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A SPATIALLY VARIED UNIT HYDROGRAPH MODEL

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In the recent past, instantaneous unit hydrographs based on geomorphology have been proposed as a tool to produce flood hydrographs from rainfall. This paper presents a flood hydrograph simulation model, formulated on the concept of a spatial unit hydrograph derived from a deterministic direct hydraulic simulation approach. The theoretical basis of the model is the time-area method for unit hydrograph derivation. The model employs a cell structure and routes the spatially distributed excess rainfall from one cell to the other following the maximum downslope direction to the watershed outlet. Application of the model is demonstrated by an example using data from a 230 km^2 watershed located on the eastern slopes of the Canadian Rockies in Alberta, Canada. The spatial unit hydrographs gave excellent results in simulating the observed flood hydrographs indicating the potential of this model as a useful tool for flood hydrograph estimation.

INTRODUCTION

For many years hydrologists have attempted to relate the hydrologic response of watersheds to watershed morphologic and topographic structures. The recently proposed geomorphologic instantaneous unit hydrograph (GIUH) method is perhaps the most promising development in this direction. The concept of the GIUH was first introduced by Rodriguez-Iturbe and Valdes (1979) and later generalized by Gupta et al. (1980). Rodriguez-Iturbe and Valdes (1979) modeled the transformation of excess rainfall to direct runoff as a continuous-time Markov process and interpreted the instantaneous unit hydrograph as the probability density function of the time that it takes for a randomly chosen drop of water to reach the watershed outlet. Rodriguez-Iturbe and Valdes (1979) and Valdes et al. (1979) linked the peak discharge and time to peak with the geomorphologic parameters of the watershed and a dynamic parameter, the velocity.

Along the same line of investigation, Rodriguez-Iturbe et al. (1982a and 1982b) proposed what they called a geomorphoclimatic instantaneous unit hydrograph (GcIUH) as a link between climate, geomorphologic structure and hydrologic response of a basin. The resultant GcIUH is a function of the excess rainfall intensity; hence the linearity restriction of the unit hydrograph theory is relaxed. The GcIUH in its present form, however, is not without problems. One of the questionable assumptions underlying the theory is the belief that the hydrologic response of the watershed depends only on some of the gross features of the watershed, and not on the details of the channel network. That is, the GcIUH depends only on the characteristics of the highest order channel and is independent of hillslope and lower-order channel processes. This assumption clearly cannot apply over a wide range of geomorphological features. Other comments on the GcIUH can be found in Bras (1990).

In this paper, a spatial unit hydrograph is presented, which is similar in concept to the GcIUH except that a cell based approach is used to describe the connectivity of flow in the channel network, which eliminates the need for using the Markovian hypothesis and probability arguments to combine the movement of water. The spatial unit hydrograph is derived from a time-area curve of the watershed using a geographic information system (GIS). A review of the literature reveals that there have been attempts to take advantage of GIS capabilities for rainfall runoff modeling (Maidment, 1993; Olivera et al., 1995; and Olivera and Maidment, 1996). Maidment (1993) presented a grid based methodology for determining a spatially distributed unit hydrograph, assuming a pure translation discharge model. Olivera et al. (1995) applied impulse response functions to model discharge and pollutant transport in a watershed. Olivera and Maidment (1996) developed a spatially distributed unit hydrograph based on linear systems theory applied to subareas within a watershed.

TIME-AREA METHODS

Time-area methods were developed in recognition of the basic importance of time distribution of rainfall on runoff (Singh, 1992). The theme of the time-area methods is the time-area histogram, which indicates the distribution of subareas of the watershed contributing to runoff at the outlet as a function of travel time. The subareas are bounded by isochrones: contour lines joining those points in the watershed that are located from the outlet by the same travel time. The time-area curve is a S-graph with the abscissa as the travel time and the ordinate as the cumulative area contributing runoff for a specified travel time.

