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Effects of Porosity on the Free Convection Flow of Non-Newtonian Fluids Along a Vertical Plate Embedded in a Porous Medium

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Abstract

In recent years, great interest has been shown in porous media. The applications of porous media in geophysics, petroleum processes, and air conditioning are well established. Many studies of porous media have been carried out, mostly dealing with constant porosity. In this study, the variable porosity effects are investigated for a vertical plate in a variable porous medium. The governing partial nonlinear differential equations were transformed into a set of coupled ordinary differential equations, which was solved using the fourth-order Runge-Kutta method. The results obtained for the different power law index n, and variable porosity of the bed, were found to be in good agreement with previous studies. Results show that when porosity increases temperature variation becomes steeper, and the Nusselt number increases almost linearly with increasing porosity.

Key Words: Non-Newtonian flow, natural convection, porous medium.

Gözenekli Ortama Düşey Olarak Yerleştirilmiş Düz Plaka Üzerindeki Newton Olmayan Akışkanın Doğal Konveksiyonuna Gözenek Yoğunluğunun Etkisi

Özet

Son senelerde gözenekli ortam ile ilgili çalışmalar ilgi çekmektedir. Jeofizik, petrol endüstrisi ve iklimlendirme ile ilgili konularda uygulama alanı bulmuştur. Gözenekli ortam üzerine yapılan çalışmaların çoğunluğu sabit gözenek yoğunluğu üzerinedir. Bu çalışmada düşey plaka üzerindeki doğal konveksiyon akışa değişken gözenek yoğunluğunun etkisi incelenmiştir. Kısmi, linear olmayan, ilgili diferansiyel denklemler normal diferansiyel denklemlere dönüştürülerek dördüncü dereceden Runge-Kutta yöntemi ile çözülmüştür. Newton olmayan akışkanların çeşitli “n” değerleri ve değişken gözenek yoğunluğu “e” için elde edilen sonuçlar daha önceki çalışmalarla karşılaştırılmış ve uyumlu olduğu görülmüştür. Gözenek yoğunluğu arttıkça sıcaklık değişim eğrisi dikleşmekte ve Nusselt sayısı artmaktadır.

Anahtar Sözcükler: Newtonien olmayan akış, doğal konveksiyon, gözenekli ortam.

Nomenclature

a, b, c :	constants
D :	packing diameter
e :	porosity of bed
f :	dimensionless stream function
g :	gravitational acceleration
h :	local-heat-transfer coefficient
k :	new bed permeability
K :	new power-law constant
K_l :	power-law constant
k_m :	thermal conductivity of the fluid-saturated porous medium
l :	length of the plate
n :	power-law index
p :	pressure
Ra :	Rayleigh number
T :	temperature
q :	heat flux
u :	velocity in x-direction
v :	velocity in y-direction
x :	transverse coordinate
y :	longitudinal coordinate

Greek Symbols

α :	equivalent thermal diffusivity
β :	coefficient of thermal expansion
η :	dimensionless similarity variable
θ :	dimensionless temperature
ρ :	density
ψ :	stream function

Subscripts

o :	initial
w :	wall
∞ :	infinity

Superscripts

* :	dimensionless property
' :	derivative with respect to η

Introduction

There has been an increase in interest in the effect of porous media, because of their extensive practical applications in geophysics, thermal insulation in buildings, petroleum resources, packed-bed reactors and sensible heat-storage beds. Many studies related to non-Newtonian fluids saturated in a porous medium have been carried out. Dharmadhikari (1985) studied experimentally the effect of non-Newtonian fluids in a porous medium. Chen & Chen (1988) investigated the free convection flow along a vertical plate embedded in a porous medium. Rees (1996) analyzed the effect of inertia on free convection over a horizontal surface embedded in a porous medium. Nakayama (1991), investigated the effect of buoyancy-induced flow over a non-isothermal body of arbitrary shape in a fluid-saturated porous medium. A ray-tracing method for evaluating the radiative heat transfer in a porous medium was examined by Argento (1996). All of these studies were dealing with uniform averaged porosity. However, variation in the porous medium is a fact in most actual cases. A better description of the temperature distribution and velocity field could be produced, if variation in porosity were taken into consideration. Vafai et.al. (1985), carried out an experimental investigation into variable porosity, finding that the Nusselt number depends on the Reynolds number and the free-stream porosity. In the study presented below, the effect of variable porosity on the free convection flow, along a vertical plate embedded in a porous medium is investigated numerically. A configuration of the porosity variation for both increasing and decreasing porosity is assumed. A similarity solution is sought for the governing equations. Then the effect of variable porosity on the temperature distribution and Nusselt number in both cases (increasing and decreasing porosity) is stated.

