Generation of *n*-Atom GHZ State via Two-sided Cavity QED*

LÜ Yuan-yuan, WANG Fa-qiang†, JIN Rui-bo, YANG Hao, LIANG Rui-sheng

(*Laboratory of Photonic Information Technology, School of Information and Optoelectronic Science and Engineering, South China Normal University, Guangzhou* 510006, *China*)

Abstract: The scheme was proposed to generate multiple-atom GHZ state via the state-of-the-art two-sided cavities. A single-photon pulse could not only be reflected but also be transmitted through the two-sided cavity with a single trapped atom, which is in certain state. Entanglement between the trapped atom and the input field was resulted in this property. The numerical simulations showed that the produced multiple- particle GHZ state had high fidelity and success probability. The intrinsic noise, such as the atomic spontaneous emission, only led to the error probability and had no influence on the fidelity. In addition, the high-Q cavity and the Lamb-Dicke condition of atom were not required, which expanded the possibility of experimental realization.

Key words: Quantum entanglement; GHZ state; Two-sided cavity

CLCN: O431 **Code Document**: A **Article ID**: 1004-4213(2009)10-2682-5

0 Introduction

Quantum entanglement plays an important role in quantum information[1-3] and quantum computation[4]. Greenberger-Horne-Zeilinger (GHZ) state, which is one of the most important kinds of multipartite entangled states, was firstly proposed by Greenberger, Horne and Zeilinger to test quantum mechanics versus local realism[5]. The preparation[6-10] of entangled states has become an attractive research subject nowadays. Ref. [8] gave an experimental generation system of quadripartite entangled states for continuous variables. Ref. [9] show that multi-photon W state was generated from EPR pairs via measurement and follow-up local operation. There have been many protocols to prepare multi-particle GHZ state in many physical systems[11-16].

Among them, the cavity QED system is an ideal system to generate entangled states[17]. Various studies have been carried out on the field. Zou et al[18]. proposed a scheme for generating GHZ state of many distant atoms trapped in cavities, while the fidelity of the generated state is largely affected by the inefficiency of photon detector. Later, Zou et al[19]. presented an improved scheme via multifold coincident detection. Although the scheme is insensitive to quantum noise, high-Q cavity is required. Man et al[20]. gave schemes for generation of Bell and W states in cavity, in which no measurements on the particles were required in the process. Recently, several schemes to get the entanglement states are based on single-sided cavity input-output process[21-22]. In this paper, we demonstrate a scheme to generate multiple-atom GHZ state via the state-of-the-art two-sided cavity[23-24]. When a field is successfully injected into the two-sided cavity, the input field could be reflected or transmitted by the two-sided cavity which is different from the single-sided cavity that just only reflects the input photon pulse. This can result in entanglement between the trapped atom and input field. Using the property, we can prepare multiple-atom GHZ state. In addition, the numerical simulations show the high fidelity and successful probability for generating a multiple-atom GHZ state. The intrinsic noise only decreases the successful probability and do not influence the fidelity of generated state. In particular, either the Lamb-Dicke condition of the trapped atom or the good cavity is required which keeps the scheme easy to be implemented in experiment.

1 The fundamental model and analysis

The basic building model involved in scheme is shown in Fig. 1. The relevant atomic levels and transitions are shown in Fig. 2. The states $|0\rangle$ and $|1\rangle$ are two ground levels of the atom, and $|e\rangle$ is an excited level. The transition $|0\rangle \rightarrow |e\rangle$ is assumed resonantly coupled to the cavity mode,

* Supported by the National Basic Research Programme of China (2007CB307001)

† Tel: 13631434379 Email: fqwang@scnu.edu.cn

Received date: 2008-11-28 Revised date: 2009-01-14

denoted w_0, which is resonantly driven by the single-photon pulse injected into the cavity.

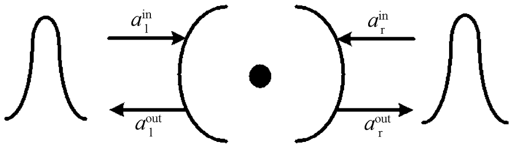


Fig. 1 Schematic setup to reflect or transmit single-photon pulse from two-sided cavity with a trapped atom

