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A Recession-Forecast Model for the Blue Nile River

A. W. Abdelhadi and Takeshi Hata
Graduate School of Science and Technology, Kobe Univ., Japan

O. E. Hamad
Ministry of Irrigation & Water Rs., Khartoum, Sudan

An analytical model for the forecasting of the Blue Nile River low flow is developed and compared with the current modified similar year method. The model is based on the general non-linear reservoir equation and the historical flow of the river for the calibration period 1912-1961. The comparison is made after the year 1961 for 6 different years in terms of the temporal 10-day flows and for the period 1962-1996 in terms of the standard error of estimates (*SEE*). The model and the modified similar year forecasted 10-day flows were also compared with the actual temporal 10-day flows during the driest and the wettest year of the whole record between 1912-1998. Results showed that the model temporal distribution of flows is more close to the actual ones compared with the modified similar year method. The model has less *SEE* in 31 years out of 38 compared with the modified similar year method.

Introduction

Over the last 50 years Sudan has witnessed a number of droughts. Mohamed (1998) examined the yearly drop in rainfall over six meteorological stations for the last 40 years. He concluded that the yearly decline in rainfall along the Blue Nile within Sudan ranges between 4 and 5 mm per year. The Blue Nile is considered as the live artery of irrigated agriculture in Sudan. The suitability of the soil, topography and water availability led to the concentration of irrigation development in the central region of the country around the Blue Nile. Currently some 1.3 million ha are irrigat-

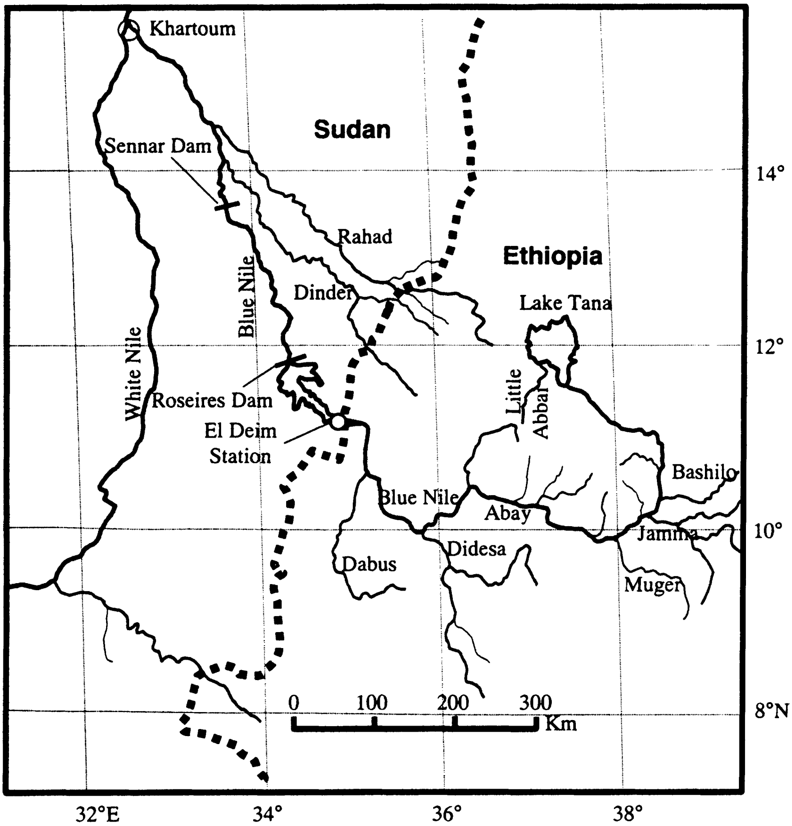


Fig. 1. Map of the Blue Nile in Ethiopia and Sudan.

ed from the Blue Nile, which represents about 46.5% of Sudan's share in the River Nile agreement with Egypt according to MIWR (1998). The Blue Nile is characterized by severe seasonality with average annual flow (1912-1997) of 47.8 billion m^3 measured at El Deim station on the Sudan borders with Ethiopia. Fig. 1 shows the Blue Nile in Ethiopia and Sudan and the location of the El Deim station. About 70% of this flow occur during the flood season between July and September, while only 4% of that flow occur during the driest period January-April. The average flow for the period 1912-1964 was 52.7 billion m^3 per year and reached the lowest during the period 1978-1986 with an average of 34 billion m^3 per year Said (1993). The hydrology of the Blue Nile is best described in Hurst *et al.* (1959). Two reservoirs were built on the river course to store water for irrigation and hydropower generation. Sennar dam operating since 1925 and Roseires dam since 1966 have combined current capacity of 2.4 billion m^3. Sennar dam was designed to store and supply the Gezira scheme (874,300 ha) with irrigation water. Since its capacity cannot supply the total requirements of Gezira scheme for two weeks it has to be supplemented from the Roseires dam upstream. But as the combined capacity of the two reservoirs fall short of half of the total requirements the gap has to be supplemented from the natural river flow during the recession time. This made the decision on the total dry-season plantation area depend merely on the expected low flow of the river. The competition between the dry-season plantation and hydropower generation has further complicated the utilization of the river water. Thus, low flow forecast for such river is deemed crucial for better planning and management. As the data on the Blue Nile catchment in Ethiopia is difficult to obtain and hindered by the political instability, the Sudanese water management staff has to depend on the flow of the river as is recorded at El Deim station. The historical records since 1912 are available on 10-day period mean discharge in millions of m^3 (mcm/period).