Governing Equations

The problem, illustrated in Figure 1, represents a non-Newtonian power-law fluid flow along a constant temperature vertical plate embedded in a porous medium. The governing equations for this study can be written as follows (Chen 1988):

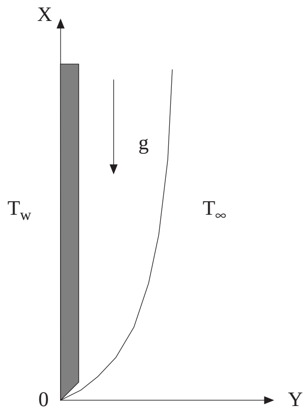

Figure 1. Physical model and its coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u^n = -\frac{k(n)}{K}\left[\frac{\partial p}{\partial x} + \rho g\right] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \tag{3}$$

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)] \tag{4}$$

the boundary conditions for these equations are:

$$v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \tag{5}$$

$$u = 0, \quad T = T_\infty \quad \text{at} \quad y \to \infty \tag{6}$$

where $K, n,$ and $k(n)$ are the power-law constant, power-law index, and the permeability of the porous medium, respectively. These are:

$$K = \frac{5n_1}{2 + 3n_1}\left(\frac{150}{32}\right)^{n_2} K_1 \tag{7}$$

$$n = n_1 + 0.3(1 - n_1) \tag{8}$$

$$n_2 = \left(\frac{3n_1}{2 + n_1}\right) \tag{9}$$

$$k(n) = \frac{2}{e}\left[\frac{De^2}{8(1-e)}\right]^{n-1}. \tag{10}$$

Outside the boundary layer, the flow of the power-law fluid remains stagnant. Thus

$$-\left(\frac{dp}{dx}\right) = \rho_\infty g. \tag{11}$$

To simplify the proceeding equations, the following dimensionless terms are introduced:

$$x^* = \frac{x}{\ell}, \qquad y^* = \frac{y}{\ell} \tag{12}$$

$$v^* = \frac{v}{\left[\rho_\infty \beta g k(n)\frac{(T_w - T_\infty)}{K}\right]^{\frac{1}{n}}}, \quad u^* = \frac{u}{\left[\rho_\infty \beta g k(n)\frac{(T_w - T_\infty)}{K}\right]^{\frac{1}{n}}} \tag{13}$$

$$Ra = \rho_\infty \beta g k(n)\frac{(T_w - T_\infty)}{K} l^n/(K\alpha^n), \quad Ra^* = \left[\rho_\infty \beta g k(n)\frac{(T_w - T_\infty)}{K} l^n/(K\alpha^n)\right]^{\frac{1}{n}} \tag{14}$$

$$\theta = \frac{(T - T_\infty)}{T_w - T_\infty)}. \tag{15}$$

Applying Eq. (11) to Eq. (2) and using the dimensionless terms, Eqs. (1-3) can be written as,

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{16}$$

$$(u^*)^n = \left(\frac{k(n)}{k_0}\right)\theta \tag{17}$$

$$u^*\frac{\partial \theta}{\partial x^*} + v^*\frac{\partial \theta}{\partial y^*} = \frac{1}{Ra^*}\frac{\partial^2 \theta}{\partial y^{*2}}. \tag{18}$$

To transform the above nonlinear partial differential equations into a set of ordinary differential equations the following dimensionless variables are defined:

$$\xi(x^*) = \left(\frac{x^*}{Ra^*}\right)^{\frac{1}{2}} \tag{19}$$

$$\eta = \left(\frac{y^*}{\xi(x^*)}\right) = y^* \left(\frac{Ra^*}{x^*}\right)^{\frac{1}{2}} \tag{20}$$

$$\psi = U(x^*)\xi(x^*)f(\eta) = \left(\frac{x^*}{Ra^*}\right)^{\frac{1}{2}} f(\eta) \tag{21}$$

by using the newly defined terms, the velocity components become

$$u^* = \frac{\partial\psi}{\partial y^*} = f'(\eta) \tag{22}$$

$$v^* = -\frac{\partial\psi}{\partial x^*} = \frac{1}{2}(f'\eta - f)/(Ra^* x^*)^{\frac{1}{2}} \tag{23}$$

