplicity, high performance and its robust character. Nowadays, this method has been successfully applied to robot control, flight control, motor control and power system control.

## 2. The Vehicle Model

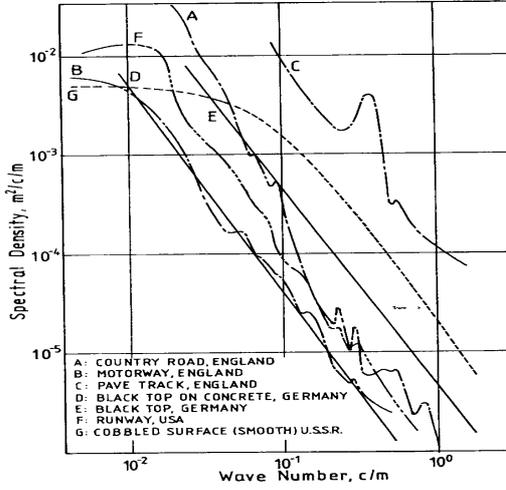
The physical model of the vehicle is presented in Figure 1. The controllers have been placed between sprung and unsprung masses in parallel. The vehicle model has four degrees of freedom which are body bounce  $z_M$ , body pitch  $\theta$ , front wheel hop  $z_{mf}$  and rear wheel hop  $z_{mr}$ . In this model,  $M$  and  $I$  represent body mass and inertia;  $k_{sf}$  and  $k_{sr}$  are front and rear suspension spring constants;  $C_f$  and  $C_r$  are front and rear damper coefficients;  $u_f$  and  $u_r$  are control force inputs to the front and the rear of the vehicle respectively;  $m_f$  and  $m_r$  are front and rear unsprung masses;  $k_{tf}$  and  $k_{tr}$  are stiffness of the front and rear



to the integral-squared value of the second by the equation,

$$\langle y_r^2(t) \rangle = cV \int_0^\infty y_1^2(t) dt \quad (12)$$

provided that the integral on the right converges. Here  $c$  is the road roughness constant, and the vehicle speed  $V$  is also assumed to be constant. Thus, the mean-squared response to the random signal at a given speed is directly proportional to the integral-squared response to the unit step input. For a given speed  $V$  it may be concluded that if a system is controlled satisfactorily for a unit step input then it can also be controlled with success for a random signal input of the integrated white noise type (Thomson and Pearce, 1998).



**Figure 2.** Some measured road profile spectra from Robson and Dodds (1970).

Sliding modes control theory has been applied in a number of non-linear systems. The main idea is to bring the error on a sliding surface such that the system is on a sliding surface and insensitive to the disturbances and parameter changes. If the system is defined as:

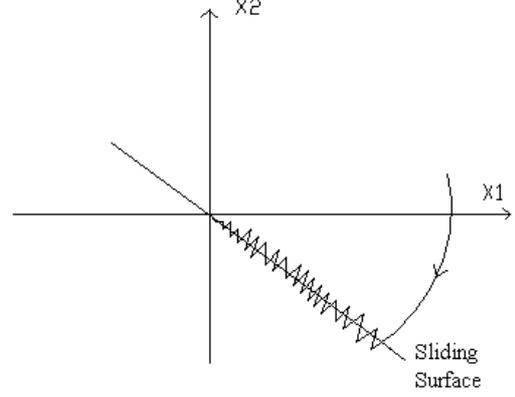
$$\dot{\underline{x}} = \underline{f}(x) + \underline{B}u \quad (13)$$

where  $\dim[\underline{B}] = n * m$ ,  $\dim(\underline{f}(x)) = n * 1$  and  $\dim(\underline{u}) = m * 1$ ;  $\underline{f}(x)$  is continuous, but  $\underline{u}(t)$  may be discontinuous. The aim is to hold the system motion on a sliding surface of  $S$ . The surface can be expressed as:

$$S = \{\underline{x} : \underline{\sigma}(x, t) = 0\} \quad (14)$$

In order to obtain a stable solution of the system, it must stay on this surface, i.e.  $\underline{\sigma}(x, t) = 0$  as shown in Figure 3. The sliding surface equation for the control of the system can be selected as follows:

$$\underline{\sigma}(x, t) = [G](\underline{x}_{ref} - \underline{x}) = [G]\underline{e} \quad (15)$$



