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A linear theory of physical properties in inhomogeneous sediments and its application to relative paleointensity determination

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1, 51–62, 2006

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Linear sediment theory and relative paleointensity

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Abstract

A linear model is developed to study the effect of variations in composition upon extensive physical properties of continuously deposited sediment sequences. By applying this model to natural and synthetic remanence acquisition, an optimal method of relative paleointensity determination is derived. The sediment is regarded as a mixture of independent components, each of which behaves uniformly in depth with respect to its physical properties. The concentration of each sediment component is assumed to independently vary linearly with an external "environmental" signal. Remanence acquisition in each sediment component is linear in external field and concentration of the component. It is demonstrated that in this case the ideal normalization procedure for relative paleointensity determination is to divide the natural remanent magnetization by a biased normalizer. Common magnetic cleaning techniques improve the relative paleointensity record by removing nonlinear behavior and by reducing the bias to the normalizer. The proposed linear sediment model for any extensive physical property clearly separates the influences of concentration of sediment components from those of environmental signals. It thus opens many possibilities for extensions to nonlinear models.

1 Introduction

Sedimentary sequences often provide densely spaced and accurately dated records of natural remanent magnetization (NRM). If the alternating field (AF) demagnetization of the NRM indicates that a single stable direction of the paleofield has been recorded throughout the sequence, a proxy record of the Earth's magnetic field intensity variation can be constructed. This is classically done by dividing the NRM at depth z by a normalization parameter (normalizer) $\nu(z)$ which is chosen such as to be proportional to the concentration of the remanence carriers at depth z. Common choices for $\nu(z)$ are anhysteretic remanent magnetization (ARM), isothermal remanent magne-

tization (IRM) or magnetic volume susceptibility (κ). Remanence parameters (ARM, IRM) are generally demagnetized by the same AF field as the NRM (Levi and Banerjee, 1976). The normalization procedure intends to remove the concentration dependence of the NRM and the remaining NRM variations are believed to mirror the pale-
⁵ ofield intensity. Therefore, the obtained normalized NRM record is denoted as *relative paleointensity* (RPI) record. The practical technique of measurement and interpretation of RPI records has been developed in a series of articles starting with Johnson et al. (1948) and continued among others in Harrison (1966); Harrison and Somayajulu (1966); Johnson et al. (1975); Levi and Banerjee (1976); King et al. (1983). Detailed
¹⁰ reviews of the development and current state of RPI techniques can be found in Tauxe (1993) and Valet (2003).

2 A linear model of sedimentary sequences

2.1 Sediment components

Sediment composition varies with depth. To quantify this variation in a linear mixing
¹⁵ model, the sediment is described as a collection of n disjoint sedimentary components P_i with $i=1,\dots,n$. While concentrations $c_i(z)\geq0$ of the components P_i change with depth z, the physical and especially magnetic properties of the components are assumed to be independent of z. The $c_i(z)$ are collected in a concentration vector $\boldsymbol{c}(z)\in\mathbb{R}^n$, where $\sum c_i\leq1$.
²⁰   For example, a component P_i may comprise a certain grain-size fraction of a magnetic mineral, e.g. all single-domain magnetite particles. Yet, it also can represent a complicated compound of matrix and magnetic minerals which has a well defined remanence acquisition behavior, e.g. slightly interacting 1 μm TM10 particles in foraminiferal ooze with 20% clay. Thus, the abstract notion of sediment components allows to deal
²⁵ with a wide range of physical properties using the same formalism. Sediments where natural remanence acquisition processes depend critically on clay or carbonate con-

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tent are modelled in the same way as sediments where only changes in grain-size or composition of the magnetic minerals are relevant for remanence acquisition.

The component grading can be arbitrarily arranged to account for nearly any factor which is important for remanence acquisition behavior.

5   The function $c(z)$ completely describes the variation of sediment composition with depth. Such a variation may have several reasons. The source region of the sediment can change, transport mechanisms may vary in efficiency, sorting effects due to wind or current velocity can modulate grain-sizes. Moreover, salinity or water temperature and biological activity can change important sediment properties like carbonate content,

10 clay mineralogy or coagulation extent. Chemical variations may also modify weathering conditions and early diagenesis.

