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Momentum and Kinetic Energy Coefficients in Symmetrical Rectangular Compound Cross Section Flumes

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Abstract

In this study a series of laboratory experiments was conducted in nine models of symmetrical rectangular compound cross section having different dimensions for a wide range of discharges in order to investigate the effects of step height and main channel width on the kinetic energy coefficient, α, and momentum correction coefficient, β. An overall kinetic energy correction coefficient and momentum correction coefficient were obtained from the data of all the models investigated. These numerical values can be used for any symmetrical rectangular compound cross section having similar geometry and similar range of Reynolds number.

Key Words: Open channel, Momentum coefficient, Energy coefficient, Flumes, Compound cross section.

Simetrik Dik Dörtgen Şeklinde olan Bileşik Kesitli Flumlardaki Moment ile Kinetik Enerji Katsayıları

Özet

Basamak yüksekliği ve ana kanal genişliğinin kinetik enerji katsayısı (α) ile moment düzeltme katsayısı (β) üzerindeki etkilerini araştırmak amacıyla, geniş bir akıtma hızı dağılımı için değişik boyutları olan dokuz simetrik dik dörtgen şeklinde olan bileşik kesitli model üzerine bir dizi laboratuvar deneyi yapılmıştır. Araştırılan bütün modellerin verilerinden kapsayıcı kinetik enerji düzeltme katsayısı ile moment düzeltme katsayısı elde edilmiştir. Bu sayısal değerler, geometrisi ve Reynold sayısı dağılımı benzer olan herhangi simetrik dik dörtgen şeklinde olan bileşik kesit için kullanılabilir.

Introduction

When energy and momentum principles are used in open channel flows, it has been customary to compute the velocity head and the momentum flux of the flow from the average velocity. In reality, the velocity distributions are not uniform over the cross section, and hence the velocity head and the momentum flux of open channel flow are generally greater than the values computed by using the average velocity. These values may be corrected by using so-called energy and momentum correction coefficients, which are always slightly greater than the limiting value of unity. These coefficients first proposed by Coriolis in 1836 and Boussinesq in 1877, respectively. In 1934, O. Brien and Johnson used a graphical method to obtain the velocity coefficients (King and Brater, 1963). For approximate values of coefficients, Rehbock, assuming a linear velocity distribution, and Chow, assuming a logarithmic velocity distribution, have proposed simple formulas which are only functions of the average and maximum velocities (Chow, 1954). King and Brater (1963) provide experimentally determined values for kinetic energy and momentum correction coefficients, α, and β, for open channels of various cross-sectional shapes but not for compound cross sections. In practice, the velocity distribution coefficients have often been assumed to be unity and the flow equations solved in an approximate way (Chen, 1993).

Different theoretical expressions for α and β based on different assumptions and conditions have been derived by many authors (Streeter, 1942; Iwasa, 1954; Rouse, 1965; Golubtsov, 1976; Benedict, 1980; Fox and McDonald, 1985; Chen, 1991)

0.1. Theoretical Considerations

In elementary hydraulics, the total energy per unit weight of the flowing fluid on any stream line passing through a channel section may be expressed as the total head, which is equal to the sum of the elevation above a datum, the pressure head and the velocity head.

In general, each stream line passing through a channel section has a different velocity head due to the nonuniform velocity distribution. On the other hand, in open channel flow when the energy principle is used, it is customary to use average velocity and write the head at a section rather than on each streamline. Hence, as a result of nonuniform velocity distribution, the velocity head of an open channel flow is generally greater than the value computed by using the average velocity. Therefore, the true velocity head may be expressed as $\alpha\frac{V^2}{2g}$, where α is known as the kinetic energy correction coefficient or Coriolis coefficient, V is the cross-sectional average velocity, and g is the gravitational acceleration. By definition, then

$$\alpha = \frac{1}{V^3A}\int_A v^3 dA \tag{1}$$

where v is the velocity distribution over the channel cross section, and A is the flow area.

Similarly, in applying the momentum equation to open-channel flows, the true momentum per unit time passing a cross section is not given by the mass passing per unit time multiplied by the average velocity, but by the integral

$$\int_A v^2 dA = \beta V^2 A \tag{2}$$

or

$$\beta = \frac{1}{V^2A}\int_A v^2 dA \tag{3}$$

The quantity β is called the momentum correction coefficient or Boussinesq coefficient.