**Figure 3.** Phase plane diagram of the state variables.

In this equation  $\underline{x}_{ref}$  represents the state vector of the reference, and the constant  $[G]$  matrix represents the slope of the sliding surface. The same equation also can be written as:

$$\underline{\sigma}(x, t) = \underline{\Phi}(t) - \underline{\sigma}_a(x) \quad (16)$$

where,

$$\underline{\Phi}(t) = [G]\underline{x}_{ref}(t) \quad \text{and} \quad \underline{\sigma}_a(x) = [G]\underline{x} \quad (17)$$

The first step in design is to select a Lyapunov function ( $\nu$ ). According to the Lyapunov Stability Criteria, the Lyapunov function must have a value greater than zero, whereas its derivative is smaller than zero. Selecting the function as in Equation (18) makes its value greater than zero:

$$\underline{\nu} = \underline{\sigma}^T(x, t)\underline{\sigma}(x, t)/2 > 0 \quad (18)$$

In order to have the derivative value of the Lyapunov Function smaller than zero:

$$d\underline{\nu}/dt = -\underline{\sigma}^T(x, t)D\underline{\sigma}(x, t) < 0 \quad (19)$$

Thus the Lyapunov Stability Criteria have been satisfied. If we equate (19) to the derivative of (18):

$$\begin{aligned} \underline{\sigma}(x, t)/dt &= -D\underline{\sigma}(x, t) \quad \text{and} \\ d\underline{\sigma}(x, t)/dt + D\underline{\sigma}(x, t) &= 0 \end{aligned} \quad (20)$$

As seen in equation (20), the sliding function becomes zero at infinity. But our goal is to make it very close to zero. If equation (16) is differentiated and (14) is used, the derivative of the sliding surface is obtained as:

$$\begin{aligned} d\underline{\sigma}(x, t)/dt &= d\underline{\Phi}(t)/dt - \{\partial\underline{\sigma}_a(x)/\partial x\} \\ d\underline{x}/dt &= d\underline{\Phi}(t)/dt - [G](\underline{f}(x) + [B]\underline{u}(t)) \end{aligned} \quad (21)$$

and  $([G][B])^{-1}$  must exist. The controller is designed as below by inserting (21) in (20):

$$\underline{u}(t) = \underline{u}_{eq}(t) + ([G][B])^{-1}D\underline{\sigma}(x, t) \quad (22)$$

$$\underline{u}_{eq}(t) = -([G][B])^{-1}([G]\underline{f}(x) - d\underline{\Phi}(t)/dt) \quad (23)$$

If the knowledge of  $\underline{f}(x)$  and  $[B]$  matrices are very poor, then the equivalent control calculated will be too far off from the actual equivalent control. In the literature, a number of approaches are proposed for the estimation of  $u_{eq}$ , rather than calculating it. In this study the approach suggested uses the fact that the equivalent control is the average of the total control. Let us design an averaging filter for calculation of the equivalent control as below.

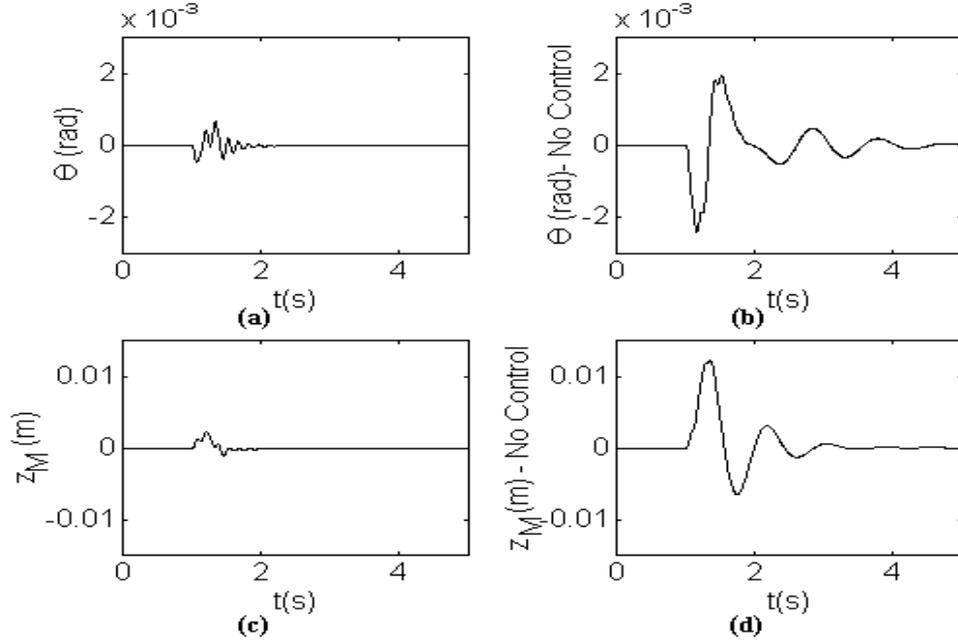
$$\hat{u}_{eq} = \frac{1}{\tau s + 1} \underline{u} \quad (24)$$