Many methods have been tried to forecast the Blue Nile low flow. One of the best methods used by the Sudanese staff is known as the similar year method. The method compares the first 10-day mean flow during October with similar period of the historical records. Then the current recession is assumed to follow the same trend of the most similar year of the record. Since no two years are exactly the same, Hamad (1981, 1987) modified the method as follows;

1) The historical 10-day mean flow rates were arranged in descending order and flows corresponding to 20, 50, 80 and 90% probability of exceedence were determined as criterion flows.
2) The mean flow during the current first period of October is compared with that of the criterion ones and the nearest criterion year is selected.
3) The ratio of the current first period of October to that of the criterion year is computed.
4) The mean flow for each 10-day period for the current year is calculated by multiplying the computed ratio above by the parallel mean flow of the selected criterion year.

Another method used by the staff is known as the *k*-value. It assumes that the recession rate for each 10-day period is constant and the logarithmic values of the successive flows are linearly related. The modified similar year method was found to be better than the *k*-value method by Hamad (1993). An analytical recession-forecast model is derived and compared with the modified similar year method.

Mathematical Development of the Model

Recently Tallaksen (1995) presented a review of base flow recession analysis, which covers Hall (1968) and the most recent techniques. The method presented in this paper uses the continuity equation and the general non-linear storage/outflow relationship to yield the outflow function thought for the Blue Nile. The continuity equation states that, the difference between the inflow, *I*, and the outflow, *Q*, is the change in the effective storage, S, of the aquifer supplying a river *i.e.*

$$\frac{dS}{dt} = I - Q \tag{1}$$

Lambert (1969) and others stated that the rate of the river flow or discharge Q is uniquely related to the effective storage S within the catchment *i.e.*

$$S \equiv f(Q) \tag{2}$$

Various forms have been used in the literature describing the relationship of Eq. (2). The linear reservoir model describes this relationship as

$$S = aQ + \text{constant} \tag{3}$$

The logarithmic form applied by Lambert (1969, 1970, 1972) for the River Ceiriog in North Wales and by O'Connell (1986) on the River Dee basin takes the form

$$S = a\ln Q + \text{constant} \tag{4}$$

On the other hand a general form of the non-linear storage/outflow relationship can be written as

$$S = aQ^{L} + \text{constant} \tag{5}$$

Checking the Blue Nile Linearity

It is important to define two terms that are used frequently in the following sections, these are the criterion years and the similar years. Following the mean 10-day flow records of the Blue Nile each year has 36 "10-day" periods, three for each month. These periods were arranged in descending order for the period 1912-1961 inclusive. Then each 10-day period was analyzed separately for probability of exceedence. Flows expected to be equaled or exceeded by 10, 50 and 90% were grouped together and named the criterion years. Thus three groups of years having the same probability of flow were determined. The second term is referred to three types of years according to their flood magnitude. These are wet, normal and dry years. The year is said to be wet when the ratio of its mean flow during the flood months (July, August and September) to the mean flow of the calibration period is more than 1.05. This ratio is abbreviated as JAS for convenience. The year is said to be dry when its JAS ratio is less than 0.95 and is a normal year when the ratio falls between the two mentioned categories. Hence three types of years were identified as wet, normal and dry.

Differentiating Eq. (4) with respect to Q we get

$$\frac{dS}{dQ} = aQ^{-1} \tag{6}$$

Eq. (1) under negligible inflow can be put in the form

$$\frac{dS}{dt} = \frac{dS}{dQ}\frac{dQ}{dt} = -Q \tag{7}$$

Combining Eq. (6) and Eq. (7) yields

$$aQ^{-1}\frac{dQ}{dt} \equiv -Q \tag{8}$$

Rearranging

$$\frac{dt}{dQ} \equiv -aQ^{-2} \tag{9}$$

Then the outflow-time relationship is obtained by integrating Eq. (9)

$$t = a(Q_t^{-1} - Q_0^{-1}) \tag{10}$$

Rearranging

$$\left(\frac{Q_t}{Q_0}\right)^{-1} = \frac{Q_0}{a}t + 1 \tag{10a}$$

This means that a plot of $(Q_t/Q_0)^{-1}$, where Q_0 is taken as the first period of October and Q_t is the flow at time period $t = 1$ after the first period of October and up to the last period of February $t = 14$, would give a straight line if the Blue Nile follows the logarithmic relationship. Fig. 2 shows that the plot is curved for the criterion and similar years suggesting that the Blue Nile recession does not follow the logarithmic relationship. Similarly the linear reservoir concept was checked by plotting log (Q_t/Q_0) against time t (not shown here) which led to the same result. A similar procedure was followed by differentiating the general non-linear reservoir Eq. (5), which gives