Usually, the values of α and β for various cross-sectional of open channels may be determined either by numerical or graphical integration methods for the measured velocity distributions according to Eqs. 1 and 3.

In this study, the kinetic energy correction coefficient, α, and the momentum correction coefficient, β, for symmetrical rectangular compound cross sections were calculated by using numerical integration as follows:

The cross sectional area A of the channel was divided into N number of elementary areas, ΔA_i, and for each area the corresponding average velocity, $\bar{v_i}$, was determined from the measured velocities. The cross-sectional average velocity, V, and the correction coefficients α, and β were calculated from the following equations:

$$V = \frac{\sum_{i=1}^{N} \bar{v}_i^2 \Delta A_i}{A} \tag{4}$$

$$\beta = \frac{\sum_{i=1}^{N} \bar{v}_i \Delta A_i}{V^2 A} \tag{5}$$

$$\alpha = \frac{\sum_{i=1}^{N} \bar{v}_i^3 \Delta A_i}{V^3 A} \tag{6}$$

1. Experimental Apparatus and Procedure

The experiments were carried out in a glass-walled horizontal laboratory flume 9.0 m long, 0.67 m wide and 0.75 m deep, at the Hydromechanics Laboratory at Middle East Technical University.

Discharge was measured volumetrically with a rectangular sharp-crested weir mounted in the inlet box of the flume. Depth measurement over the crest for this weir was conducted by point gauge reading to the nearest 0.1 mm accuracy, and the predetermined calibration curve of the weir was used to determine the discharges. The maximum capacity was around 110 lt/sec.

In the course of the experiments, a point gauge was used for head measurements along the centreline of the flume. All depth measurements were done with respect to the bottom of the flume. A Pitot tube of circular section with an external diameter of 7 mm was used to measure the static and total pressures, which were used for velocities and shear stresses at required points in the experiments conducted throughout this study.