This is actually a low-pass filter. The value of  $1/\tau$  gives the cut-off frequency. The logic behind designing a low-pass filter is that low frequencies determine the characteristics of the signal and high frequencies come from unmodelled dynamics. Then:

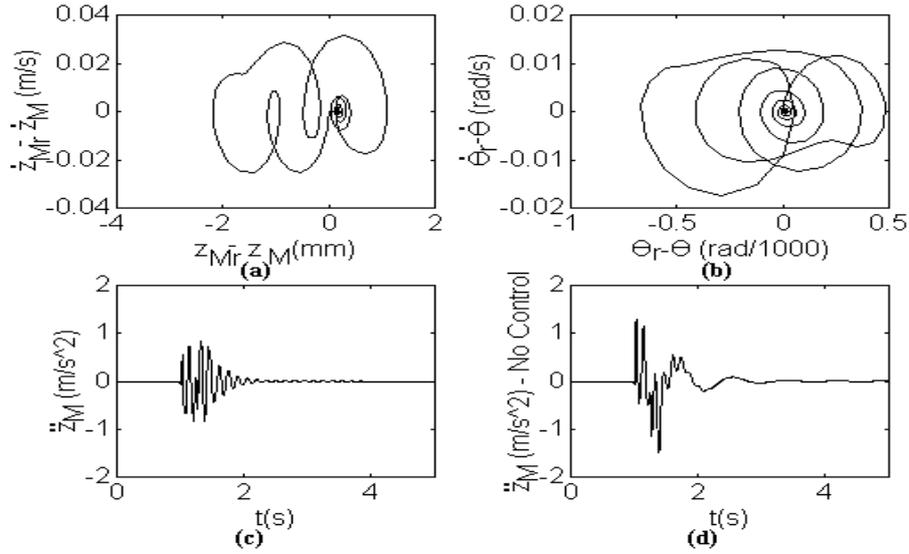
$$\underline{u}(t) = \hat{u}_{eq} + ([G][B])^{-1}D\underline{\sigma}(x, t) \quad (25)$$

#### 4. Simulation

By defining the mathematical model of the system, the simulation has been realized. Road disturbance  $z(t)$  has been chosen as a single rectangular bump having height  $0.01m$  between  $t = 1s$  and  $t = 1.25s$ . It must be noted that there are two road inputs to the system which are to the front wheel and with a time delay  $\delta t$  to the rear wheel. As demonstrated in Figures 4a and 4c the motion of the passengers both in vertical and angular directions quickly follow the "0" reference. When the body bounce is compared with the uncontrolled ones in Figures 4b and 4d, the success of the controllers becomes obvious.



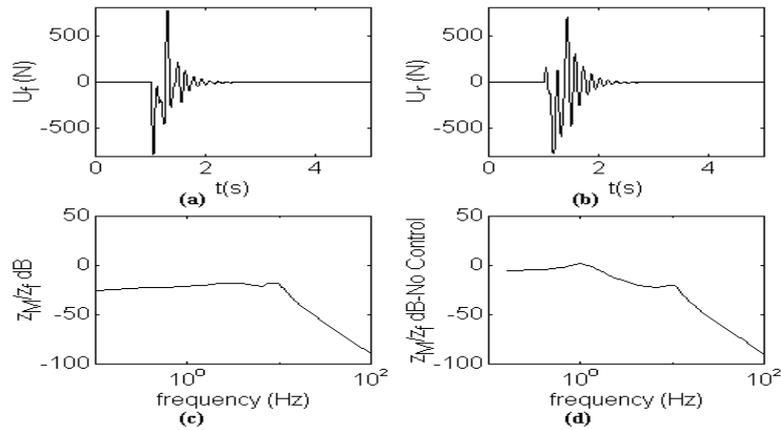
**Figure 4.** The system response of the vehicle in the case of a rectangular bump road surface input. (a) Pitch Motion. (b) Pitch Motion without controllers. (c) Body bounce. (d) Body bounce without controllers.