$$\frac{dS}{dQ} = aLQ^{L-1} \tag{11}$$

Substituting in Eq. (7) gives

$$aLQ^{L-1}\frac{dQ}{dt} \equiv -Q \tag{12}$$

Rearranging and integrating gives

$$\int_0^t dt = -aL\int_{Q_0}^{Q_t} Q^{L-2}\,dQ \tag{13}$$

Then

$$t = -\frac{aL}{L-1}(Q_t^{L-1} - Q_0^{L-1}) \tag{14}$$

provided that $L \neq 1$ and Q_0 and Q_t are the rates of flow at the time of starting the forecast and after time t respectively. Then putting $b = 1-L$ gives

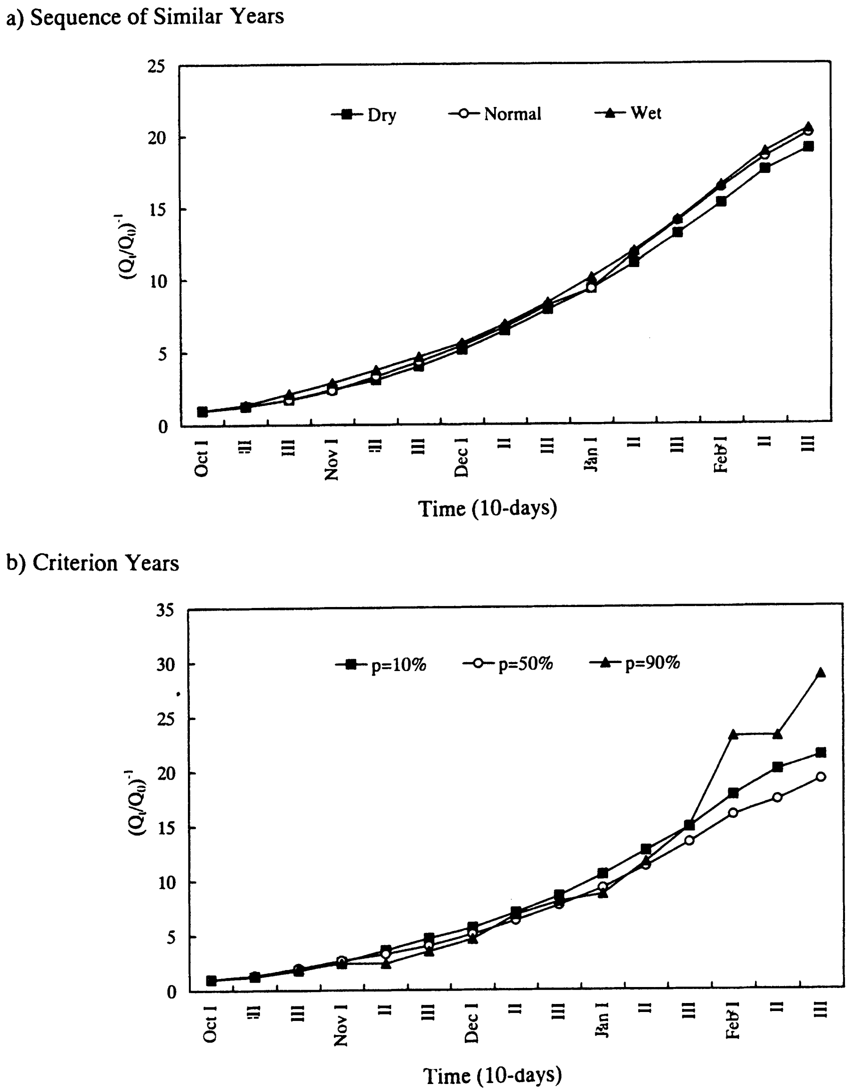


Fig. 2. Checking the Blue Nile recession patterns with $b = 1$.

$$t \equiv \frac{a(1-b)}{b}\left(Q_t^{-b} - Q_0^{-b}\right) \tag{15}$$

Rearranging

$$Q_t^{-b} \equiv \frac{b}{a(1-b)}\, t + Q_0^{-b} \tag{15a}$$

Eq. (15a) is a straight line form with the slope $m = b/a(1\text{-}b)$ and the intercept is Q_0^{-b}

where Q_0 is the initial flow at the time of forecast (first period of October). It can be seen that when b is zero and so L is 1, is that of the linear reservoir case. When b approaches 1, the slope approaches infinity. Using the slope m in Eq. (15a) gives

$$Q_t^{-b} = \frac{mQ_0^b t+1}{Q_0^b} \tag{16}$$

Hence the equation for the Blue Nile non-linear recession behavior would be

$$Q_t = \frac{Q_0}{(1+mQ_0^b t)^{1/b}} \tag{17}$$

Knowing the appropriate values of b and m and the initial flow rate Q_0 the flow at any time t can be determined.