2.2   Uni-causality and sediment linearity

Many of the above influences are not easily detected by standard geophysical or sedimentological methods. Yet, concentration variations of the components P_i due to

15 the above effects often are highly correlated: they all mirror the same environmental changes. At geologically and climatically "quiet" locations, it can be assumed that a predominant single environmental influence modulates *all* concentration changes. Therefore, the simplest *uni-causal* linear sediment model is obtained by assuming that $c(z)$ depends linearly on a single (environmental) master signal $s_1(z)$. For convenience,

20 $s_1(z)$ is chosen to have zero mean and standard deviation of 1. Thus, for constant vectors $\overline{c}$ and a the concentration vector is given by

$$c(z) = \overline{c} + a\, s_1(z). \tag{1}$$

For component P_i, the average concentration then is $\overline{c}_i$ and its amplitude with respect to $s_1(z)$ is a_i (see Fig. 1).

25 In more complex environments, it may be necessary to take into account several independent external influences. This can be obtained by using a *multi-causal* linear model with k different master signals $s_1(z), \ldots, s_k(z)$. In practical applications these

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master signals and their respective influence can be reconstructed by principal component analysis of the investigated physical properties. The concentration vector in a multi-causal model is

$$c(z) = \overline{c} + \mathbf{A}\,s(z),\tag{2}$$

where $\mathbf{A}$ is a $k \times n$ matrix and $s(z) \in \mathbb{R}^k$. Nonlinear response to a single master signal $s(z)$ is mathematically equivalent to a multi-causal model. One way to see this is to interpret the j-th component of $s(z)$ in Eq. (2) as $s^j(z)$ to obtain a k-th order polynomial dependence of $c(z)$ upon $s(z)$. Nonlinear response is especially important when in the studied sedimentary sequence relevant phases vanish completely for some time or reach a state of saturation. However, the following investigation will focus on the case of uni-causal linear response.

2.3 Linear sediment properties

A linear sediment property is a parameter $p(z)$ which depends linearly upon the concentration of the sediment components such that for some constant vector p,

$$p(z) = \langle p, c(z) \rangle = \sum_{i=1}^{n} p_i\, c_i(z).\tag{3}$$

A simple example is clay content. In this case, p_i denotes the volume fraction of clay within the i-th component. The same can be done for the volume fraction of any sediment constituent and any extensive physical property. Accordingly, many magnetic parameters are linear sediment properties. If $p_i = \kappa_i$ denotes volume susceptibility of the i-th component, then $\kappa(z) = \langle \kappa, c(z) \rangle$ is a linear sediment property if magnetostatic interaction between the components is negligible.

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3   Linear theory of relative paleointensity

3.1   An explicit model of NRM acquisition

The sediment model of the previous section is now applied to natural and synthetic remanence acquisition. This will help to understand the formation and reconstruction of relative paleointensity records and gives insight into their possible error sources.

To model NRM, it is assumed that each sediment component P_i has its specific DRM acquisition constant m_i representing magneto-mineral mobility, primary remanence intensity as well as relevant matrix properties. It is further assumed that the component acquires a DRM which depends linearly on both, field $H(t(z))$ and mechanical activation $A(z)$. The activation term in the following will be assumed to be constant $A(z){=}A_0$ over the whole sequence. Yet, it is kept in the formulae to emphasize the related uncertainty. In accordance with the above definitions the total NRM can be written as

$$
\begin{aligned}
\mathrm{NRM}(z) &= \sum_{i=1}^{n} \mathrm{NRM}_i(z) \\
&= A(z)\,H(t(z)) \sum_{i=1}^{n} m_i\,c_i(z) \\
&= A(z)\,H(t(z))\,\langle \boldsymbol{m}, \boldsymbol{c}(z) \rangle.
\end{aligned}
\tag{4}
$$

Thus, the susceptibility $\boldsymbol{m}(z){=}\langle \boldsymbol{m}, \boldsymbol{c}(z) \rangle$ of DRM acquisition with respect to external field and mechanical activation is a linear sediment property.

3.2   Synthetic remanences as linear sediment properties

In the same spirit it is assumed that each component P_i contributes with its own proportionality factor γ_i linearly to the normalizer $v(z)$. Accordingly, $v(z)$ also is a linear sediment property

$$
v(z) = \langle \boldsymbol{\gamma}, \boldsymbol{c}(z) \rangle.
\tag{5}
$$

By choosing a sufficiently fine grading of components, the linear model allows for arbitrary and independent grain-size dependence of both, DRM and normalizer. Therefore, considerable shift in lithology, mobility or grain-size of the magnetic fraction can be present without corrupting the following conclusions.

