

## Study of Aeration Efficiency at Weirs

**Ahmet BAYLAR**

*Fırat University, Civil Engineering Department, Elazığ-TURKEY*

**Tamer BAĞATUR**

*Fırat University, Environmental Engineering Department, Elazığ-TURKEY*

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### Abstract

The amount of dissolved oxygen (DO) in the waters of rivers and streams is very important to the quality and existence of aquatic life. Hydraulic structures have an impact on the amount of dissolved oxygen in a river system, even though the water is in contact with the structure for only a short time. The same quantity of oxygen transfer that normally would occur over several kilometers in a river can occur at a single hydraulic structure. The primary reason for this accelerated oxygen transfer is that air is entrained into the flow, which produces a large number of bubbles. These air bubbles greatly increase the surface area available for mass transfer. Plunging overfall jets from weirs are a particular instance of this, and the aeration properties of such structures have been studied widely in the laboratory and field over a number of years. This study investigates weirs having different cross-sectional geometry and how they affect the aeration performance. It is demonstrated that the aeration efficiency of the triangular notch weir is generally better than the other weirs.

**Key Words:** Oxygen transfer, Dissolved oxygen, Aeration, Aeration efficiency, Weirs

## Savaklarda Havalandırma Veriminin İncelenmesi

### Özet

Nehir ve akarsulardaki çözülmüş oksijen miktarı, hem suyun kalitesini gösteren bir özellik olarak hem de suda yaşayan canlıların yaşamlarını devam ettirebilmeleri için gereken çok önemli bir kriterdir. Hidrolik yapılar, akan su ile kısa bir süre için temasta olmalarına rağmen, bir nehir sistemindeki çözülmüş oksijen miktarı üzerinde önemli bir etkiye sahiptirler. Bir nehirde doğal olarak birkaç kilometrede meydana gelebilecek oksijen transferi, tek bir hidrolik yapı ile hızlı bir şekilde meydana getirilebilir. Bu hızlandırılmış oksijen transferinin asıl sebebi, çok miktarda kabarcık meydana getirerek akım içerisine havanın sokulmasıdır. Bu hava kabarcıkları, kütle transferi için mevcut yüzey alanını çok miktarda artırır. Savaklardan serbest düşen jetler, bunun özel bir örneğidir ve birkaç yıldır laboratuvar ve arazide geniş bir şekilde incelenmektedir. Bu çalışmada farklı enkesit geometrilerine sahip savaklar ve bunların, havalandırma verimini nasıl etkiledikleri incelenmiştir. Çalışmanın sonucunda üçgen enkesite sahip savakların diğer enkesitli savaklardan daha iyi havalandırma verimine sahip olduğu tespit edilmiştir.

**Anahtar Sözcükler:** Oksijen transferi, Çözülmüş oksijen, Havalandırma, Havalandırma verimi, Savaklar

## Introduction

Currently there is much emphasis placed on water quality and maintaining water quality parameters in our freshwater hydrosphere (rivers, lakes, and reservoirs). One of the most widely cited parameters is that of dissolved oxygen (DO) concentration. DO is often used as an indicator of the quality of water used by humans or serving as a habitat for aquatic flora and fauna. It is maintained by many natural chemical and biological processes that either increase or decrease local oxygen concentrations. Respiration by aquatic life serves to reduce DO, as does biodegradation of organic material in the sediments, along with a host of the other oxygen-consuming chemical reactions. Photosynthesis by aquatic plant life can be a significant source of oxygen to a water body, as can oxygen transfer with the atmosphere.

Weir aeration occurs in rivers, fish hatcheries, and water treatment plants. Often, the hydraulic head is naturally available and incurs no operating cost. In some cases, however, weir aeration is economically competitive with alternative aeration technology such as surface aeration, even when energy costs for pumping the water are included.