The recession forecast is needed immediately after the flood has tailed off. This because the decision on the winter plantation has to be taken as early as October second period. Thus Q_0 in this paper refers to the flow of the first period of October.

Determination of *b* for the Blue Nile

To find the proper value of b, Eq. (15a) is put in the form

$$\left(\frac{Q_t}{Q_0}\right)^{-b} = \frac{bQ_0^b}{a(1-b)}t+1 \tag{15b}$$

Then several values of b were tried while $(Q_t/Q_0)^{-b}$ is plotted against time t for the criterion years and the sequence of similar years. It was found that b is not sensitive between 0.4 and 0.3 for both the criterion and the sequence of similar years. Regression analysis was carried out for this range of b resulted in coefficient of determinations greater than 0.997. However Hamad (1993) took b as 0.35. In this work b is taken as 0.4 as it resulted in higher coefficients of determinations for the similar years as shown in Table 1. It also resulted in less standard error of estimates compared with 0.35 with regard to the forecasted and the actual flows. Fig. 3 shows different values of b for the group of similar years. Taking b as 0.4 Eq. (17) can be written as

$$Q_t = \frac{Q_0}{(1+mQ_0^{0.4}\,t)^{2.5}} \tag{17a}$$

Knowing that m is a constant and so the initial flow Q_0 Eq. (17a) is similar to the approximated two-parameter equation presented by Griffiths and Clausen (1997). Griffiths and Clausen (1997) combined the continuity equation with the Muskingum method of stream routing to yield their outflow function.

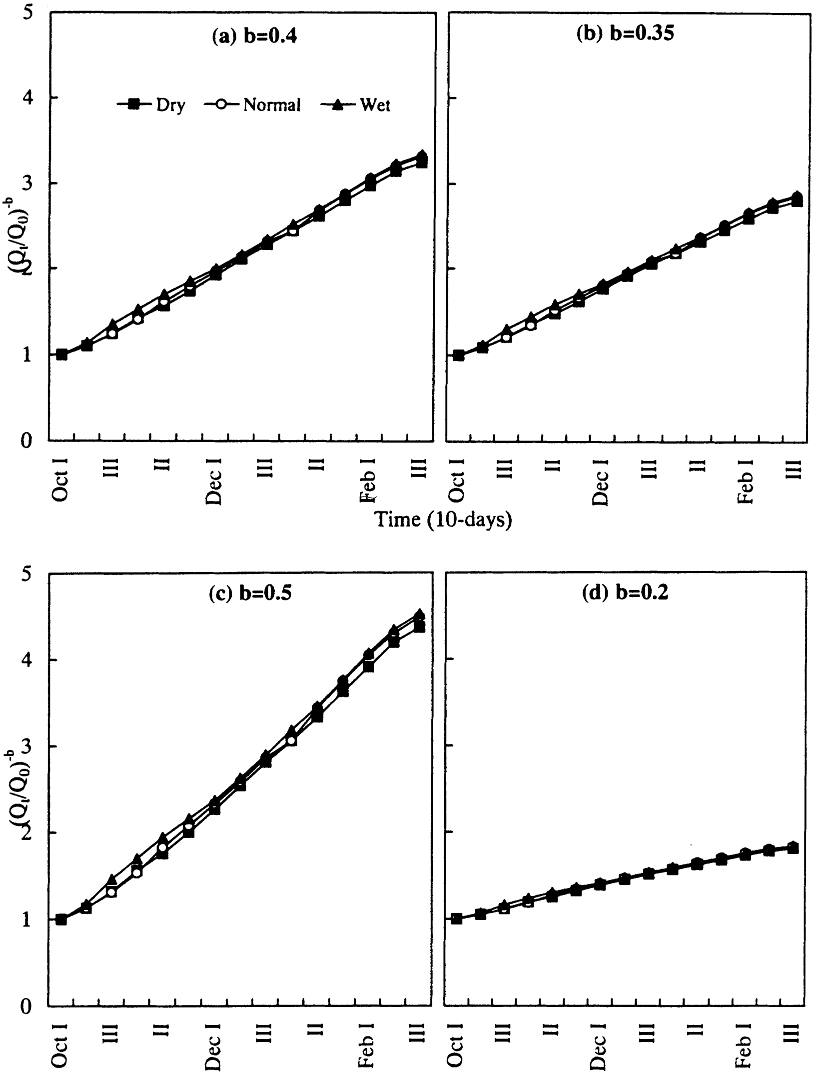
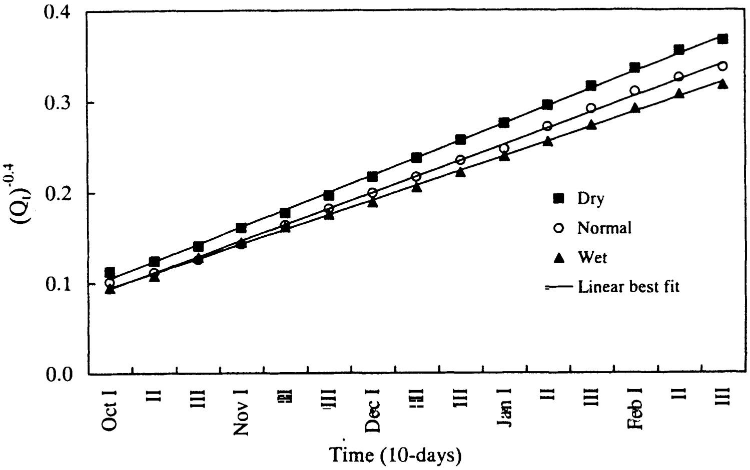


Fig. 3. Different values of *b* for the sequence of similar years.