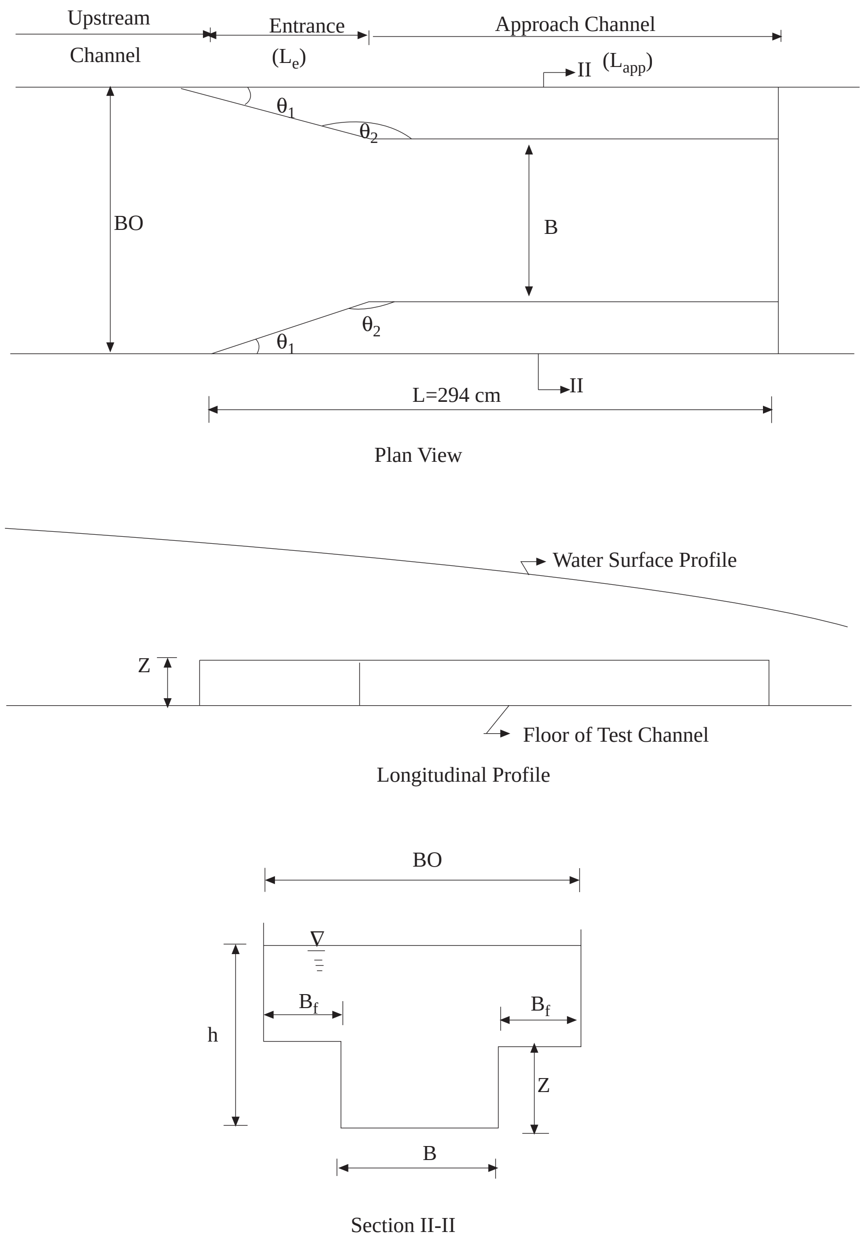


Figure 1. Definition Sketch of the Flume used in the Experiments.

Table 1. Dimensions and Dimensionless Values of Models

Types of models	B (cm)	Z (cm)	B_f (cm)	B_0 (cm)	θ_1 (degree)	θ_2 (degree)	BO/B_f (-)	BO/Z (-)	BO/B (-)	B_f/Z (-)	B_f/B (-)	B/Z (-)
B1Z1	20	5	23.5	67	26.57	153.43	2.85	13.40	3.35	4.70	1.18	4.00
B1Z2	20	10	23.5	67	26.57	153.43	2.85	6.70	3.35	2.35	1.18	2.00
B1Z3	20	15	23.5	67	26.57	153.43	2.85	4.47	3.35	1.57	1.18	1.33
B2Z1	30	5	18.5	67	26.57	153.43	3.62	13.40	2.23	3.70	0.62	6.00
B2Z2	30	10	18.5	67	26.57	153.43	3.62	6.70	2.23	1.85	0.62	3.00
B2Z3	30	15	18.5	67	26.57	153.43	3.62	4.47	2.23	1.23	0.62	2.00
B3Z1	45	5	11.0	67	26.57	153.43	6.09	13.40	1.49	2.20	0.24	9.00
B3Z2	45	10	11.0	67	26.57	153.43	6.09	6.70	1.49	1.10	0.24	4.50
B3Z3	45	15	11.0	67	26.57	153.43	6.09	4.47	1.49	0.73	0.24	3.00

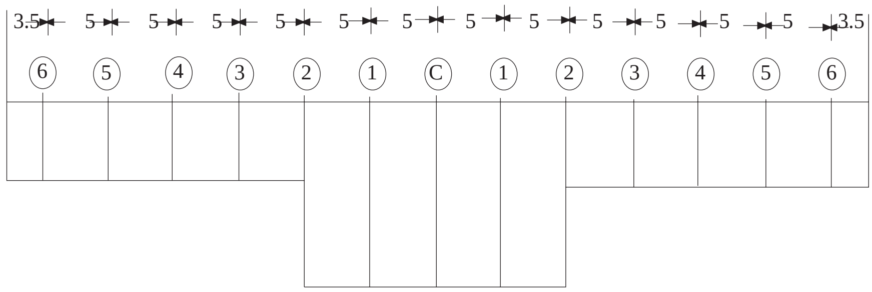
Table 2. Kinetic Energy Correction Coefficient, α, for Various Types of Models

	Types of Models								
Q (lt/s)	B1Z1	B1Z2	B1Z3	B2Z1	B2Z2	B2Z3	B3Z1	B3Z2	B3Z3
3.194	1.059	1.059	1.059	1.054	1.054	1.054	1.045	1.045	1.045
9.459	1.062	1.057	1.057	1.043	1.045	1.045	1.054	1.054	1.054
13.180	1.043	1.048	1.063	1.051	1.043	1.068	1.057	1.063	1.063
21.368	1.031	1.049	1.044	1.037	1.047	1.041	1.044	1.046	1.046
31.421	1.048	1.035	1.052	1.032	1.055	1.052	1.053	1.050	1.050
42.819	1.054	1.026	1.024	1.054	1.054	1.031	1.043	1.044	1.051
62.262	1.024	1.024	1.037	1.045	1.041	1.039	1.042	1.041	1.041
80.459	1.033	1.026	1.026	1.036	1.037	1.050	1.051	1.043	1.040
96.357	1.041	1.038	1.023	1.043	1.053	1.043	1.043	1.031	1.032
110.706	1.029	1.025	1.024	1.046	1.047	1.042	1.041	1.042	1.031