DERIVATION OF SPATIAL UNIT HYDROGRAPH

The unit hydrograph derived is similar to the derivation proposed by Maidment (1993). For an excess rainfall of intensity i over the watershed, the runoff at the outlet is given by $Q(t)=iA(t)$, which is a S-hydrograph of runoff tending to an equilibrium discharge of *iA*, where *A* is the total contributing area of the watershed. The unit hydrograph of a watershed is derived from the watershed's time-area curve by the method analogous to the S-hydrograph procedure. Lagging the S-hydrograph by Δt, taking the difference between the S-hydrograph value at time *t* and its value lagged by time Δt, and dividing by $i\Delta t$, yields the Δt-time unit hydrograph ordinates as:

$$U(t)=\frac{A(t)-A(t-\Delta t)}{\Delta t} \tag{1}$$

Because the time-area curve values are known only at time coordinates t=0, Δt, $2\Delta t$, ..., $n\Delta t$, ..., it follows that the Δt-time unit hydrograph ordinates at the corresponding time coordinates are given by:

$$U_n=\frac{A(n\Delta t)-A\left[(n-1)\Delta t\right]}{\Delta t}=\frac{A_n}{\Delta t} \tag{2}$$

where $A_n=A(n\Delta t)-A[(n-1)\Delta t]$ are the incremental areas. Equation (2) states that the Δt-time unit hydrograph ordinate at the end of the nth time interval is equal to the incremental area A_n, whose drainage first reaches the outlet during that time interval divided by the duration of the interval, Δt. This is analogous to stating that the Δt-time unit hydrograph ordinate at time $n\Delta t$ is given by the slope of the time-area curve over the time interval $[(n-1)\Delta t, n\Delta t]$.

EXCESS RAINFALL

Spatial variations of hydrologic abstractions occur due to differences in soils and land use/land cover. In the cell based system, each grid cell has its respective curve number (CN) value corresponding to its soil type and land use/land cover characteristics. The excess rainfall in each cell is computed according to the Soil Conservation Service (SCS) runoff CN method (Chow et al., 1988), and routed through the channel network to the watershed outlet. The excess rainfall is then used in the computation of overland flow, channel flow and channel storage times.

CHANNEL DISCHARGE

Each cell will have one or more channels passing through it. The channel discharge is produced by the cell's excess rainfall as overland flow and channel flows from one or more upstream contributing cells. The flows are calculated based on equilibrium conditions. The procedure adopted to compute channel discharges is as follows. If Q_1 and Q_2 are inflows from two upstream contributing cells, and q_c is the locally produced overland flow, then the outflow from the cell Q_3 is computed as $Q_3=Q_1+Q_2+q_c$ and the average discharge, Q in channel 1-3 is computed as $Q=(Q_3+Q_1)/2$. The average depth of flow, y is calculated using Manning's equation:

$$Q=\frac{1}{n}\frac{\left[A(y,B)\right]^{\frac{5}{3}}}{\left[P(y,B)\right]^{\frac{2}{3}}}S^{\frac{1}{2}} \tag{3}$$

where $A(y,B)$ is the cross-sectional area of the channel, $P(y,B)$ is the wetted perimeter of the channel, S is the slope of the channel and n is the Manning's roughness coefficient. The channel bottom width, B and n are estimated from aerial photographs and field survey data.

TRAVEL TIME

The estimation of overland flow times is based on the kinematic wave equation. The time to equilibrium, t_{eq} for an overland flow plane is given by:

$$t_{eq} = \frac{6.9}{i^{0.4}} \frac{N^{0.6} L^{0.6}}{S_o^{0.3}} \tag{4}$$

where i is the excess rainfall intensity on the overland flow plane, L is the overland flow length, S_o is the slope of the overland flow plane and N is the Manning's overland flow roughness coefficient.

Computation of travel time through the channels require an estimate of the average flow velocities. The average flow velocity, V in the channel is computed using Manning's equation. The estimated average flow velocity is used to calculate the travel time through the channel 1-3 under equilibrium conditions.

Channel storage time is computed by calculating the time required to attain equilibrium storage volume based on its corresponding channel inflow. For channel 1-3, the average depth of flow is used to calculate the storage time in channel 1-3.

The total travel time from a location within the watershed to the outlet is given by:

$$T = t_{eq} + \sum_{n=1}^{n_s} \left(t_{ch} + t_{st} \right) \tag{5}$$

where t_{eq} is the overland flow travel time, t_{ch} is the channel flow travel time, t_{st} is the channel storage time and n_s is the number of cells along a specific flow path to the watershed outlet. Since the flow pathway from a cell to the outlet is inferred from the DEM analysis, using equation (5) a grid of travel times from different locations within the watershed to the outlet is computed.