**Figure 5.** The system response of the vehicle. (a)Phase plane of the body bounce. (b)Phase plane of the body pitch. (c)Vertical acceleration of the body with controller. (d)Vertical acceleration of the body without controller.

The phase planes presented in Figures 5a and 5b show the stability of the system. Also there is a slight improvement in the acceleration of the body (Figures 5c and 5d). This is preferable when the inertia forces acting on passengers are considered. The maximum value of the control forces is around 700N as seen in Figures 6a and 6b. When we check the frequency response of the system without controllers,

two overlapping resonance frequencies are observed around 1.1Hz and 10Hz of body motions and wheel hops in Figure 6d. On the other hand, when the controllers are active, the resonance frequencies of body motions almost vanish and the amplitude of the motion throughout the related frequency range gets considerably smaller as presented in Figure 6c.

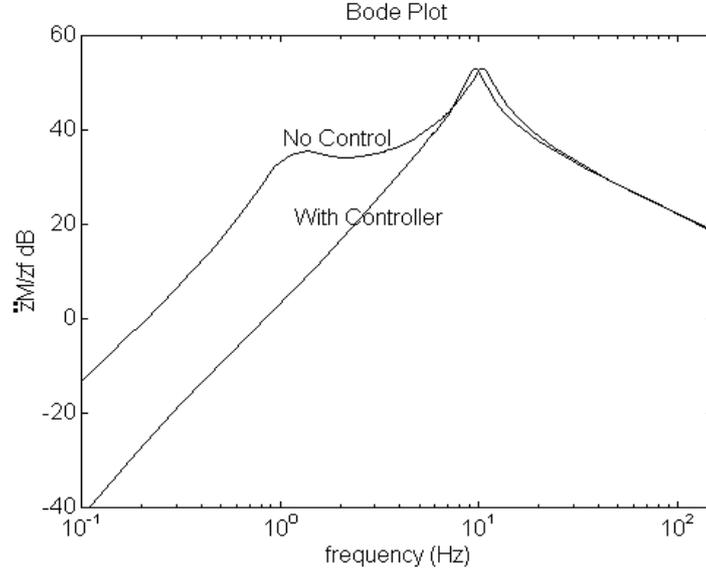


**Figure 6.** The system response of the vehicle. (a) Front suspension control force. (b)Rear suspension control force. (c)Frequency response of the body. (d) Frequency response of the body without controllers.

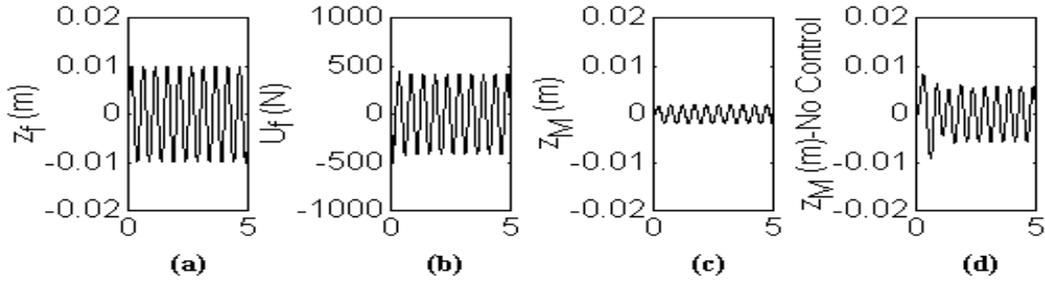
Since the aim of the controller was to control the body bounce and pitch motions, the improvement has been realized particularly around  $1.1Hz$  at low frequencies. This is also demonstrated by plotting the frequency response of controlled and

uncontrolled body bounce acceleration in Figure 7.

In Figure 8, the performance of the active suspension has also been checked for harmonic road surface input.