whereas Eqs. (17-18) and their boundary conditions become

$$\theta = \left(\frac{k_0}{k(n)}\right) f'^n \tag{24}$$

$$\theta'' + \frac{1}{2}\theta' f = 0 \tag{25}$$

$$\theta = 1, \quad f = 0 \quad \text{at} \quad \eta = 0 \tag{26}$$

$$\theta = 0, \quad f' = 0 \quad \text{at} \quad \eta \to \infty \tag{27}$$

from Eq. (10),

$$k(n) = \frac{2}{e}\left[\frac{De^2}{8(1-e)}\right]^{n-1} \tag{28}$$

and

$$e = a[1 - b(1 - e^{-c\eta})] \tag{29}$$

where a, b and c are constants.

$$k_0 = \frac{2}{e_0}\left[\frac{De_0^2}{8(1-e_0)}\right]^{n-1}. \tag{30}$$

If e is constant then $k(n) = k_0 =$ constant and the original equations for constant porosity are obtained. The variation of the porosity is shown in four different cases in Figure 2.

The local heat flux at the wall is

$$q_w = -k_m\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{31}$$

$$= -k_m(T_w - T_\infty)\theta'(0)\frac{1}{l}\left(\frac{Ra^*}{x^*}\right)^{\frac{1}{2}} \tag{32}$$

where the local Nusselt number is

$$Nu_x = \frac{hx}{k_m} = \frac{q_w x}{k_m(T_w - T_\infty)}. \tag{33}$$

Substituting Eq. (33) into (32) gives

$$\frac{Nu_x}{\left(\frac{Ra^*}{x^*}\right)^{\frac{1}{2}}} = -\theta'(0). \tag{34}$$

Solution Procedure

Eqs. (1-3) and their boundary conditions have been non-dimensionalized, then solved using the similarity method. The resulting equations, namely (24-25) with the boundary conditions (26-27) are solved numerically using the fourth-order Runge-Kutta method, with step size $\Delta\eta = 0.1$ and $\eta_\infty = 9$.

Table 1. Dimensionless temperature distribution and dimensionless velocity distribution when $n = 1$.

	Successive approximation	Chen & Chen study	present study
η	$f' = \theta^{1\backslash n} = u^*$	$f' = \theta^{1\backslash n} = u^*$	$f' = \theta^{1/n} = u^*$
0.000	1.00000	1.00000	1.00000
1.000	0.58715	0.58722	0.58719
4.000	0.06624	0.06643	0.06631
7.000	0.00600	0.00624	0.00609

Results and Discussion

The results of Eqs. (24-27) using the Runge-Kutta method are shown in Figures 3-10, for Newtonian and non-Newtonian fluids in different variable porosity media. These results have been compared to previous studies and have been found to be in good agreement, as shown in Table 1. Figures 3-6 show the dimensionless temperature versus the similarity variable η for different porosity variations (e=0.3 to 0.1, 0.3 to 0.2, 0.3 to 0.4, and 0.3 to 0.5) and at the same time with a different power-law index n (0.5, 1.0, and 2.5). It is clear that as the porosity increases the temperature variation becomes steeper, which is explained by the increase in the porous media, and thus the increase in the heat transfer rate. Figures 7-9 show again the dimensionless temperature versus the similarity variable η, but for a variable power-law index n (0.5, 1.0, 2.5), each showing the four different porosity variations for the same power-law index. It is interesting to note that the lines of the dimensionless temperature approach each other as the porosity increases. This gives the impression that there is an optimum porosity for each power-law index. Figure 10 represents the values of $\theta' = (Nu/Ra^*x^*)^{1/2}$ versus power-law index, with different porosity variations. It may be stated clearly that as the porosity increases the ratio $(Nu/Ra^*x^*)^{1/2}$ increases. Figure 11 shows the variation of $(Nu/Ra^*x^*)^{1/2}$ with porosity. The ratio shows almost a linear increment as the porosity increases.