Table 1 – Coefficients of determinations for different values of *b* obtained from linear fitting on the temporal distribution of $(Q_t/Q_0)^{-b}$ for each group of similar years.

Group of Similar Years	$b = 0.2$	$b = 0.35$	$b = 0.4$	$b = 0.5$
Wet Years	0.992	0.997	0.999	0.997
Normal Years	0.994	0.997	0.998	0.995
Dry Years	0.995	0.998	0.998	0.996

a) Sequence of Similar Years



b) Criterion Years

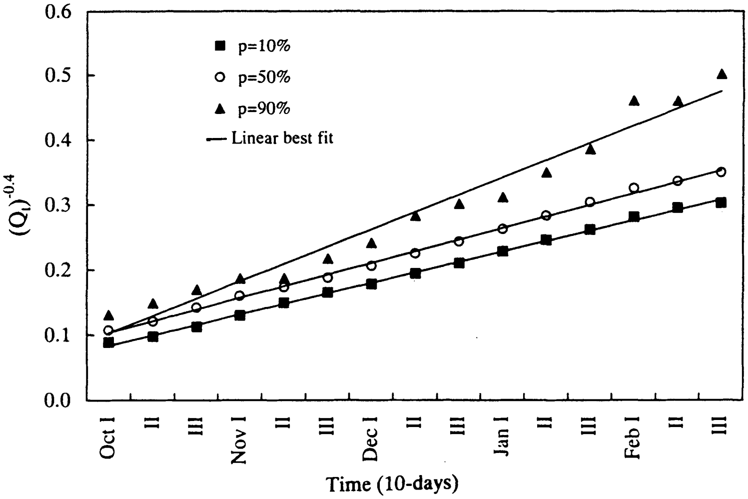


Fig. 4. Variation of *m* with the criterion years and the sequence of similar years.

Determination of the Slope *m*

Values of $Q_t^{-0.4}$ for each of the criterion years and similar years were plotted against time t to see whether *m* stays the same or changes with the type of the year. Fig. 4 shows that *m* did not stay the same, it went steeper as the year got drier. Similar observation was noticed for the sequence of similar years. Taking three values of m according to the type of the year in question would only be suitable for the years close to those categories, but would not be suitable for those between the categories or towards the extremes. Multiple regression and correlation analysis was performed to compute the proper value of m. The analysis considered the effects of the antecedent floods by including the previous year's annual flow ratio named as Pannual. It is computed as the ratio of the previous annual flood to the mean annual flood of the period 1912-1961. The ratio of the mean flow during August, September and the first period of October of the current year to that of the same period for the previous year was also computed and named CurASO1/PreASO1. Finally the ratio of the current year's mean flow during August and September to the mean flow of July, August and September during the period 1912-1961 was computed and named AgSp/JAS. The values of m over the 10-day period were computed for each year from 1912-1961 inclusive. The following relationship was then obtained for m with multiple correlation coefficient equal to 0.80

$$m = 0.02+0.0027\times\mathrm{Log}\left(\frac{\mathrm{CurASO1}}{\mathrm{PreASO1}}\right)-0.021\times\mathrm{Log}\left(\frac{\mathrm{AgSp}}{\mathrm{JAS}}\right)-0.005\times\mathrm{Log}(\mathrm{Pannual}) \quad (18)$$

This may be compared with the relationship used by Hamad (1993) with b taken as 0.35 as;

$$m = 0.0186-0.018\mathrm{Log}(\mathrm{JAS})-0.0078\mathrm{Log}(\mathrm{Pannual}) \quad (19)$$