Before breaking up into drops, the flow over a weir or waterfall would be classified as a free jet as shown in Fig. 1. Typically, most of the oxygen transfer is accomplished in this type of structure during the breakup of the jet, and the free jet's subsequent collision with the bottom of the channel. If the free jet plunges into a downstream water pool, air entrainment and turbulent mixing will contribute to oxygen transfer. Furthermore, the depth

of the downstream water pool can enhance the absorption because of the increased hydrostatic pressure on the entrainment air bubbles. Avery and Novak (1978) found that the transfer efficiency is maximum at a tailwater depth of approximate 0.6 times the drop height, indicating that a trade-off exists between bubble residence time, pressure, and turbulence levels. Oxygen absorption efficiencies vary widely, but for low-head overflow weirs, efficiencies of up to 70 % have been measured.

Gameson (1957) was the first to report on the aeration potential of weirs in rivers. Since then a number of laboratory investigations into weir aeration have been carried out, notably by Van der Kroon (1969a, b), Apted and Novak (1973), Avery and Novak (1978), and Nakasone (1987). Investigations also have been reported on the aeration performance of existing hydraulic structures and these are reviewed by Wilhelms et al. (1992). Gulliver and Rindels (1993), in particular, discuss problems associated with field measurements of oxygen transfer and the degree of uncertainty involved. Much of this work has dealt with straight weirs and free overfalls, among other structures, and none has concentrated specifically on the aeration performance of different shaped weirs.

This paper describes an experimental investigation into the performance of sharp-crested weirs (Fig. 2), and in particular, the effect of varying the shape of the weir. The shape of the weir dictates the behavior of the jet. This in turn is believed to alter the air entrainment and contact time in both the jet itself and the downstream water pool and hence the aeration performance of the weir as a whole.

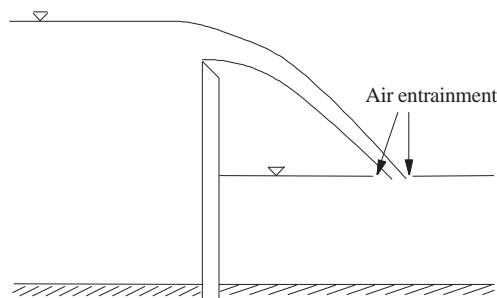


Figure 1. Free Jet over Weir

## Background

Oxygen is a highly volatile compound with a gas-water transfer rate that is controlled entirely by the liquid phase. Thus, the change in oxygen concentra-

tion over time in a parcel of water as the parcel travels through a hydraulic structure can be expressed as

$$\frac{dC}{dt} = K_L \frac{A}{V} (C_s - C) \quad (1)$$

where  $C$  = dissolved oxygen concentration;  $K_L$  = liquid film coefficient for oxygen;  $A$  = surface area associated with the volume  $V$ , over which transfer occurs;  $C_s$  = saturation concentration, or the dissolved oxygen concentration at which equilibrium with the gas phase is achieved; and  $t$  = time. The term  $A/V$

is often called the specific surface area,  $a$ , or surface area per unit volume. Eq. (1) does not consider sources and sinks of oxygen in the water body because their rates are relatively slow compared to the oxygen transfer that occurs at most hydraulic structures due to the increase in free-surface turbulence and the large quantity of air that is normally entrained into the flow.

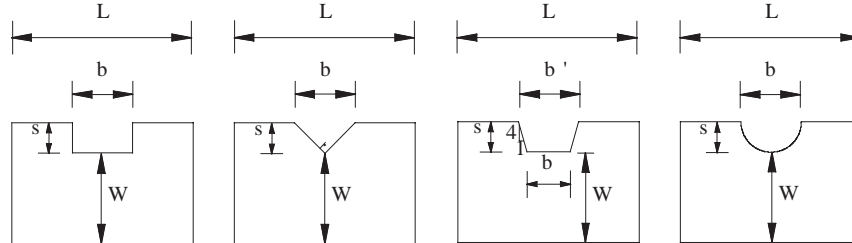


Figure 2. Weir Types Used for Experiments

The predictive relations described herein all assume that  $C_s$  is constant and determined by the water-atmosphere partitioning. If that assumption is made,  $C_s$  is constant with respect to time, and (1) can be integrated in a moving coordinate system to result in an oxygen transfer efficiency,  $E$  (Gulliver et al., 1990).

$$E = \frac{C_d - C_u}{C_s - C_u} = 1 - \frac{1}{r} \quad (2)$$

where subscripts  $u$  and  $d$  = upstream and downstream locations, respectively and  $r$  = oxygen deficit ratio. A transfer efficiency value of 1.0 means that the full transfer up to the saturation value has occurred at the structure. No transfer would correspond to  $E = 0.0$ . The saturation concentration is normally assumed to be known from charts or equations, and is typically chosen at the local atmosphere value. This is not always the proper choice because the saturation DO concentration for natural waters is often different from that of distilled, deionized water due to the salinity effects.