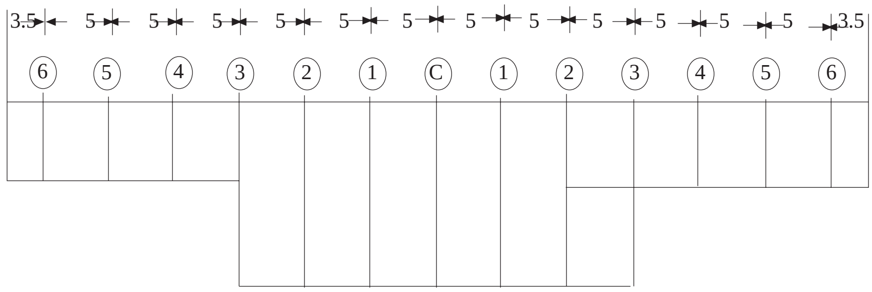
Models of rectangular compound cross sections were manufactured from Plexiglas and placed at about mid length of the laboratory flume. Fig. 1 shows the plan view, longitudinal profile and cross section of the models with symbols designating important dimensions of model elements. The dimensions of the various models used in the experiments are given in Table 1. Model types tested are denoted by BIZI (I=1,2,3). Here, B and Z are the width and step height of the main channel of the compound cross section, respectively.

The required experiments first were conducted in the models with the smallest B (= 20 cm) and varying Z values (=5 cm, 10 cm and 15 cm), and then B was increased to 30 cm at the required amount of Z=5 cm, 10 cm and 15 cm, and finally for B=45 cm with the same three values of Z. The entrance angles, θ_1 and θ_2, were 26.565 and 153.435 degrees, respectively. The entrance length, L_e, was twice the floodplain width, B_f. All the compound cross section models were constructed on a horizontal bottom slope channel.

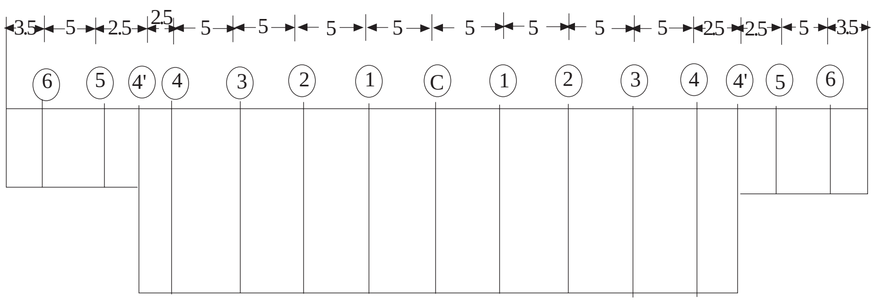
In order to determine the kinetic energy and momentum correction coefficients in the rectangular compound cross sections, the channel cross section was divided by a number of successive lines normal to the direction of flow. Then the total and static heads were measured at several points along these normal lines by Pitot (Preston) tube. More points were taken close to the channel boundary. Towards the free surface, the distances were increased between the points where the velocities were measured. Fig. 2 shows a definition sketch for vertical lines over which velocity measurements were made in models BIZI (I= 1,2,3).



(a) BIZI Model types, I=1,2,3



(b) B2ZI Model types, I=1,2,3



(c) BIZI Model types, I=1,2,3

Figure 2. Definition Sketch for Vertical Linesl over Which Velocity Measurements were Made for the Different Models, (Dimensions are in cm).

2. Presentation and Discussion of Results

For ten different discharges, the computed values of kinetic energy and momentum correction coefficients of nine models are given in Tables 2 and 3 and presented in Figs. 3-8, respectively.