3.3 Reconstruction of relative paleointensity

Under the above premises, $\text{NRM}(z)$ and $v(z)$ can be calculated independently yielding

$$\text{NRM}(z) = A(z)\, H(t(z))\, \left(\langle \boldsymbol{m}, \overline{\boldsymbol{c}} \rangle + s_1(z) \langle \boldsymbol{m}, \boldsymbol{a} \rangle \right) \tag{6}$$

and similarly

$$v(z) = \langle \boldsymbol{\gamma}, \overline{\boldsymbol{c}} \rangle + s_1(z) \langle \boldsymbol{\gamma}, \boldsymbol{a} \rangle. \tag{7}$$

The latter equation can be solved for $s_1(z)$, if $\langle \boldsymbol{\gamma}, \boldsymbol{a} \rangle \neq 0$. By substituting the result into Eq. (6) one obtains

$$\text{NRM}(z) = A(z)\, H(t(z))\, k_v\, (\overline{v} + v(z)), \tag{8}$$

where

$$k_v = \langle \boldsymbol{m}, \boldsymbol{a} \rangle / \langle \boldsymbol{\gamma}, \boldsymbol{a} \rangle, \tag{9}$$

and

$$\overline{v} = k_v^{-1} \langle \boldsymbol{m}, \overline{\boldsymbol{c}} \rangle - \langle \boldsymbol{\gamma}, \overline{\boldsymbol{c}} \rangle, \quad \text{when } k_v \neq 0. \tag{10}$$

It follows that not even in the linear first order multi-component model the "normalized" NRM is necessarily proportional to the external field. If $\overline{v} \neq 0$ it still depends on $v(z)$:

$$\text{NRM}(z)/v(z) \;=\; A(z)\, H(t(z))\, k_v \left(1 + \frac{\overline{v}}{v(z)}\right). \tag{11}$$

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The correct normalization has to take into account the constant offset $\bar{v}$ to the normalizer and is given by

$$\frac{\mathrm{NRM}(z)}{v(z) + \bar{v}} = k_v \, A(z) \, H(t(z)). \tag{12}$$

For constant $A(z)$ and known $\bar{v}$, Eq. (12) can be used to recover the paleofield.

Although more complicated than a simple division of NRM by the normalizer, Eq. (12) is remarkable because it doesn't require the normalizer to mirror the DRM acquisition with respect to grain-size as is requested by the current RPI paradigm for homogeneous sediments. Also the NRM is allowed to be carried by a complex mixture of phases, each responding individually to external field and mechanical activation. In this respect, requiring the validity of the linear sediment model is a much weaker restriction upon the sediment record than the rigid homogeneity requirements of the current RPI paradigm.

3.4 Validity of the classical normalization procedure

The above formulation allows to state the necessary conditions for the validity of the classical RPI normalization procedure. The NRM(z) actually is only proportional to $v(z)$ if $\bar{v}$=0 in Eq. (10), which is equivalent to

$$\langle \boldsymbol{m}, \boldsymbol{a} \rangle \, \langle \boldsymbol{\gamma}, \overline{\boldsymbol{c}} \rangle = \langle \boldsymbol{\gamma}, \boldsymbol{a} \rangle \langle \boldsymbol{m}, \overline{\boldsymbol{c}} \rangle. \tag{13}$$

There are two independent possibilities for the validity of Eq. (13). The first is that $\boldsymbol{m} = \kappa\,\boldsymbol{\gamma}$ for some $\kappa \in \mathbb{R}$, which corresponds to the physical condition that each phase must contribute in exactly the same way to both, normalizer v and NRM. This is related to the request of Levi and Banerjee (1976) that the normalizer must activate the same spectrum of magnetic particles which is responsible for NRM, an argument which has been further elaborated by Amerigian (1977) and King et al. (1983). A proposed test for this condition is to check for similarity of the demagnetization curves of NRM and

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normalizing remanence in function of sediment depth (Levi and Banerjee, 1976). In case of perfect parallelism both demagnetization curves may vary with depth but in exactly the same way.

A second possibility is that $\overline{c} = \lambda\,a$ for some $\lambda \in \mathbb{R}$. The physical interpretation of this condition is that the relative composition of the sediment doesn't change with depth. All concentration changes of the sediment phases have to be perfectly parallel. Shifts in grain size or composition must not occur. This is the request of perfect lithological homogeneity which can be tested by comparing the downcore variation of the shape of ARM demagnetization curves (Levi and Banerjee, 1976) or more general by checking whether all concentration independent magnetic properties (shape of any isothermal magnetization loop, quantities like H_{cr} or H_c) are invariant with depth.

Mathematically, both possibilities are independent. One of them alone suffices to guarantee the validity of the classical RPI normalization procedure. Therefore, a perfect normalizer should yield a good RPI record even for a less homogeneous sedimentary sequence, whilst even a mediocre normalizer should work well in a perfectly homogeneous lithology.