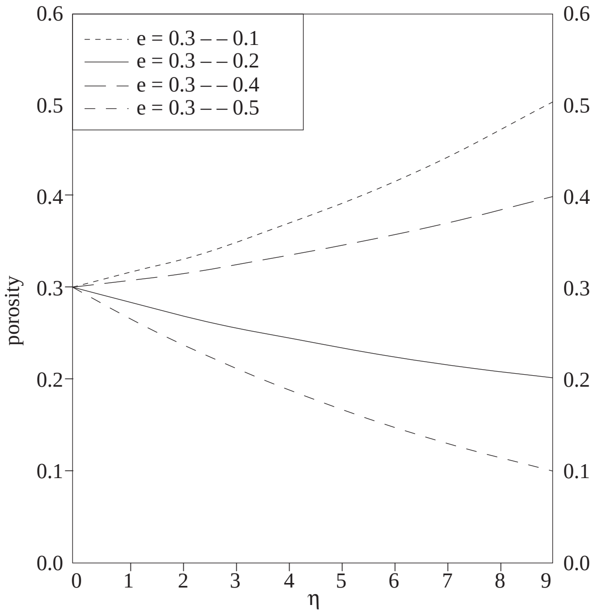

Figure 2. The variation of porosity, in four different cases e=0.3 to 0.1, 0.3 to 0.2, 0.3 to 0.4, 0.3 to 0.5.

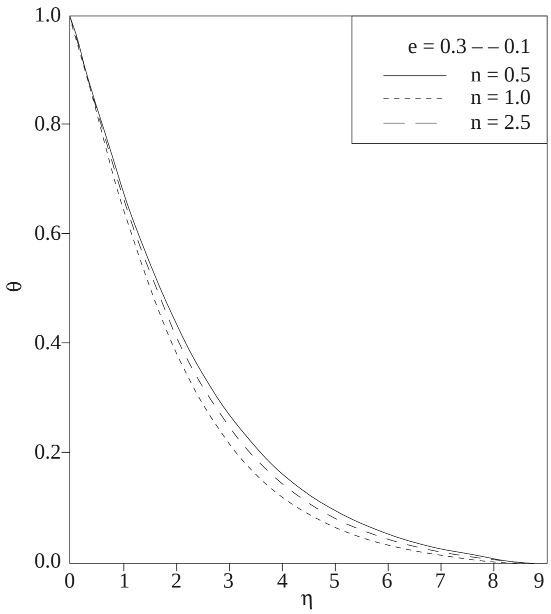

Figure 3. Temperature distribution along the plate in terms of the similarity variable.

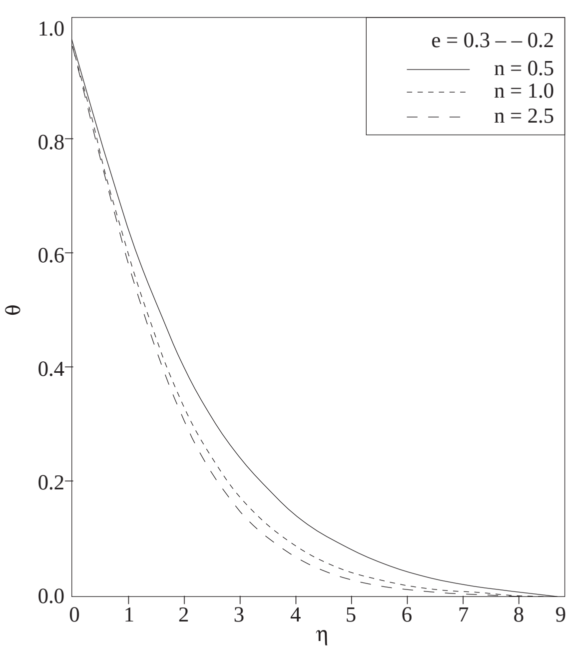

Figure 4. Temperature distribution along the plate in terms of the similarity variable.

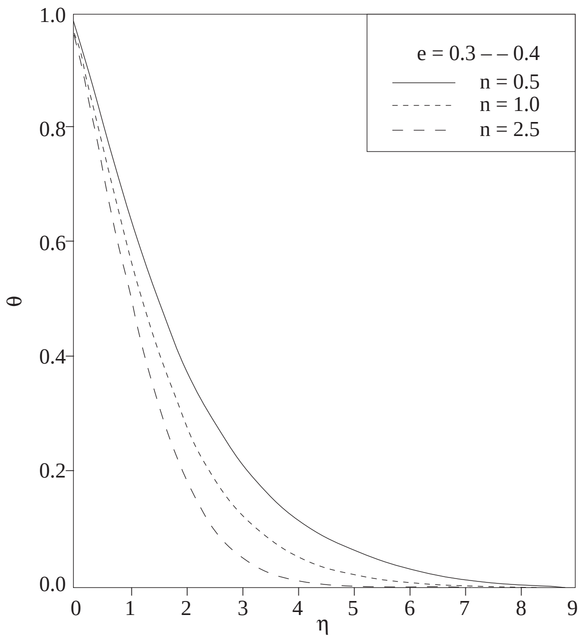

Figure 5. Temperature distribution along the plate in terms of the similarity variable.