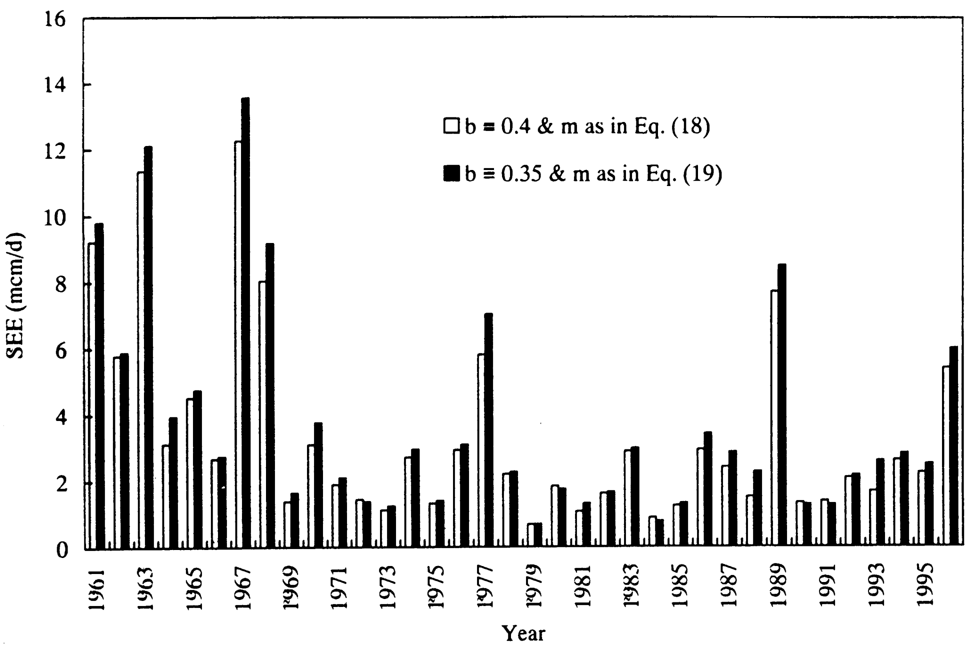


Fig. 5. The standard error of estimates for the forecasted versus the actual flows using two values of b and m.

Other relationships were tried but resulted in less coefficient of multiple correlation. Thus the recession forecast model can be applied as far as the 10-day flow rates up to the first period of October for the year in concern were known.

The recession for the whole records (1913-1996 inclusive) from October second period to February third period was determined with b taken as 0.35 in Eq. (17) and m of Eq. (19). The recession was also calculated for the same period with b taken as 0.4 and m of Eq. (18). The two methods were compared with the actual recession of the mentioned period and the standard error of estimates (*SEE*) is calculated for each year of the whole period (83 years), the later method gave less *SEE* in 70 years compared with the former one. The number of years with smaller *SEE* during the verification period (1962-1996), was 30 out of 35 as shown in Fig. 5.

Results

Eight years were chosen for the comparison all outside the calibration period used in the development of the model except the driest and the wettest year of the record, which occurred during 1913/14 and 1946/47 respectively. The years were chosen in such a way to represent different type of years together with the effects of prolonged periods of droughts. Table 2 shows the details of these years. The comparison was made in terms of the temporal distribution of flow between the second period of October and the last period of February for the two methods against the actual flow. In addition the *SEE* of the forecasted flows against the actual ones for each year during the period 1962-1996 were also calculated.

Figs. 6 and 7 show the temporal distribution of the actual and forecasted flows during the recession period for the chosen years. It is clear that the model is better than the modified similar year method in all the chosen years. There are some years in the records with abnormal recession patterns such as the year 1963/64 where the consecutive 10-day flows during November and December out yielded the previous flows. The *SEE* during such year was very high as shown in Fig. 8. Fig. 8 also indi-

Table 2 – General description of the years chosen for the comparison

Year	Year Type	JAS	Position in the Record (1912-1997)
1913/14	The Driest Year	0.43	Driest year of the record after a dry year
1946/47	The Wettest Year	1.41	Wettest year of the record after 2 dry years
1970/71	Normal Year	1.00	Normal year after a wet year
1975/76	Wet Year	1.22	Wet year after a wet year
1981/82	Dry Year	0.90	Dry year after 3 dry years
1884/85	Dry Year	0.63	Dry year after 6 dry years
1985/86	Normal Year	0.98	Normal year after a series of 7 dry years
1995/96	Dry Year	0.77	Dry year after a wet year