In this study, the saturation concentrations were determined by the chart of McGhee (1991). The salinity effect would be insignificant because the salt content of tap water used for the experiments was consequently low.

### Factors Affecting Aeration Efficiency

The oxygen transfer that occurs at a given structure is sensitive to water temperature, water quality, tailwater depth, drop height, weir discharge, and

dissolved-oxygen deficit.

### Water Temperature

Oxygen transfer efficiency is sensitive to water temperature, and investigators have typically employed a temperature correction factor. For hydraulic structures, the most often used temperature correction factor has been that of Gameson et al. (1958), although some investigators have chosen to use an Arrhenius-type of water temperature correction (Holler 1970). Gulliver et al. (1990) applied the theories of Levich (1962), Hinze (1955), and Azbel (1981) to mass transfer similitude and developed the relationship

$$1 - E_{20} = (1 - E)^{1/f} \quad (3)$$

where  $E$  = transfer efficiency at the water temperature of measurement and  $E_{20}$  = transfer efficiency at the 20°C. The exponent,  $f$ , was found to be described by

$$f = 1.0 + 0.02103(T - 20) + 8.261 \times 10^{-5}(T - 20)^2 \quad (4)$$

### Water Quality

The presence of surface active agents, organic substances, and suspended solids in water have all been observed to affect the aeration process. Surface active agents in particular appear to modify the process by reducing surface tension, forming diffusion inhibiting films at the air-water interface, and affecting

the hydrodynamic characteristics of the flow. The effect of water quality often is generalized by the use of a "water quality factor" in equations for the deficit ratio, for instance in Gameson (1957) and Markofsky and Kobus (1978). Avery and Novak (1978) used a similar constant to allow for the affects of different concentrations of sodium nitrate in water.

Tap water was used for all of the experiments reported in this paper. Salt content was consequently low and was monitored constantly during the experiments to prevent any buildup of residues caused by the deoxydant chemicals added to the water. Therefore, the presence of chemicals or pollutants did not affect the results.

### Tailwater Depth

The residence time of entrained air bubbles in a water body directly affects the oxygen mass transfer. The residence time is related to the bubble flow path and hence the bubble penetration depth into the downstream water pool. Tailwater depth would be an important factor with regard to weir aeration and aeration efficiency would increase with increasing tailwater depth. There should be a limit, however, because the penetrating air bubbles will not go to infinite depths. Actually, for each combination of discharge and fall height, there would be an approximate maximum depth to which the bubbles would penetrate, thus limiting the aeration efficiency and possibly even defining its maximum value. Avery and Novak (1978) found that the tailwater depth of weirs should be approximate 0.6 times the drop height. They indicated that the aeration efficiency remained stable for tailwater depths greater than 0.6h. For consistency, all tests reported in this paper were carried out under these conditions. In all of the experiments at all four weir types the writers determined that air bubbles did not generally reach the floor of the downstream water pool.

### Drop Height

The oxygen transfer that occurs at weirs is sensitive to drop height across the structure. Initially, water jets with relatively smooth surfaces issue from the weir and entrain air mainly at the surface of the downstream water pool. As the drop height increases, the surface of the jets first becomes roughened and then the jet oscillates during the fall, entraining air. This results in greater air flow into the downstream water pool. With increasing drop

height, the jet eventually breaks up into discrete droplets and air entrainment prevails. The breakup of the jet reduces its penetration depth into the pool and hence also the depth of the biphasic zone. This effectively reduces contact time  $t_c$  between the bubbles and the surrounding water, and so aeration is observed to have little effect. It should be noted that the "breakup length" of the jet (i.e., the difference in level between the weir sill and the point of breakup) is not at all well defined and the jet breaks up over a considerable length. Thus, the change of the jet to discrete droplets is sudden and takes place over a range of drop heights. It does not entail a reduction in aeration efficiency, but a significant decline in the rate of increase in aeration efficiency with drop height.