Table 3. Momentum Correction Coefficients, β, for Various Types of Models

Q (lt/s)	Types of Models								
	B1Z1	B1Z2	B1Z3	B2Z1	B2Z2	B2Z3	B3Z1	B3Z2	B3Z3
3.194	1.029	1.029	1.029	1.031	1.031	1.031	1.027	1.027	1.027
9.459	1.022	1.027	1.027	1.029	1.025	1.025	1.016	1.016	1.016
13.180	1.023	1.028	1.023	1.019	1.018	1.018	1.017	1.013	1.013
21.368	1.031	1.019	1.024	1.014	1.009	1.011	1.014	1.010	1.010
31.421	1.028	1.025	1.022	1.009	1.015	1.012	1.007	1.010	1.010
42.819	1.034	1.016	1.020	1.013	1.016	1.011	1.012	1.014	1.015
62.262	1.024	1.024	1.018	1.015	1.008	1.009	1.010	1.013	1.006
80.459	1.023	1.016	1.021	1.016	1.006	1.010	1.007	1.005	1.011
96.357	1.021	1.018	1.018	1.013	1.012	1.006	1.009	1.011	1.005
110.706	1.019	1.015	1.012	1.006	1.010	1.007	1.008	1.010	1.009

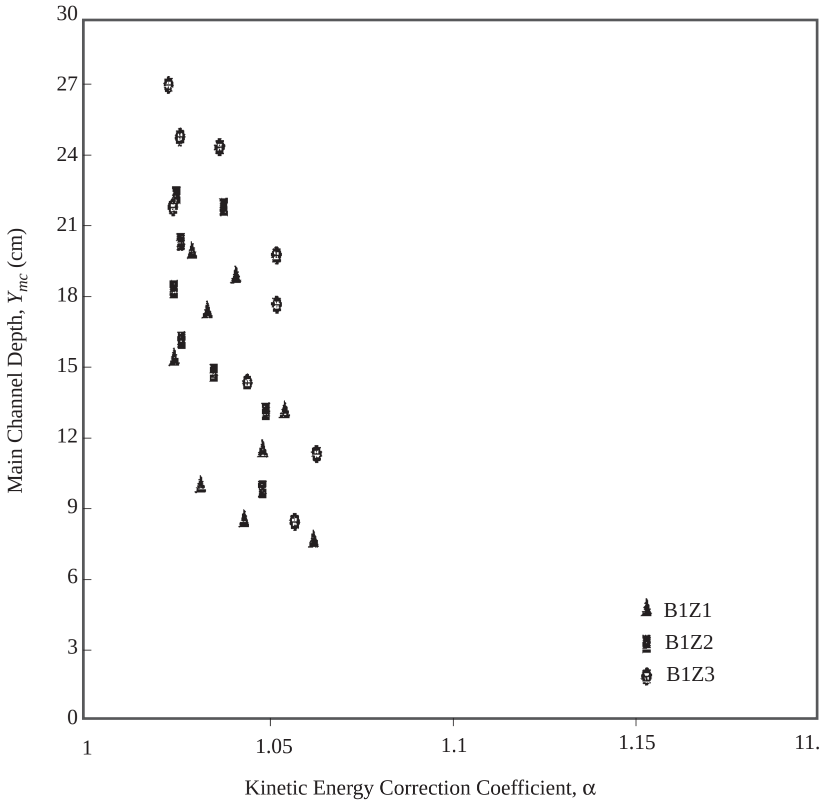


Figure 3. Variation of Kinetic Energy Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

As expected from the general uniformity of the velocity distribution, the values of α and β are quite small. It can also be seen from Tables 2 and 3 and from Figs. 3-8 that the values of the correction coefficients α and β slightly decrease with increasing depth of flow, as in the case of fairly straight prismatic channels (Chow, 1959). The influence of the main channel width and step height on α values is almost negligible, whereas β values slightly decrease as the main channel width increases, although they do not significantly vary with increasing main channel height.

Values of the Reynolds number, Re, for the different models are presented in Table 4, where

$$Re = VD/\nu \tag{7}$$

where D is the hydraulic depth which is defined as the cross-sectional area of the water normal to the direction of flow in the channel divided by the width of the free surface; ν is the kinetic viscosity of water.

As is clearly seen in Table 4, as the discharge increases, the values of the Reynolds number increase, this is true for all tested models.

The range of the values of the kinetic energy and momentum correction coefficients mentioned in this study (Tables 2 and 3) can be utilized for practical purposes for conditions (Table 1) under which velocity measurements have been conducted in symmetrical rectangular compound cross sections and the same range of Reynolds number. These values can be used for models of the same geometry and the same range of Reynolds number in order to calculate the energy and momentum flux of the flow at a given section.