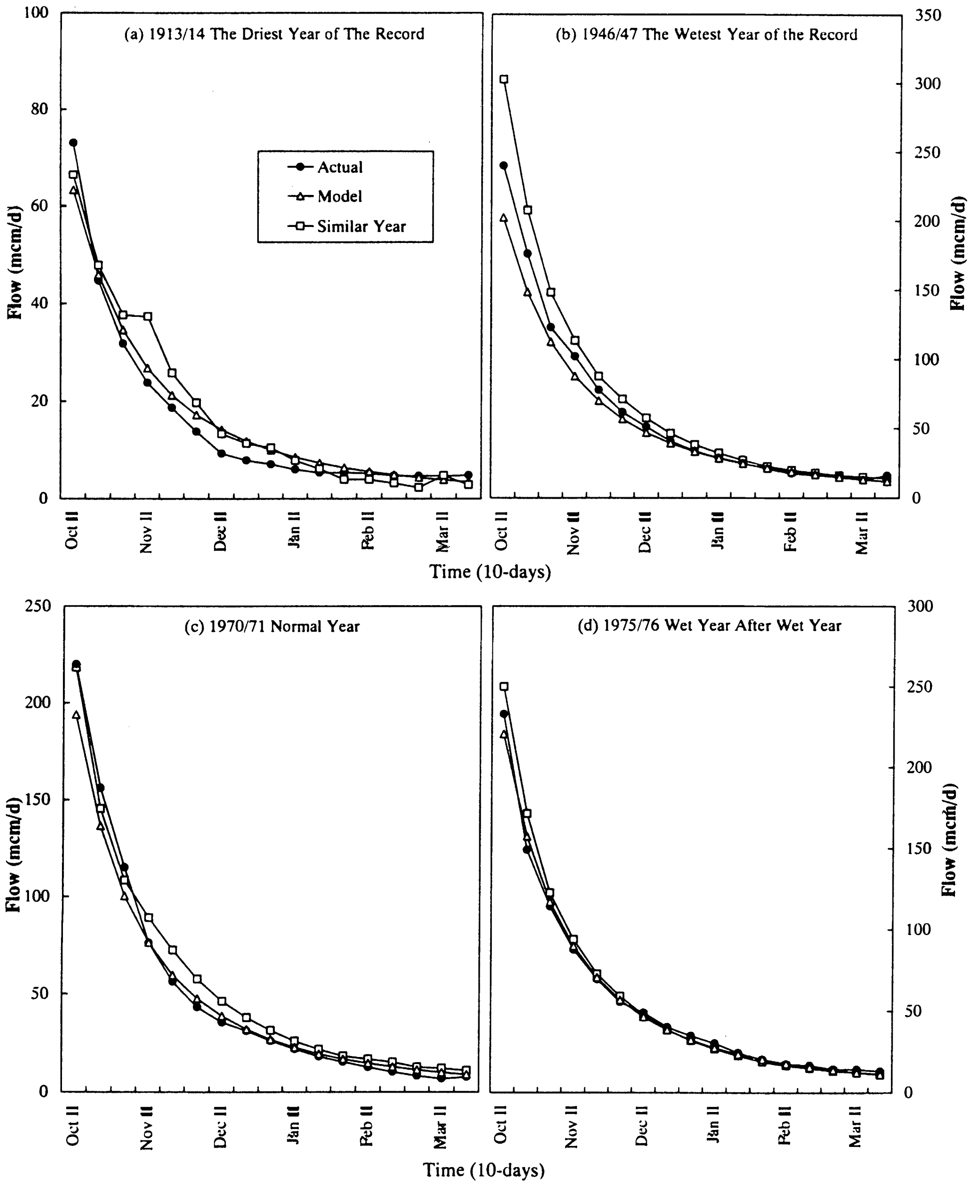


Fig. 6. Temporal distribution of the forecasted and the actual flows for the first four years chosen for comparison.