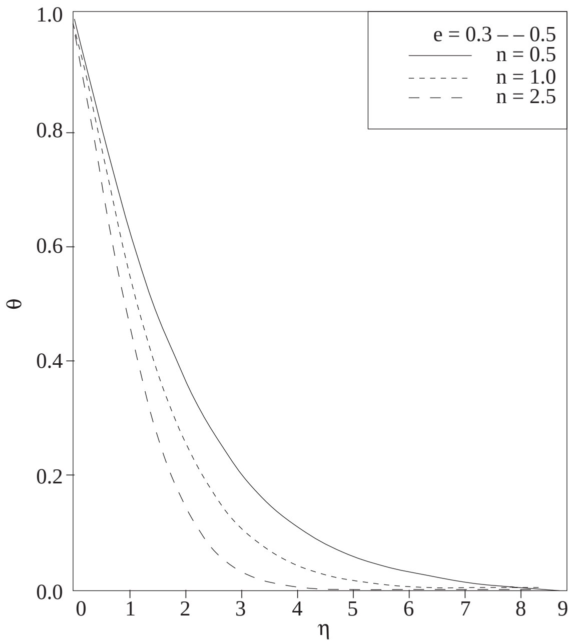

Figure 6. Temperature distribution along the plate in terms of the similarity variable.

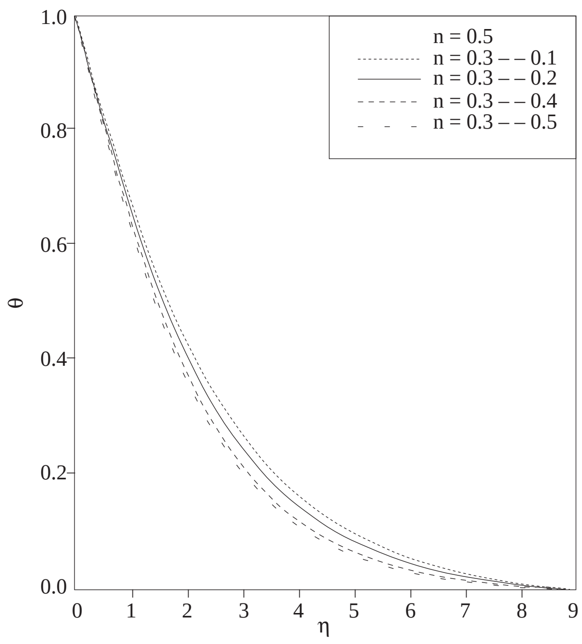

Figure 7. Temperature distribution along the plate in terms of the similarity variable.

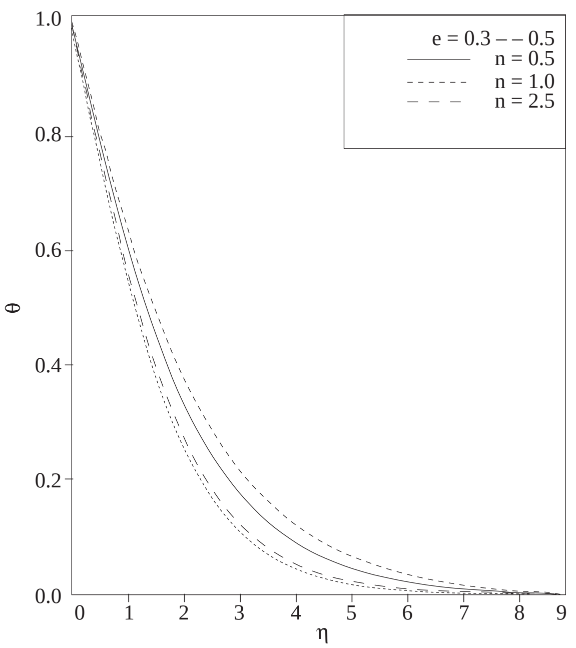

Figure 8. Temperature distribution along the plate in terms of the similarity variable.