### Weir Discharge

The aeration efficiency for weirs varies with discharge. The aeration efficiency decreases with an increase in discharge. Novak (1973 and 1978) and Van der Kroon (1969a and 1969b) reported a constant increase in the aeration efficiency with decreasing discharge. At low discharges, on the other hand, breakup of the jet is observed as drop height increases. This leads to reduced penetration and bubble contact time into the downstream water pool and so reduced aeration efficiency.

### Dissolved-Oxygen Deficit

Oxygen-transfer measurements are typically required at a hydraulic structure to assess the potential for low DO concentrations in the upstream reservoir to continue downstream. For this situation, Murphy's law dictates that the difference between the upstream DO concentration and saturation concentration (the upstream DO deficit) will not be large on the day of the measurement, even though it may be large at other times. From (2) it can be seen that the measurement of transfer efficiency becomes quite sensitive to measurement errors with a low DO deficit upstream. Gulliver and Wilhelms (1992) have stated that an upstream DO deficit of greater than 2.5 mg/L is normally required for any respectable accuracy in an oxygen-transfer efficiency measurement. The primary source of measurement uncertainty was found to be uncertainty in the oxygen-saturation concentration. In summer, when saturation approximates 7 mg/L in most areas, this specification results in an upstream DO of less than 4.5

mg/L. Wilhelms et al. (1992) found that a substantial portion of the oxygen-transfer measurements at hydraulic structures given in the literature suffered from the low upstream deficit problem. They were dropped from the database because an analysis of measurement uncertainty propagation indicated that the uncertainty in these measurements was above a useful value.

DO deficit ratio,  $r[(C_s - C_u)/(C_s - C_d)]$  and hence oxygen transfer efficiency  $E$  are independent of the upstream DO value  $C_u$ . Wormleaton and Soufiani (1998) investigated the independence of oxygen transfer efficiency and upstream DO level. A set of readings was taken of deficit ratio for a model linear weir, with 320 mm sill length, under constant drop height, discharge, tailwater depth, and temperature conditions. The upstream DO concentration  $C_u$  was varied over a range from 0 to 80 % of its saturation value and variation in the downstream DO value  $C_d$  was noted. The results showed a linear relationship between  $C_u$  and  $C_d$ . A relationship between  $C_u$  and  $C_d$  was derived from Eq. (2) as

$$C_d = (1 - E)C_u + EC_s \quad (5)$$

The best-fit line between  $C_u$  and  $C_d$  was

$$C_d(\%) = 0.289C_u(\%) + 69.53 \quad (6)$$

By comparison with Eq. (5), this gives values for oxygen transfer efficiency  $E$  of 0.711 and for  $C_s$  of 97.8 %, confirming that the oxygen transfer efficiency is sensibly independent of the upstream DO

deficit. It also reinforces the use of oxygen transfer efficiency as a useful indicator of the aeration behavior of structures.

In this study, to insure that a minimum upstream DO deficit of 2.5 mg/L was maintained, sodium sulfite ( $\text{Na}_2\text{SO}_3$ ) was added to the water. Cobalt chloride ( $\text{CoCl}_2$ ) was used as a catalyst.

### Experimental Setup

Aeration experiments were conducted using an experimental channel in the Hydraulic Laboratory at the Civil Engineering Department of Fyrat University, Elazığ, Turkey. The experimental channel used in this study was 3.4 m long, 0.60 m wide, and 0.50 m deep with a maximum water flow rate of approximately 4 L/s (Fig. 3). The water jet from the test weir plunged into a downstream water pool, whose height could be adjusted using a pulley arrangement. The water depth in the downstream water pool was controlled by an adjustable weir. The plan-view dimensions of the downstream water pool were 0.6 × 0.6 m. The system included a 3 m<sup>3</sup> storage tank.

The test weir featured four exchangeable weir elements: rectangular weir, triangular notch weir, trapezoidal (Cipolletti) weir, and semi-circular weir, as shown in Fig. 2.