Of course Eq. (13) can be fulfilled by other combinations of $\boldsymbol{m}, \boldsymbol{\gamma}, \overline{\boldsymbol{c}}, \boldsymbol{a}$, but these are unlikely to occur by chance.

Numerical modeling indicates that even if a linear sediment has a small non-zero bias $\overline{\nu}$ in Eq. (12), the classically inferred RPI record deviates only slightly from the correct field variation. In this case a better RPI estimate may be obtained by choosing an approximation of $\overline{\nu}$ which minimizes the average amplitude of the normalized record. This typically leads to only a minor amplitude reduction in comparison to the classically normalized record and may explain why the latter in inhomogeneous sediments often works by far more better than should be expected from considering the rigid prerequisites which are usually demanded for its validity (Haag, 2000).

The above discussion relies on the linear uni-causal sediment model and therefore disregards highly nonlinear effects like VRM or diagenetic overprint. Perhaps the main outcome of this study is, that linearity and uni-causality already imply a sufficient reg-

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ularity of the sedimentary sequence to make it useful for RPI reconstruction. Quality checks therefore should focus on testing to which degree the assumptions of uni-causality and linearity are fulfilled in a given sediment core and need not concentrate too much on idealistic homogeneity requests. Especially the choice of magnetic cleaning techniques should also be interpreted as a means to restrict RPI determination to linearly behaving sediment fractions.

4 Conclusions

1. The presented linear sediment model describes a stratigraphic sequence of an undisturbed sediment as a weighed sum of independent sediment components. The depth variation of the respective concentrations depends on the main environmental signals. It is thus possible to logically separate between varying physical properties of the components and environmental variation. Here only the most simple case of linear variations and dependence upon a single signal is elaborated, but an extension to more complex behavior is conceptional straightforward.

2. Applying the model to the problem of relative paleointensity determination leads to a simple theory for linear inhomogeneous sediments which by and large coincides with the commonly used normalization procedures. However, the presented theory interprets the role of the normalization parameters in a new way: they serve to remove a linear environmental signal common to all concentration variations. The linear theory thereby introduces the principal necessity of a constant bias to any used normalization parameter and relates zero bias to sediment homogeneity or to a perfect coincidence of activation between NRM and normalizer. Although both conditions are rarely fulfilled in real sediments, a small bias doesn't change the inferred RPI signal noticeably. It is therefore concluded that any linear uni-causal sediment is a good RPI recorder.

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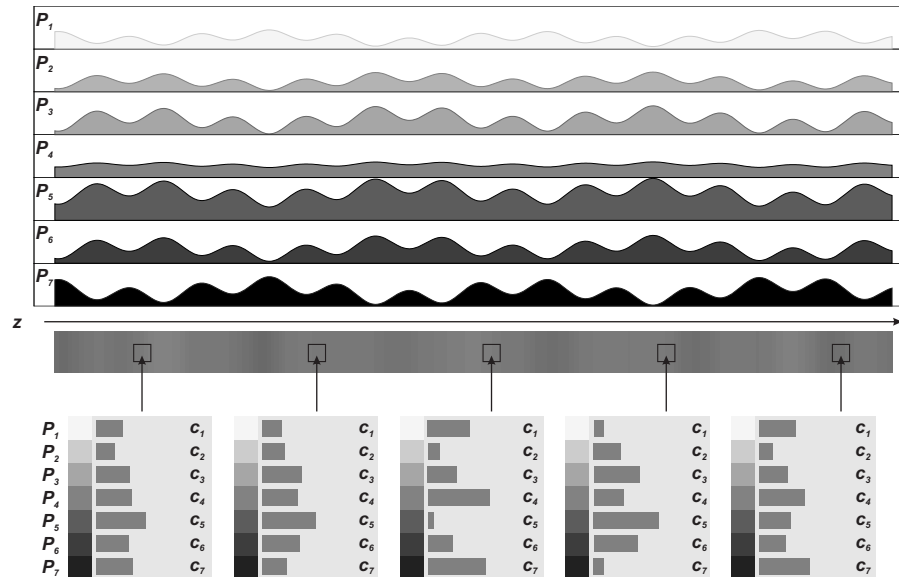Tables | Figures

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Fig. 1. Linear sediment model with seven phases $P_1, \dots, P_7$. At each depth z the sediment is a mixture of these phases in varying concentrations $c_1(z), \dots, c_7(z)$. If a single climate signal $s_1(z)$ controls all sediment phases, each concentration is an affine combination $c_i(z) = \overline{c}_i + a_i\, s_1(z)$.