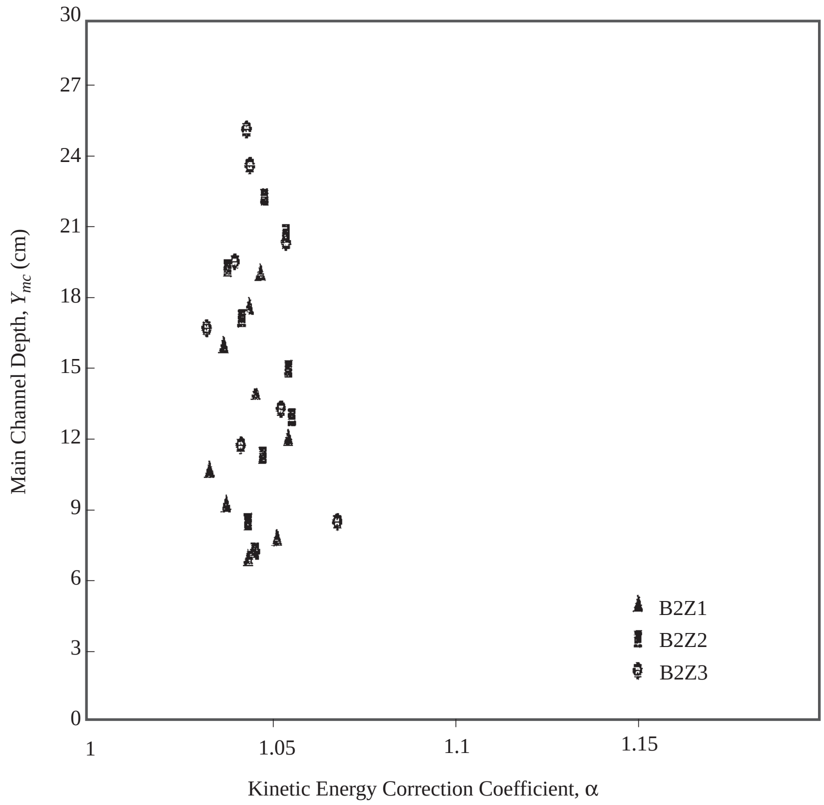


Figure 4. Variation of Kinetic Energy Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

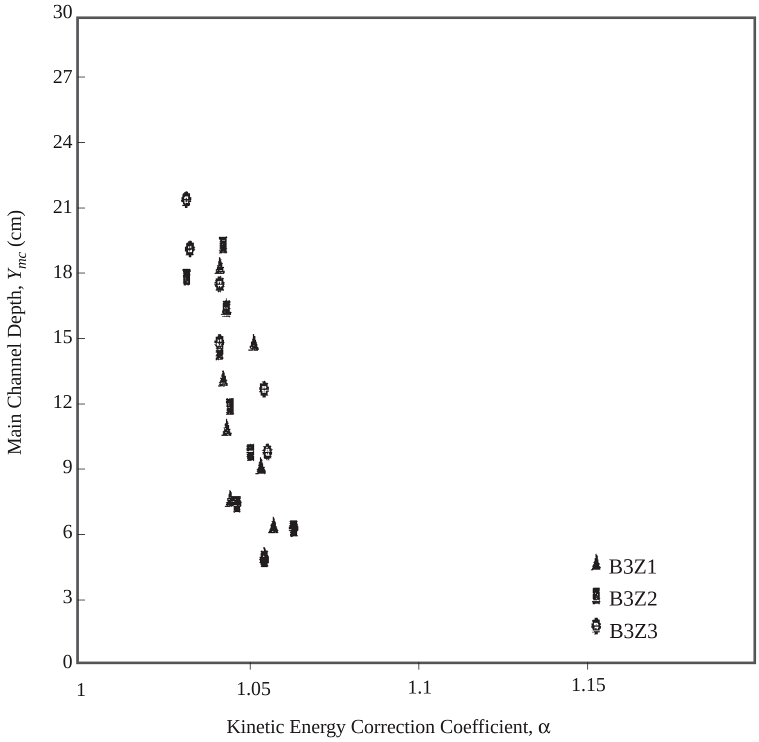


Figure 5. Variation of Kinetic Energy Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