Each experiment was started by filling the storage tank with clean water. Sodium sulfite and cobalt chloride were added to the water to increase the upstream DO deficit ( $C_s - C_u$ ) to  $\approx \times 2.5$  mg/L.

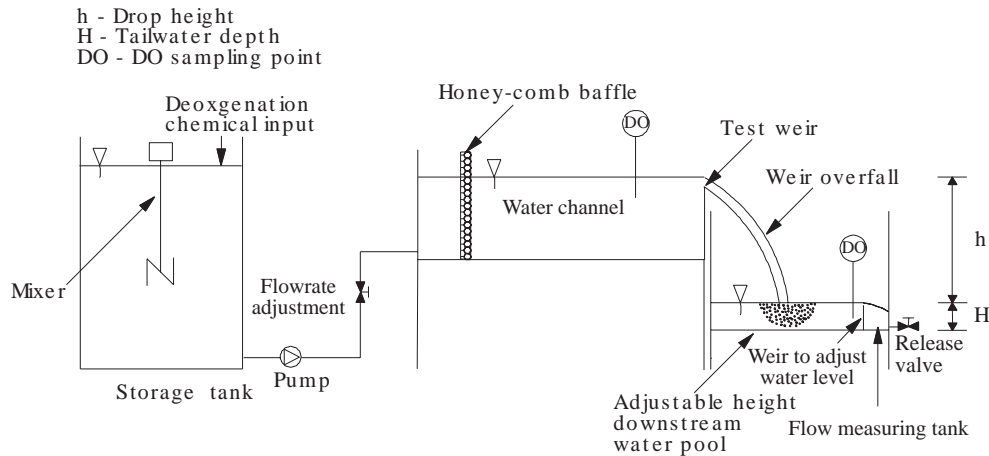


Figure 3. Laboratory Weir Aeration Apparatus

During the experiments, dissolved oxygen and temperature measurements upstream and downstream of the weir were taken using a calibrated portable HANNA Model HI 9142 oxygen meter at the locations identified in Fig. 3. The stirrer was necessary to obtain accurate and reproducible water-phase measurements. The DO meter was calibrated daily, prior to use, by the air calibration method. Calibration procedures followed those recommended by the manufacturer. The calibration was performed in humid air under ambient conditions.

## Experimental Program

The dimensions of the weirs tested are given Table 1. Each weir configuration was tested under flow rates  $Q$  varying from approximately 1.0 to 4.0  $L/s$ . The drop height  $h$ , defined as the difference between the water levels upstream and downstream of the weir, was varied between 0.15 and 0.90 m. The depth in the downstream water pool was maintained throughout at greater than the bubble penetration depth to ensure optimum aeration conditions.

**Table 1.** Experimental Program and Details

Weir Type	L (cm)	b (cm)	s (cm)	W (cm)
Rectangular weir	60	20	10	40
Triangular notch weir	60	20	10	40
Trapezoidal (Cipolletti) weir	60	15 ( $b'=20$ )	10	40
Semi-circular weir	60	20	10	40

## Results

An experimental run consisted of establishing target values for  $Q$ ,  $h$ , and  $H$  within the experimental channel followed by measurement of  $T$ ,  $C_u$ , and  $C_d$ . Experimental values of  $E_{20}$  were calculated from measured values using (2) and (3).