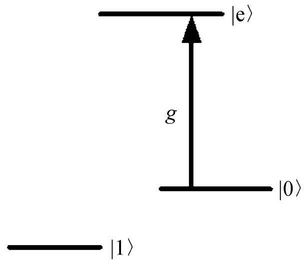


Fig. 2 Configuration of relevant atomic levels

The cavity mode a is driven by the input fields a_l^{in} and a_r^{in} from both the left and right sides of the cavity. The Heisenberg equations of motion for the cavity field operator a and atom operator σ_-, given by[23-25]

$$da/dt = -ig\sigma_- - (i\omega_0 + k)a + \sqrt{k}(a_l^{in} + a_r^{in}) \quad (1)$$

$$d\sigma_{-n}/dt = ig\sigma_z a - (i\omega_0 + \gamma/2)\sigma_- + \sqrt{\gamma}\sigma_z \hat{N} \quad (2)$$

where k describes the cavity energy decay rate, g is atom-cavity coupling rate, the parameter γ denotes the spontaneous emission rate of the atomic level $|e\rangle$, $\sigma^z \equiv |e\rangle\langle e| - |0\rangle\langle 0|$, and $\hat{N}$ is the vacuum noise operator which preserves the commutation relation. Assume $\langle \sigma^z(t) \rangle \approx 1$ in the weak excitation limit.

The output fields a_μ^{out} ($\mu = l, r$) are related to the input fields by

$$a_\mu^{out} + a_\mu^{in} = \sqrt{k} a_\mu \quad (3)$$

They satisfy the following commutation relation $[a_\mu^{in}(t), a_\mu^{in+}(t')] = [a_\mu^{out}(t), a_\mu^{out+}(t')] = \delta_{\mu\mu'}\delta(t-t')$. Taking the Fourier transforms, the waveguide output can be solved and given by the expression

$$a_\mu^{out} = R(\omega) a_\mu^{in} + T(\omega) a_{\bar{\mu}}^{in} \quad (4)$$

Where

$\{\mu, \bar{\mu}\} \equiv \{l, r\}$ or $\{r, l\}$,

$$R(\omega) = \frac{i\omega + g^2/(i\omega - \gamma/2)}{k - i\omega - g^2/(i\omega - \gamma/2)},$$

$$T(\omega) = \frac{k}{k - i\omega - g^2/(i\omega - \gamma/2)},$$

and ω measures the frequency detuning of the bare cavity mode.

Consider the case that the pulse bandwidth δ (the range of ω) $\ll k, g^2/k$. When the atom in the state $|1\rangle$, the field is resonant with the cavity and $g=0$, so we have $a_\mu^{out} = a_{\bar{\mu}}^{in}$. In the opposite regime, when the noise satisfies the condition $2g^2/\gamma \gg k$, we can get $a_\mu^{out} = -a_\mu^{in}$, and we can also get the condition under $\gamma \ll k$ in weak regime of smaller value of g.

2 Generation of *n*- Atom GHZ State

Now we send the single-photon pulse $|\alpha\rangle$ down to one cavity, and the atom is assumed to be initially in the state $(|0\rangle + |1\rangle)$. In the ideal case, we can get

$$(|0\rangle + |1\rangle)|\alpha\rangle_\mu \to -|0\rangle|\alpha\rangle_\mu + |1\rangle|\alpha\rangle_{\bar{\mu}} \quad (5)$$

That is to say, the pulse will be kept in the same side of the cavity if the atom is the state $|0\rangle$ or be flipped to the other side if the atom is in the state $|1\rangle$.

In Fig. 3, the optical paths from $a_l^{in} \to D_1$ and

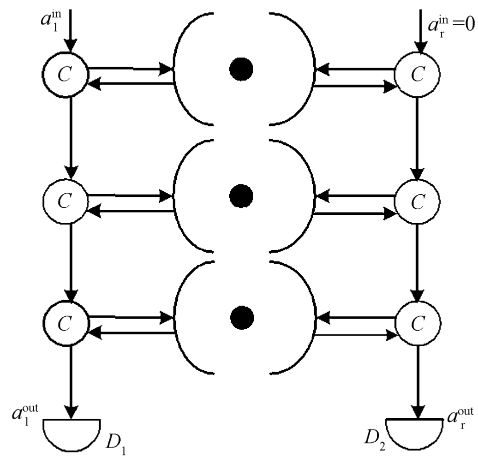


Fig. 3 Schematic setup for generation of three-atom GHZ state

from $a_l^{in} \to D_2$ are assumed to be equal. The single pulse $|\alpha\rangle$ is sent down to the left side of the cavity, and the other side has no input pulse. The three atoms are initially in the state

$$|\Phi_0\rangle_{123} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_1(|0\rangle + |1\rangle)_2 \cdot (|0\rangle + |1\rangle)_3 \quad (6)$$