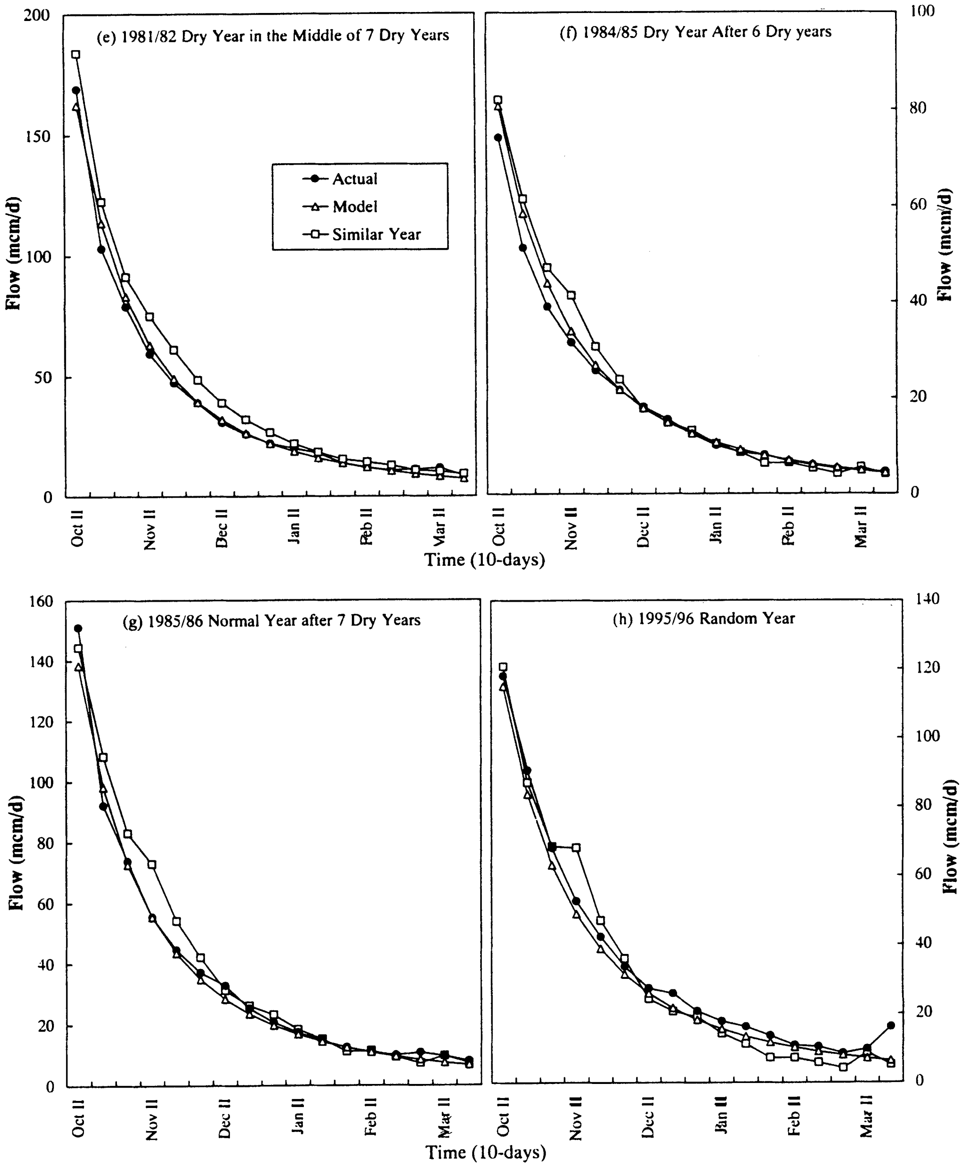


Fig. 7. Temporal distribution of the forecasted and the actual flows for the second four years chosen for comparison.

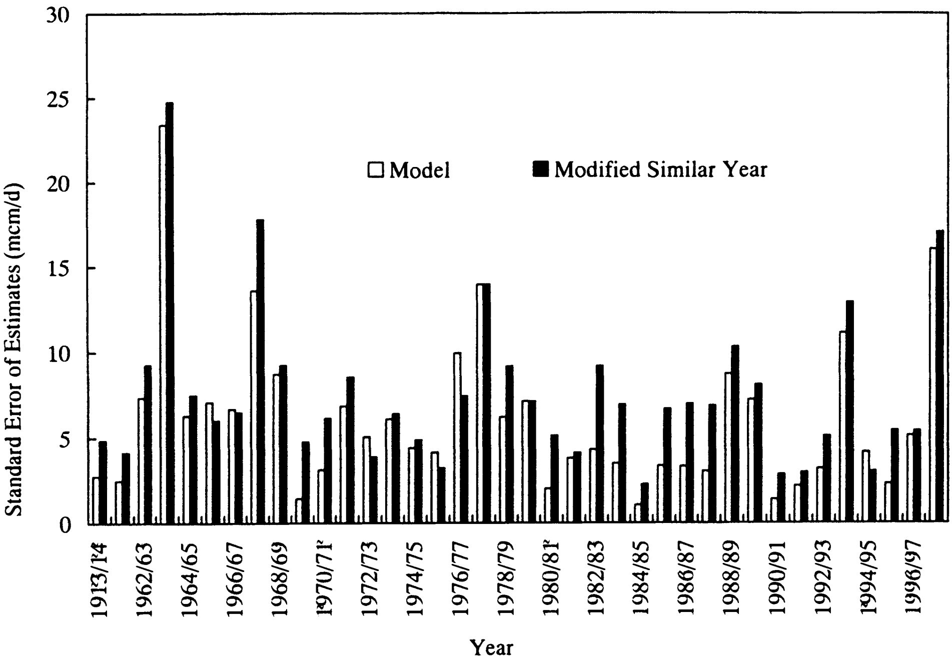


Fig. 8. The standard error of estimates for the model and the modified similar year method.

cates that out of the 38 years used in the calculation of the *SEE* between 1962-1998 plus the driest and the wettest years of the whole record, 31 years were forecasted with less *SEE* compared with the modified similar year method. The model performed better than the similar year method during the most important period where the forecast was deemed necessary for better planning. That was during the series of 7 dry years between 1978/79 and 1984/85. During this period the model produced less *SEE* in all of the dry years except the year 1979/80. Even though the difference in *SEE* between the modified similar year method and the model was very small (0.02 mcm for the whole recession period) which is less than 0.003% of the actual total flow during the recession period.

Conclusions

A model was developed to forecast the recession flows of the Blue Nile from the tail of the flood during the second period of October to the last period of March. This period is considered vital for the planning of the winter crop area in the Gezira scheme. The winter crop usually wheat is grown between November and March every year. No major crops are grown after March till the beginning of the next flood. The model will support the decision concerning the determination of the winter crop total area and the expected hydropower generation during the period of scarcity. The model performed better than the modified similar year method in terms of the temporal distribution of the 10-day flows and the *SEE.*

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Address:
A. W. Abdelhadi and Takeshi Hata,
The Graduate School of Science and Technology,
Department of Regional Environment,
Kobe University,
Rokkodai, Nada-Ku,
Kobe 657-8501, Japan,
Email: hadi103@hotmail.com
Email: hata@kobe-u.ac.jp

O. E. Hamad,
Ministry of Irrigation and Water Resources,
Technical Water Resources Organ,
P. O. Box 878,
Khartoum, Sudan,