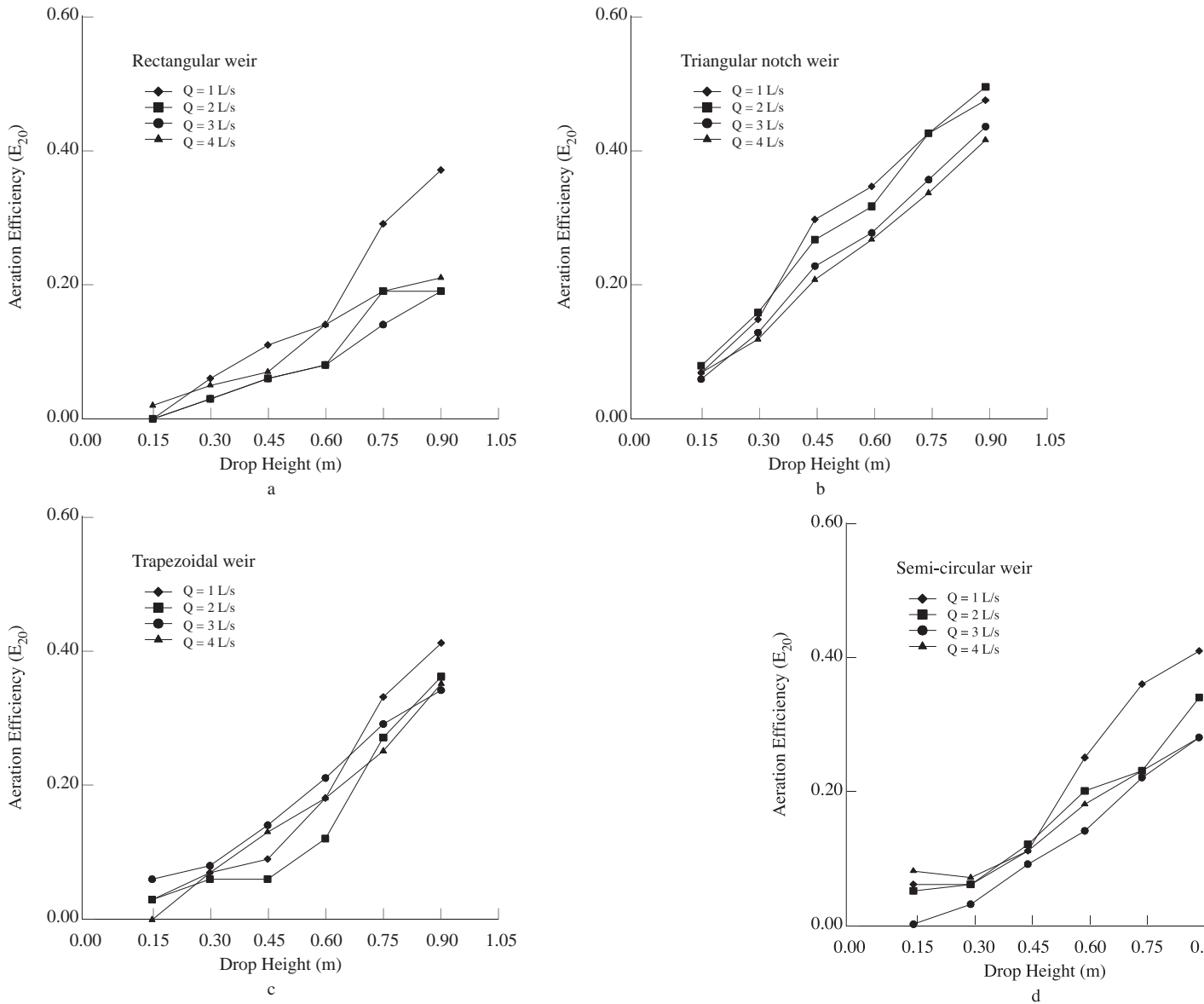
The following sections discuss the oxygen transfer efficiency ( $E_{20}$ ) results, which vary with drop height ( $h$ ), and discharge ( $Q$ ) (Fig. 4.).

Experiments with all four weir types indicate that the drop height is the most important factor influencing aeration efficiency. Fig. 4 shows the oxygen transfer efficiency observed during experiments as a function of drop height and discharge for four different weir types. Fig. 5 also shows variation in aeration efficiency of four different weir types with drop height while the change in discharge is constant. All of these graphs show an increase in aeration efficiency with drop height. Generally, a greater drop height leads to greater bubble penetration depths into the downstream water pool and longer contact times  $t_c$ . This increases aeration efficiency. On the other hand, breakup of the jet was observed as the drop height

increased more than 90 cm. Because the jet eventually breaks up into discrete droplets, bubble penetration depth and contact times  $t_c$  decrease and hence aeration efficiency decreases.

The results of experiments involving changing weir discharge were far less explicit than those involving drop height. Fig. 4 shows that weir discharge influencing oxygen uptake seems to be closely related to the cross-sectional weir geometry. The aeration efficiency of the triangular notch weir was reduced as the discharge increased over the whole range of drop heights tested. In the other weirs the aeration efficiency was generally greatest at a discharge of 1  $L/s$  and the lowest values of the aeration efficiency were observed at different discharge values. At all four weir types, for the lower discharge, breakup of the jet was observed as the drop height increased. This decreases aeration efficiency.