The total process can be depicted as follow

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_1(|0\rangle + |1\rangle)_2(|0\rangle + |1\rangle)_3|\alpha\rangle_l \xrightarrow{\text{cavity}1,2,3} -\frac{1}{\sqrt{2}}(|000\rangle_{123} + |011\rangle_{123} + |101\rangle_{123} + |110\rangle_{123})|\alpha\rangle_l + \frac{1}{\sqrt{2}}(|001\rangle_{123} + |010\rangle_{123} + |100\rangle_{123} + |111\rangle_{123})|\alpha\rangle_r \quad (7)$$

If the left detector clicks, the state of the system of atom 1,2,3 is projected into

$$|\Phi\rangle = \frac{1}{2}(|000\rangle + |011\rangle + |101\rangle + |110\rangle)_{123} \quad (8)$$

If the right detector clicks, it is projected into

$$|\Phi_2\rangle = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)_{123} \quad (9)$$

We perform the Hadamard operation on each atom via Raman pulses that transforms $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$ or $|1\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$. After normalization, the states $|\Phi_1\rangle$ and $|\Phi_2\rangle$ become

$$|\Phi_3\rangle=\frac{1}{\sqrt{2}}(|000\rangle-|111\rangle)_{123} \tag{10}$$

$$|\Phi_4\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{123} \tag{11}$$

Obviously, both states $|\Phi_3\rangle$ and $|\Phi_4\rangle$ are three-atom GHZ states.

Now, we can extend the scheme to prepare n-atom GHZ state with n cavities with trapped atoms in Fig. 3, and each atom in cavities is initially in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$. In the similar way, a single pulse passes through the cavities. After the left and right detectors click, we perform the Hadamard operation on each atom. Then we can get

$$|\Phi_5\rangle=\frac{1}{\sqrt{2}}(|0\cdots\cdots0\rangle-|1\cdots\cdots1\rangle)_{1\cdots\cdots n} \tag{12}$$

$$|\Phi_6\rangle=\frac{1}{\sqrt{2}}(|0\cdots\cdots0\rangle+|1\cdots\cdots1\rangle)_{1\cdots\cdots n} \tag{13}$$

So n-atom GHZ states are prepared.

3 Simulation and discussion

In the following, we quantify the quality of the three-atom GHZ state through the numerical simulation method. Assuming that the input single-photon pulse is taken to be a Gaussian pulse $f(\omega)=\exp(-\omega^2/\delta^2)/(\sqrt{\pi}\delta)$. The fidelity $F=|\langle\Psi_{\mathrm{ideal}}|\Psi_{\mathrm{real}}\rangle|^2$, where $|\Psi_{\mathrm{ideal}}\rangle$ refers to the state of the atomic system in the ideal case; $|\Psi_{\mathrm{real}}\rangle$ is the final state tracing over the noise. The fidelity can be written finally as $F=\frac{1}{64}|\xi_0^3+3\xi_0^2\xi_1+3\xi_0\xi_1^2+\xi_1^3|^2$, where

$$\xi_0=-\frac{\int_{-\infty}^{\infty}\mathrm{d}\omega|f(\omega)|^2R(\omega)}{\sqrt{r}\sqrt{\int_{-\infty}^{\infty}\mathrm{d}\omega|f(\omega)R(\omega)|^2}},$$

$$\xi_1=\frac{\int_{-\infty}^{\infty}\mathrm{d}\omega|f(\omega)|^2T(\omega)}{\sqrt{r}\sqrt{\int_{-\infty}^{\infty}\mathrm{d}\omega|f(\omega)T(\omega)|^2}},$$

$r=\int_{-\infty}^{\infty}\mathrm{d}\omega|f(\omega)|^2$. We take the practical parameter in Ref. [26], i.e. $g=6k$, $(k,r)/2\pi=(4.2, 2.6)$ MHz. The results show that the fidelity is very high, and is up to 0.996 4. Fig. 4 shows the fidelity F associated with g/k.