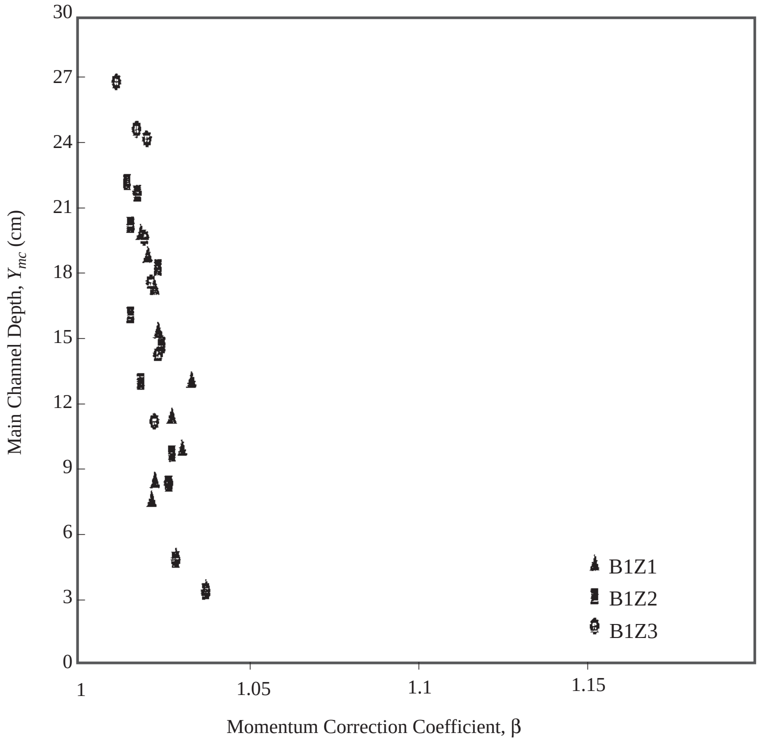


Figure 6. Variation of Momentum Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

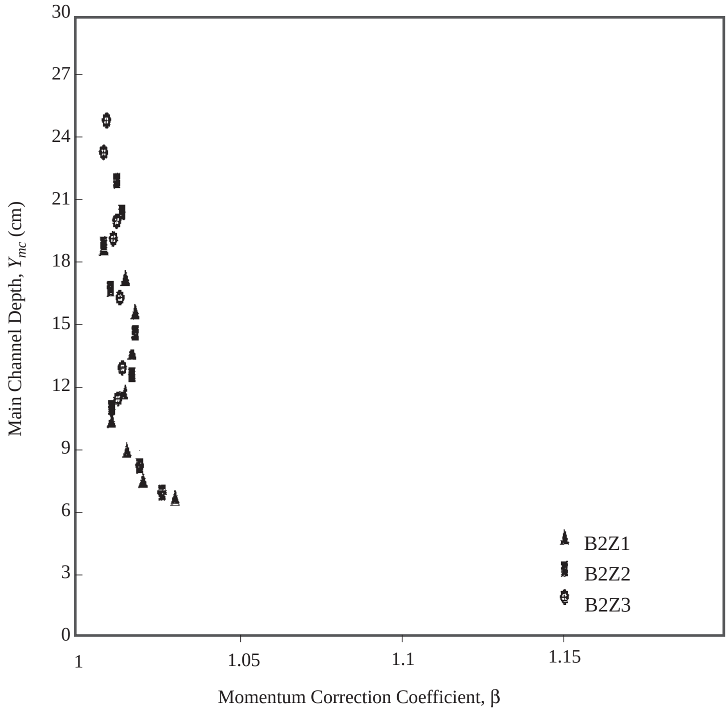


Figure 7. Variation of Momentum Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

Table 4. Reynolds Number for Various Types of Models

	Types of Models								
Q (lt/s)	B1Z1	B1Z2	B1Z3	B2Z1	B2Z2	B2Z3	B3Z1	B3Z2	B3Z3
3.194	4.767×10^3	1.597×10^4	1.597×10^4	1.065×10^4	1.065×10^4	1.065×10^4	7.098×10^3	7.098×10^3	7.098×10^3
9.459	1.412×10^4	4.730×10^4	1.470×10^4	1.412×10^4	3.153×10^4	3.153×10^4	1.470×10^4	2.102×10^4	2.102×10^4
13.180	1.967×10^4	6.590×10^4	6.590×10^4	1.967×10^4	4.393×10^4	4.393×10^4	1.967×10^4	2.929×10^4	2.929×10^4
21.368	3.189×10^4	3.189×10^4	1.068×10^4	3.189×10^4	3.189×10^4	7.123×10^4	3.189×10^4	4.748×10^4	4.748×10^4
31.421	4.690×10^4	4.690×10^4	4.690×10^4	4.690×10^4	4.690×10^4	1.047×10^4	4.690×10^4	6.982×10^4	6.982×10^4
42.819	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4	6.391×10^4
62.262	9.293×10^4	9.293×10^4	9.293×10^4	9.293×10^4	9.293×10^4	9.293×10^4	9.293×10^4	9.293×10^4	1.384×10^5
80.459	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5	1.201×10^5
96.357	1.438×10^5	1.438×10^5	1.438×10^5	1.438×10^4	1.438×10^5	1.438×10^5	1.438×10^5	1.438×10^5	1.438×10^5
110.706	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5	1.652×10^5