MODEL APPLICATION AND RESULTS

Study Area

The Waiparous creek watershed is used to demonstrate the derivation of a spatial unit hydrograph and its application in simulation of an observed flood hydrograph. The watershed has an area of 230 km^2, which is mostly forested and is located on the eastern slopes of the Canadian Rockies in Alberta, Canada. The average annual rainfall over the summer months is 400 mm. Soils in the area generally belong to the B or C hydrologic soil group, according to the SCS classification (National Engineering Handbook, 1972). Runoff curve numbers determined from standard SCS tables for antecedent moisture condition II, range from 65 to 85.

Channel Network

Starting from the digital elevation model (DEM), hydrologic features of the landscape such as flow direction, flow accumulation and channel network can be determined. A simple digital elevation model of the watershed is constructed by discretizing the watershed into 1km x 1km grid cells. The elevation of each grid cell is evaluated based on assigning an average elevation over the specified section of the watershed. Assuming that a unique downstream cell can be defined for each cell, then a unique connection from each cell to the watershed outlet is determined. In the case of water draining under gravity, the criterion defining the unique downstream cell is given by the direction of the

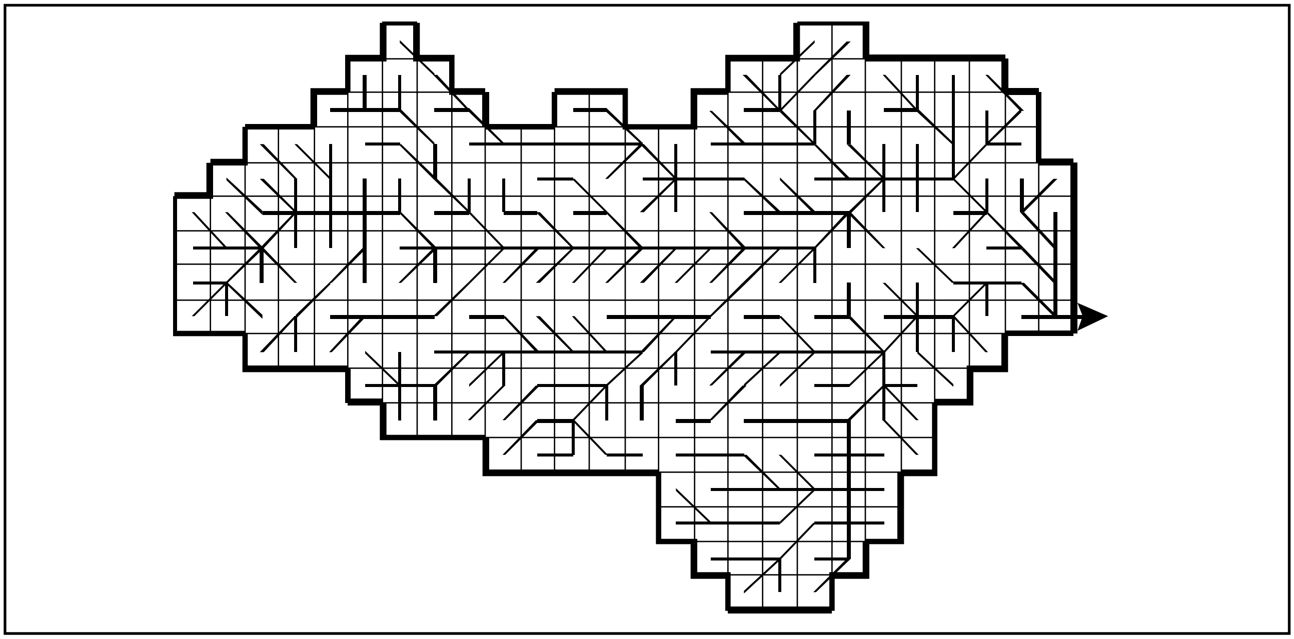

Figure 1. Channel network.

maximum downslope, that is, each cell flows to the lowest of its immediate eight neighbor cells. Connecting the cell centers along the direction of maximum downslope, the channel network is created as shown in Figure (1). This produces a cell-network, with the shape of a spanning tree, that represents the watershed channel network.