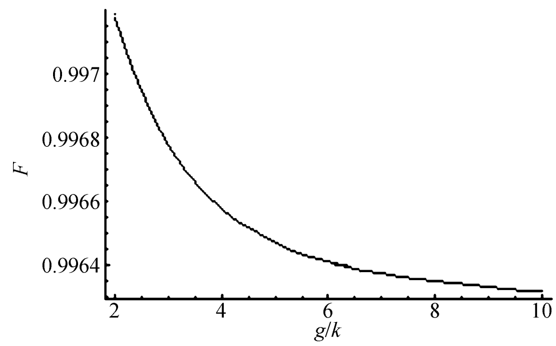


Fig. 4 Fidelity F as a function of g/k with $\delta=0.05k$

From the simulation, we can see the change of F is very small as long as the variation of g, which means the produced GHZ state keeps high fidelity even when the atom is brought out of the Lamb-Dicke regime. The dominate noise in our scheme is photon loss, which is especially aroused from atomic spontaneous emission. The photon loss can be calculated by $P_{\mathrm{loss}}=1-\langle\Psi_{\mathrm{real}}|\Psi_{\mathrm{real}}\rangle$. Fig. 5 shows P of the success probability for generation three-particle GHZ state as a function of δ/k, which is simulated by the formula $P=\frac{1}{32}[(\xi_0^3+3\xi_0\xi_1^2)^2+(\xi_1^3+3\xi_0^2\xi_1)^2]$. The photon loss caused by intrinsic noises only affects the success probability but has no influence on the fidelity of the state since the single-photon detection will never occur when the photon is lost. Moreover, we do not need the high-Q cavity either. The previous condition $2g^2/\gamma\gg k$ can be satisfied just when $\gamma\ll k$ in the solid-state cavity[18], which makes the process easier to be implemented in actual experiment.

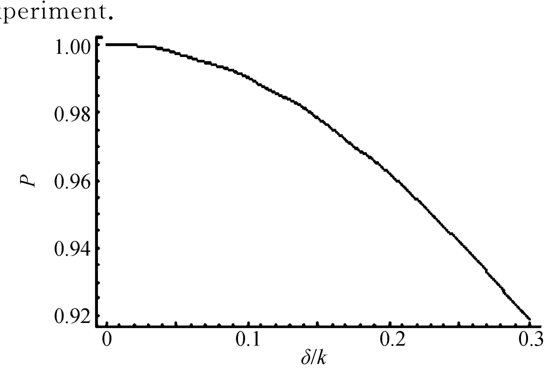


Fig. 5 Success probability P as a function of the scaled bandwidth δ/k with $g=6k$ and $k\approx\gamma$ taken for simplicity

4 Conclusion

In summary, a new scheme to generate multiple-atom GHZ state is proposed. In numerical simulations, the scheme has high fidelity even if the Lamb-Dick condition of the trapped atom is not required. The intrinsic noise, such as the atomic spontaneous emission, only reduces the success probability and has no effect to the fidelity. In the ideal case, the success probability of our protocol can approach unity. In particular, the high-Q cavity also is not needed, which means the scheme is better for experimental realization.

References

[1] BENNETT C H, BRASSARD G, CREPEAU C, *et al*. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels[J]. *Phys Rev Lett*, 1993, **70**(13):1895-1899.

[2] LIU X S, LONG G L, TONG D M, *et al*. General scheme for superdense coding between multiparties[J]. *Phys Rev A*, 2002,**65**(2):022304.

[3] EKERT A K. Quantum cryptography based on Bell's theorem [J]. *Phys Rev Lett*, 1991, **67**(6): 661-663.

[4] DING Sheng-chao, JIN Zhi. Review on the study of entanglement in quantum computation speedup[J]. *Chinese Science Bulletin*, 2007, **52**(16): 2161-2166.

[5] GREENBERGER D M, HORNE M A, ZEILINGER A. Bell's theorem without inequalities[J]. *Am J Phys*, 1990, **58**(12): 1131-1143.

[6] DAVIDOCICH L, ZAGURY N, BRUNE M, *et al*. Teleportation of an atomic state between two cavities using nonlocal microwave fields[J]. *Phys Rev A*, 1994, **50**(2): R895-R898.

[7] GUO Guang-can, ZHANG Yong-sheng. Scheme for preparation of the W state via cavity quantum electrodynamics [J]. *Phys Rev A*, 2002, **65**(5):054302.

[8] SU Xiao-long, JIA Xiao-jun, XIE Chang-de, *et al*. Generation of GHZ-like and cluster-like quadripartite entangled states for continuous variable using a set of quadrature squeezed states [J]. *Science in China Series G: Physics Mechanics& Astronomy*, 2008, **51**(1): 1-13.

[9] XU Wen-hu, ZHAO Xin, LONG Gui-lu. Efficient generation of multi-photon W states by joint-measurement[J]. *Progress in Natural Science*, 2008, **18**(1): 119-122.