3. Conclusions and Recommendations

This study has been experimentally conducted in a symmetrical rectangular compound cross section to investigate the effect of main channel width and step height on the kinetic energy and momentum correction coefficients. From the analysis of the experimental results, the following conclusins can be drawn:

1. The values of the correction coefficients α and β slightly decrease with increasing depth of flow.
2. The influence of the main channel width and step height on α values is almost negligible.
3. β values slightly decrease as the main channel width increases,
4. α and β do not significantly vary with increasing main channel height.
5. For all models tested, the range of kinetic energy coefficient and momentum correction coefficient values can be used for practical purposes for models of the same geometry and the same range of Reynolds number.

The following recommendations can be made for future studies on this topic:

For future studies; higher step heights and wider main channel widths than those used in this study should be tested. Different compound cross sections should be investigated to be compared with those obtained in this study.

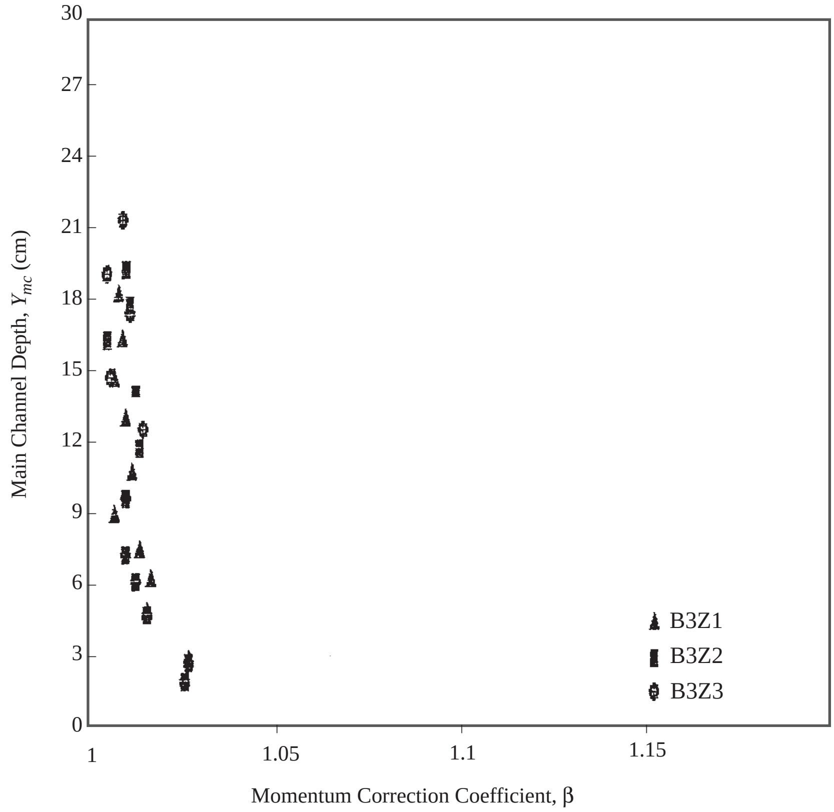


Figure 8. Variation of Momentum Correction Coefficient with Main Channel Depth for Models of Constant Main Channel Widths

4. Notation

The following symbols are used in this paper:

B	=	bottom width of the approach channel;
B_f	=	floodplain channel width;
B0	=	bottom width of the upstream channel;
g	=	gravitational acceleration;
L_{app}	=	approach channel length;
L_e	=	entrance channel length;
N	=	number of small areas, ΔA_i,
Q	=	volume rate of flow;
Re	=	Reynolds number;
V	=	average full cross-sectional velocity;
v	=	velocity distribution over the channel cross section;
$\bar{v}_i$	=	average velocity corresponding to each small area
Y_f	=	floodplain water depth;
Y_{mc}	=	main channel water depth;
Z	=	step height;
α	=	Kinetic energy correction coefficient;
β	=	momentum correction coefficient;
θ_1, θ_2	=	entrance angles.

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