The compiled channel network is used to compute the average flow velocities through the channels and subsequently the travel time to the watershed outlet from different locations within the watershed. The computed travel times in the 1km x 1km grid are used in the interpolation to obtain the travel times in the 100m x 100m grid. Using the GIS: IDRISI, the histogram of the area of cells falling within one-hour travel time increments are compiled, as well as their cumulative distribution, which is the time-area curve.

Figure (2) shows the time-area curves from different rainfall intensities over the watershed. The time-area curves obtained are in the form of S-hydrographs. The channel responds gradually to rain

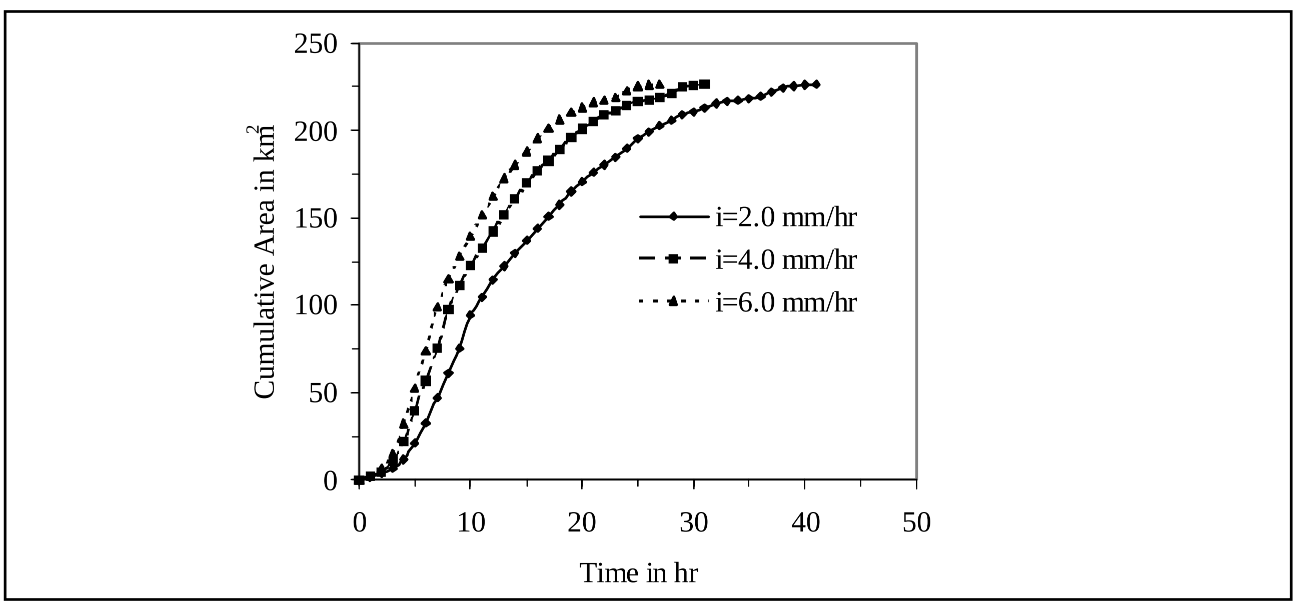

Figure 2. Time-area curves.

falling on the nearby areas and the channel itself. As time progresses, rain falling on remote overland segments contribute to the channels and traverses along the channels to the outlet. At the time to equilibrium the whole basin is responding and the entire storage capacity is used and the inflow is equal to the outflow. Moreover, as the rainfall intensity increases the slope of the S-hydrograph increases, and the time to equilibrium decreases. The excess rainfall generated from this rainfall input is spatially nonuniform, due to the varying runoff curve numbers of the cells.