The rectangular weir produced the lowest values of oxygen transfer efficiency. The greatest rectangular weir oxygen transfer efficiency was 0.37, at a discharge of 1  $L/s$ , and drop height of 0.90 m. The rectangular weir was found to have a poor performance as an aerator.

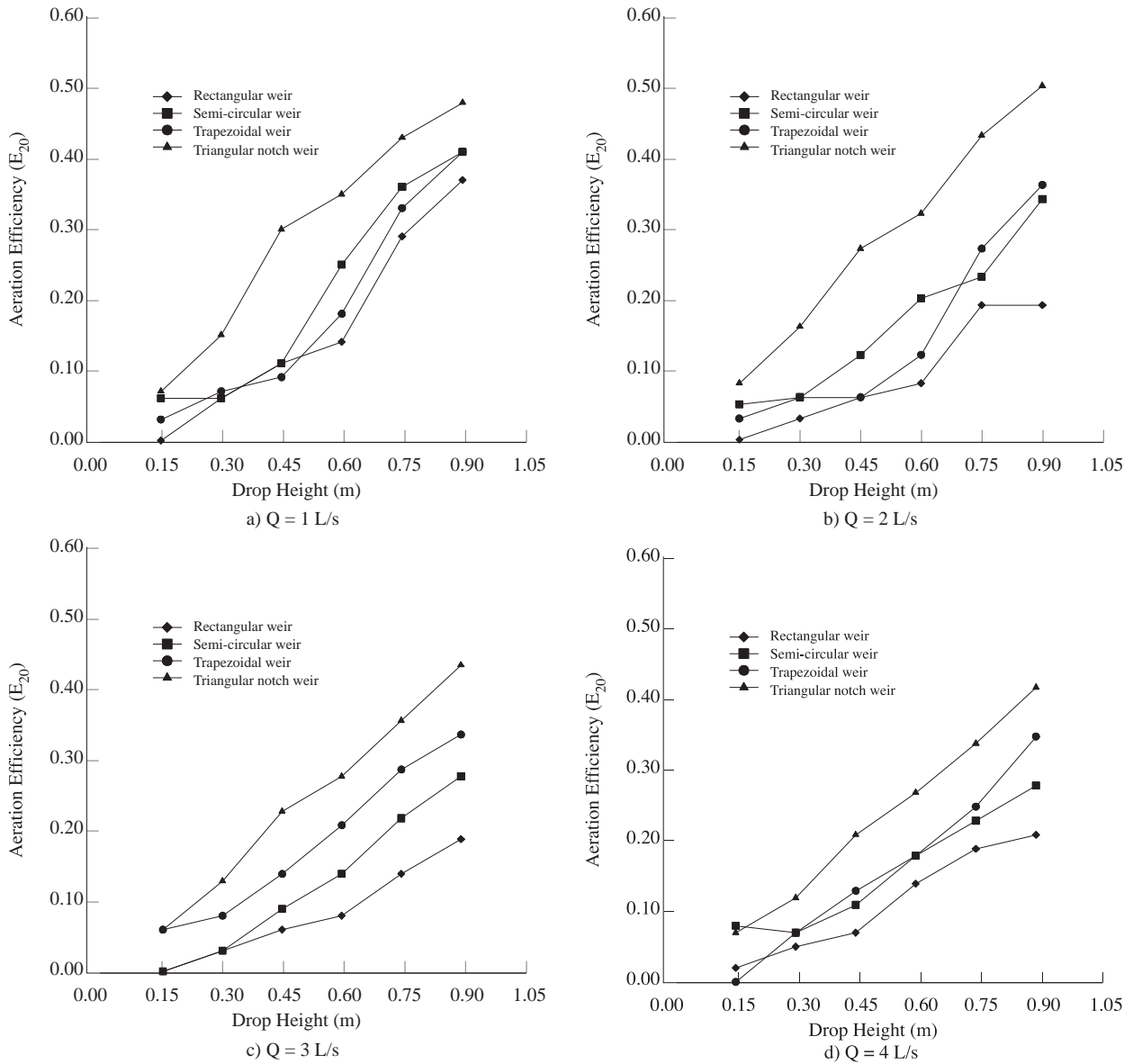


**Figure 4.** Variation in Aeration Efficiency with Drop Height and Discharge for (a) Rectangular Weir; (b) Triangular Notch Weir; (c) Trapezoidal (Cipolletti) Weir; (d) Semi-Circular Weir