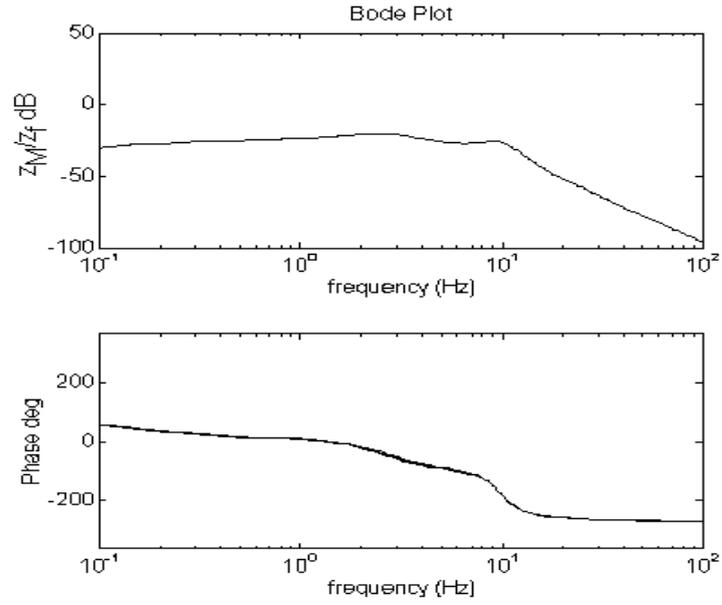
**Figure 7.** Frequency response of the body bounce acceleration for controlled and uncontrolled case.



**Figure 8.** The system response in the case of harmonic road surface input. (a)Front wheel harmonic road surface input. (b)Front suspension control force. (c)Body Bounce. (d)Body Bounce without controllers.

If the control method is not robust, then for different load and road conditions, the control method cannot function as desired. In order to demonstrate the robust character and success of the sliding mode

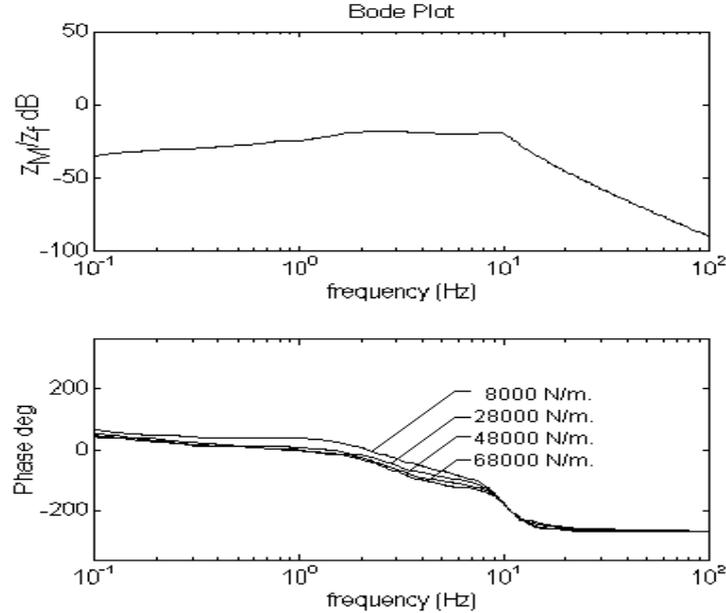
control, it is assumed that the vehicle, which is  $1000kg$  (unladen) is loaded with an additional  $500$  and  $1000kg$  of masses representing new passengers and luggage (Figure 9).



**Figure 9.** Frequency response of body bounce for vehicle mass of 1000,1500 and 2000 kg all coinciding with each other.

It is observed that the active suspensions keep on functioning successfully with no change in frequency response. Then it is assumed that a problem started in the front suspension springs because of the road conditions and its stiffness changed dramatically instead of having its actual value of  $28000N/m$

as shown in Figure 10. Again it is observed that active suspension functions successfully without any change in the magnitude of the controlled vehicle body vertical motion and with a slight change in phase, which is of no importance.



**Figure 10.** Frequency response of body bounce for front suspension stiffness of 8000, 28000, 48000, and 68000 N/m.

## 5. Conclusions

In this study, a sliding mode controller for a vehicle has been designed and simulation results have been presented. The main idea behind proposing this controller is its robustness and the ability of using these types of controller on vehicles with developing technology. Since vehicle dynamics change with load and road conditions, this method is of great im-

portance. The results of this study prove that the performance of the active suspension of this type is highly superior to that of the passive one. The passengers are almost unaware of the unevenness of the road and it is foreseen that they feel the ride as if on an excellent road surface. The extraordinary improvement in resonance values and decrease in vibration amplitudes support this result.

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