SPATIAL UNIT HYDROGRAPH

The time-area curve is lagged by Δt=1 hr and subtracted from the original curve, and the resulting values are divided by Δt, to yield the 1-hr spatial unit hydrograph. Figure (3) shows the 1-hr spatial unit hydrographs for different rainfall intensities. As the rainfall intensity increases the peak discharge increases and the time to peak decreases. The spatial unit hydrograph is no longer a lumped model, since it accounts for the internal dynamics of flow distribution within the watershed. It is computed on the basis of watershed hydraulics, and as such, allows the derivation of watershed responses to inputs of varying magnitudes, thus eliminating the assumption of proportionality of input and output.

HYDROGRAPH SIMULATION

An observed flood hydrograph produced by a 21-hr storm recorded at two neighboring rainfall gaging stations is simulated using the derived 1-hr spatial unit hydrographs. The Thiessen polygon method is used to obtain the spatially averaged rainfall hyetograph of the watershed. The rainfall hyetograph, discretized into 1-hr rainfall increments is applied uniformly over the watershed's grid cells. Each cell generated an excess rainfall hyetograph according to its assigned CN value. The 1-hr excess rainfall increments are spatially averaged to obtain a representative excess rainfall hyetograph of the watershed. This excess rainfall hyetograph is convoluted with the corresponding 1-hr spatial unit hydrographs to obtain the simulated hydrograph shown in Figure (4). The superposition of outputs is retained in simulating the flood hydrograph by convolution, since it has been shown that some quasi-linear systems satisfy the principle of superposition. The agreement

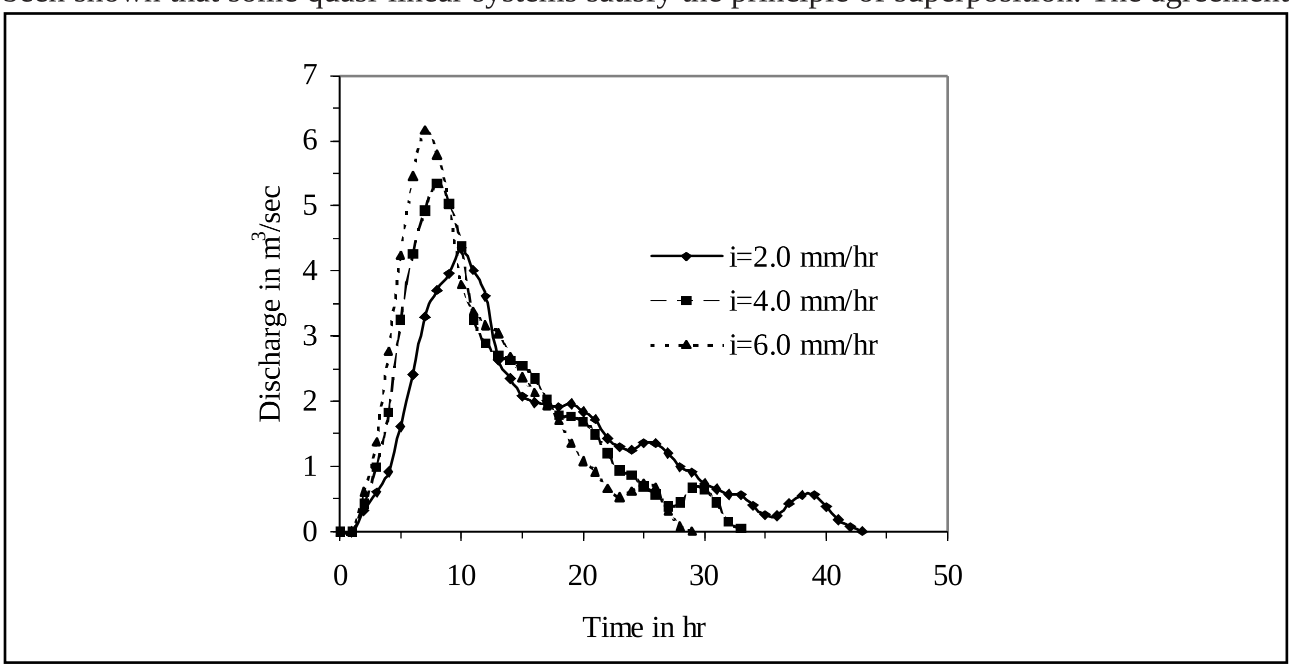

Figure 3. 1-hr spatial unit hydrographs.