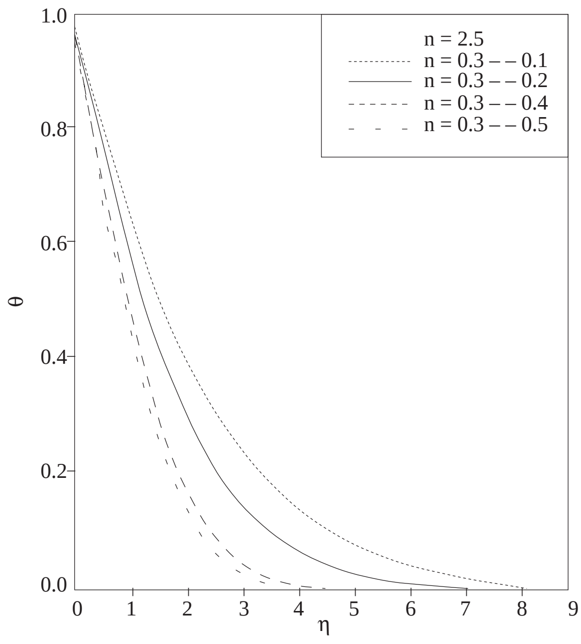

Figure 9. Temperature distribution along the plate in terms of the similarity variable.

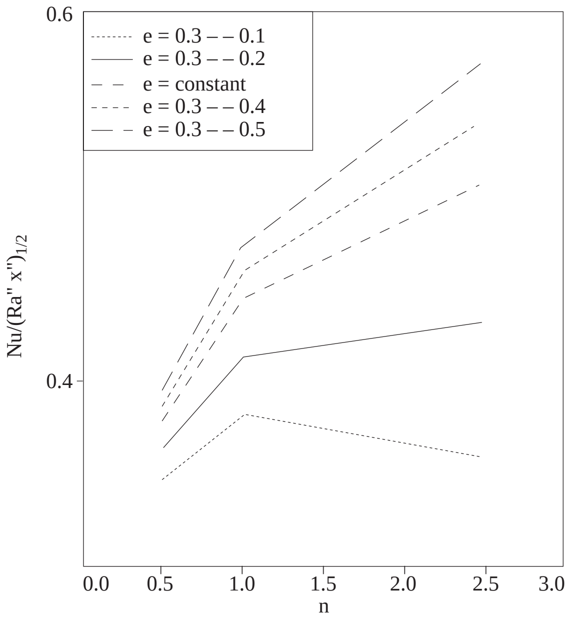

Figure 10. The value of θ^* versus n for variable porosity.

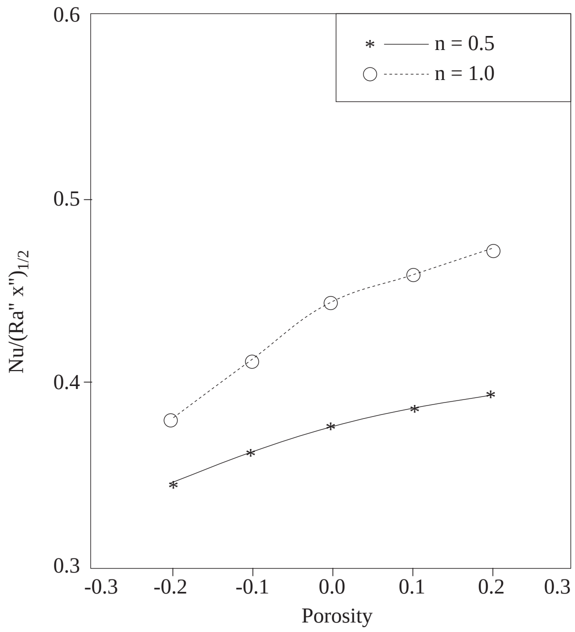

Figure 11. The variation of $Nu/(Ra''x'')^{1/2}$ with porosity.

Conclusions

In this study the effect of porosity on free convection flow along an isothermal vertical plate embedded in a porous medium was investigated. The result of this study was in good agreement with previous studies of Newtonian fluid, i.e., $n = 1$. The results show that as the porosity increases the temperature variation becomes steeper, that is, the heat transfer rate increases as expected, the Nusselt number increases almost linearly with increasing porosity, and increasing the porosity increases the Nusselt number, especially with non-Newtonian fluids with $n > 1$.

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