[10] CHEN Mei-feng, MA Song-she. Generation of W-type entangled coherent states of three-cavity field by raman interaction[J]. *Acta Photonica Sinica*, 2007, **36**(5): 950-954.

[11] BOUWMEESTER D, PAN Jian-wei, DANIELL M, *et al*. Observation of three-photon greenberger- horne- zeilinger entanglement[J]. *Phys Rev Lett*,1999, **82**(7):1345-1349.

[12] ZHAO Zhi, YANG Tao, CHEN Yu-ao, *et al*. Experimental violation of local realism by four- photon greenberger-horne-zeilinger entanglement[J]. *Phys Rev Lett*, 2003, **91**(18): 180401.

[13] RAUSCHENBEUTRL A, NOGUES G., OSNAGHI S, *et al*. Step-by-step engineered multiparticle entanglement[J]. *Science*, 2000, **288**(5473):2024-2028.

[14] CIRAC J I, ZOLLER P. Preparation of macroscopic superpositions in many-atom systems[J]. *Phys Rev A*, 1994, **50**(4):R2799-R2802.

[15] LEIBFRIED D, BARRETT M D, SCHAETZ T, *et al*. Toward heisenberg-limited spectroscopy with multiparticle entangled states[J]. *Science*, 2004, **304**(5676):1476-1478.

[16] ZHENG Shi-Biao. One-step synthesis of multiatom greenberger-horne-zeilinger states[J]. *Phys Rev Lett*, 2001, **87**(23): 230404.

[17] RAIMOND J M, BRUNE M, HAROCHE S. Manipulating quantum entanglement with atoms and photons in a cavity [J]. *Rev Mod Phys*, 2001, **73**(3):565-582.

[18] ZOU Xu-bo, PAHLKE K, MATHIS W. Conditional generation of the greenberger-horne-zeilinger state of four distant atoms via cavity decay[J]. *Phys Rev A*, 2003, **68**(2):024302.

[19] ZOU Xu-bo, PAHLKE K, MATHIS W. Conditional generation of quantum entanglement of many distant atoms via multifold coincidence detection[J]. *Phys Rev A*, 2004, **69**(1):013811.

[20] MAN Zhong-xiao, SU Fang, XIA Yun-jie. Efficient generation of Bell and W-type states in cavity QED[J]. *Chinese Science Bulletin*, 2008, **53**(15):2410-2413.

[21] CHO J, LEE H W. Generation of atomic cluster states through the cavity input-output process[J]. *Phys Rev Lett*, 2005, **95**(16):160501.

[22] LIN Xiu-min, XUE Peng, CHEN Mei-ying, *et al*. Scalable preparation of multiple-particle entangled states via the cavity input-output process[J]. *Phys Rev A*, 2006, **74**(5): 052339..

[23] WAKS E, VUCKOVIC J. Dipole induced transparency in drop-filter cavity-waveguide systems[J]. *Phys Rev Lett*, 2006, **96**(15):153601.

[24] WANG B, DUAN L M. Implementation scheme of controlled SWAP gates for quantum fingerprinting and photonic quantum computation[J]. *Phys Rev A*, 2007, **75**(5): 050304.

[25] WALLS, MILBURN G. Quantum optics[M]. 2nd ed. Berlin: Springer Verlag,1994: 121-127.

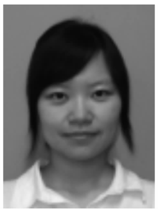
[26] MCKEEVER J, BUCK J R, BOOZER A D, *et al*. Determination of the number of atoms trapped in an optical cavity[J]. *Phys Rev Lett*, 2004, **93**(14):143601.

利用双面腔制备 *n* 原子 GHZ 态

吕园园，王发强，金锐博，杨昊，梁瑞生
（华南师范大学 信息光电子科技学院 光子信息技术实验室，广州 510006）

摘 要：提出了一种利用双面腔制备多原子 GHZ 态的方法.当腔中囚禁原子处于特定态时，腔可能反射入射的单光子脉冲，也可能透射它.这个特性可以引起囚禁原子和输入腔肠的纠缠.数值模拟显示制备的多原子 GHZ 态具有很高的保真度和成功率.而且原子自发辐射等内禀噪声只对成功率有影响，而对保真度几乎没有影响.另外，对高 Q 腔和原子的 L-D 条件的不要求，提升了试验实现的可行性.

关键词：量子纠缠；GHZ 态；双面腔



LÜ Yuan-yuan was born in 1983. Now she is pursuing her M. S. degree at School of Information and Optoelectronic Science and Engineering, South China Normal University. Her research interests focus on quantum optics.