For the trapezoidal weir, the values of oxygen transfer efficiency were in general agreement with the values of the semi-circular weir. The greatest trapezoidal weir and semi circular weir oxygen transfer efficiency was 0.41, at a discharge of 1  $L/s$ , and drop height of 0.90 m.

The triangular notch weir was found to have the greatest values of oxygen transfer efficiency. The greatest triangular notch weir oxygen transfer efficiency was 0.50, at a discharge of 2  $L/s$ , and drop

height of 0.90 m and 0.48, at a discharge of 1  $L/s$ , and drop height of 0.90 m. Aeration efficiency was greatest with the triangular notch weir because in this weir air entrainment and turbulent mixing which will contribute to the oxygen transfer were greater than in the other weirs. The primary reason for this difference may be jet shapes. The weir geometry defines jet shapes that are unique to each weir, and the oxygen transfer seems to strongly depend on these jet shapes.



**Figure 5.** Variation in Aeration Efficiency of All Four Weir Types with Drop Height for (a)  $Q=1$  L/s; (b)  $Q=2$  L/s; (c)  $Q=3$  L/s; (d)  $Q=4$  L/s

**Conclusions**

A series of laboratory experiments were carried out to measure the aeration performance of different shaped weirs over a range of flows between 1 and 4 L/s with drop heights from 0.15 - 0.90 m. The total weir length was kept constant at 0.60 m. The following conclusions may be drawn about weirs.

- The drop height was confirmed to be the most important parameter influencing oxygen transfer at weirs. The aeration efficiency increased

with drop height in all cases.

- The results of experiments involving changing weir discharge were far less explicit than those involving drop height. The aeration efficiency of the triangular notch weir was reduced as the discharge increased over the whole range of drop heights tested. In the other weirs the aeration efficiency was generally greatest at a discharge of 1 L/s and the lowest values of the aeration efficiency were observed at different



discharge values.

- At all four weir types, for the lower discharge, breakup of the jet was observed as the drop height increased. This decreases aeration efficiency.
- The weir shape was found to be an important factor influencing the aeration efficiency. The weir geometry defines jet shapes that are unique to each weir, and the oxygen transfer seems to strongly depend on these jet shapes.
- The experimental values of the trapezoidal weir for the oxygen transfer efficiency ( $E_{20}$ ) are in general agreement with the results of the semi-circular weir experimental values.
- The oxygen transfer efficiency was greatest with the triangular notch weir and lowest with the rectangular weir. The rectangular weir generally would not therefore be recommended.
- Tailwater depth as well as drop height and discharge is important for weir aeration. Therefore, there should be a tailwater depth that air bubbles will penetrate to an approximate maximum depth.

### Acknowledgements

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### Symbols

$a$	: the specific surface area ( $A/V$ ), or surface area per unit volume
$A$	: surface area associated with the volume $V$ , over which transfer occurs
$b$	: crest width of weir
$C$	: dissolved oxygen concentration
$C_d$	: dissolved oxygen concentration downstream of a hydraulic structure
$C_s$	: saturation concentration
$C_u$	: dissolved oxygen concentration upstream of a hydraulic structure
$E$	: transfer efficiency at the water temperature of measurement
$E_{20}$	: transfer efficiency at the 20°C
$f$	: term to adjust from 20°C to T°C
$h$	: drop height
$H$	: tailwater depth
$K_L$	: liquid film coefficient for oxygen
$L$	: the experimental channel width
$Q$	: weir discharge
$r$	: oxygen deficit ratio
$s$	: difference between crest and top of weir
$t$	: time
$T$	: water temperature
$W$	: difference between base and crest of weir

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