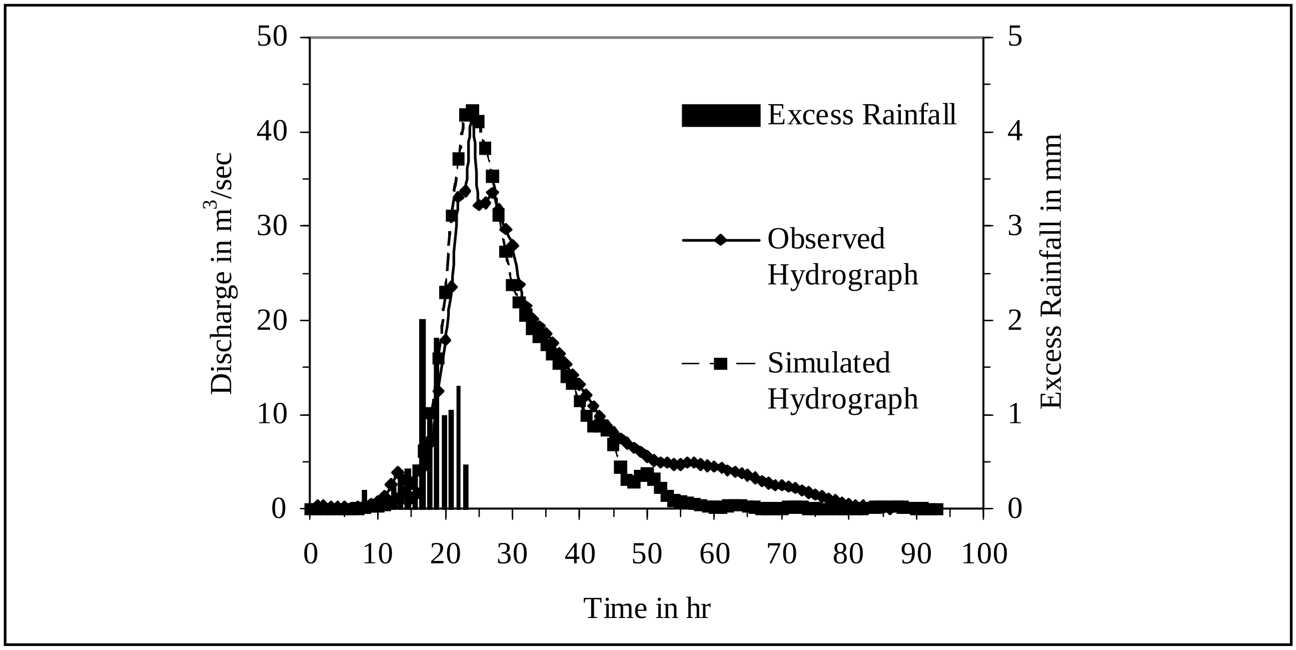

Figure 4. Observed and simulated flood hydrographs

between the simulated and the observed flood hydrographs are very good, considering that no parameter optimization is performed.

CONCLUSIONS

The spatial unit hydrograph is a hybrid between a lumped and a distributed model. For natural watersheds where the amount of available data is limited in distributed form, this model formulation and methodology should prove to be a good compromise.

The spatial unit hydrograph can be derived for ungaged watersheds without observed rainfall and runoff data, since the time-area curve is computed on the basis of watershed hydraulics, for which data may be obtained by field survey.

The existence of a set of time-area curves based on the grid cell approach implies the presence of a velocity field that varies spatially during the occurrence of a storm event. Therefore, it follows that if this velocity field is known a priori, the spatial unit hydrographs are completely specified without the need for any arbitrary mathematical functions or empirical formulas to be used as response functions. Moreover, the velocity field is dependent on the rainfall intensity. This implies the existence of response functions that exhibit a non-linear behavior, which is in contrast to the classical unit hydrograph approach.

The spatial unit hydrograph model of a watershed can be efficiently implemented by the use of a GIS. The GIS facilitates to capture and to utilize a number of watershed and rainfall parameters in distributed form, which is not possible when using the classical unit hydrograph approach. Experience with the spatial unit hydrograph modeling indicates it may be a promising tool for prediction of flood hydrographs.

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