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Base Flow and Water Supply

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The interaction between the base flow created by seepage through the unsaturated zone in a region between two parallel rivers and the flow caused by the gaining of water from wells positioned at arbitrary points over the region is analysed. Normally both flows are non-steady but, for the sake of simplicity, the base flow is here assumed to be steady whereas the yield of water from the single well is assumed to be a free function of time defined by the actual consumption which assumption leads to a non-steady flow towards the wells. A special result of the analysis is that it becomes possible to define the catchment area of the single well.

Introduction

The problem of non-steady flow towards a well yielding a steady or non-steady amount of water has been treated by several authors. Here it is essential to mention the fundamental solution given by Theis (1935). On the basis of this solution a number of problems especially related to the interaction between the flow in a nearby river and the flow towards a well have been discussed by Theis (1941), Hantush (1964 and 1965) – in other cases the aim has been to discuss the application of the solutions to more practical problems such as the testing of wells etc. (Jenkins 1968 and 1970).

The aim of this paper is fourfold: 1) to combine the (steady) base flow between two parallel rivers with the non-steady flow towards one (or more) wells 2) to demonstrate the transition from the original flow to a new steady flow if the yield of the well is kept constant or a quasi-steady flow, if the yield of the well is a rhytmic function of time 3) to calculate the amount of water withheld from each of the two rivers, and 4) to define the catchment area of each well. The last-mentioned result will be valuable when discussing pollution problems related to groundwater resources.

All solutions will be given in form of mathematical functions suited for computational work.

Base Flow

The base flow is here defined as the plane, horizontal flow in an aquifer resting on an impermeable layer between two rivers, Fig. 1. The flow is created by a steady seepage P through the unsaturated zone and is upwards bounded by the phreatic surface which as a vault connects the water surfaces of the two rivers; the levels h_1 and h_2 of these water surfaces are assumed to be constant. Moreover it is assumed that the banks of the rivers are vertical and penetrating the aquifer so that no special resistance in the vicinity of the rivers has to be taken into account. Finally we assume that the transmissivity T of the aquifer is constant.

Fig. 2 shows the definition of the (s,l) coordinate system, and using the distance L between the rivers as a unit of length we define the dimensionless system (x,y) by

$$s = Lx \qquad l = Ly \tag{1}$$

in which system the level h_b of the phreatic surface is given by

$$h_b = h_1 - (h_1 - h_2)x + \frac{PL^2}{2T}(1-x)x \tag{2}$$

and where the discharge (per unit of length) q_b in the direction of the x-axis is given by

$$q_b = \frac{T}{L}(h_1 - h_2) - PL(\frac{1}{2} - x) \tag{3}$$

which gives the discharges to the two rivers

$$q_{b,1} = -q_b = -\frac{T}{L}(h_1-h_2) + \frac{1}{2}PL \qquad (x = 0) \tag{4}$$

$$q_{b,2} = q_b = \frac{T}{L}(h_1-h_2) + \frac{1}{2}PL \qquad (x = 1) \tag{5}$$

Finally, we note that the dimensionless distance from river I to the watershed is given by

$$x_w = \frac{1}{2} - \frac{T}{PL}(h_1-h_2) \tag{6}$$

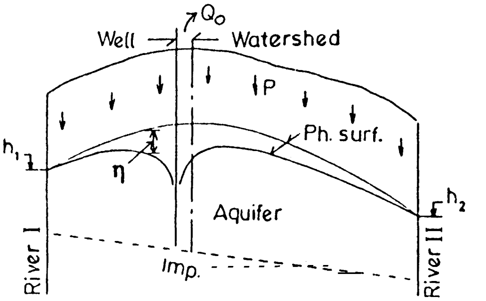

Fig. 1. Cross section between river I and II.

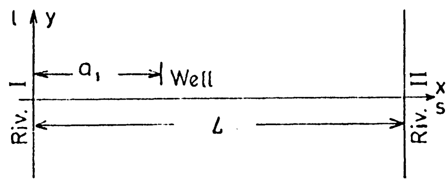

Fig. 2. Definitions of coordinate systems (s,l) and (x,y).

The well-known solution quoted here satisfies the boundary conditions $h_b \equiv h_1$ and $h_b \equiv h_2$ at the two rivers. The solution holds true only if the seepage P is independent of time; as will be known P is normally an almost periodical function over the year and sometimes P is varying over the region. This case of non-steady base flow is analyzed in Dahl (1980 and 1981).

Gaining of water from wells located at arbitrary points in the region will cause a depression η of the phreatic surface, Fig. 1, and the local depression will become a function of the local coordinates and of the time. This function must satisfy the boundary condition $\eta = 0$ for $x = 0$ and $x = 1$ and the initial condition $\eta = 0$ over the whole region; moreover, the complete solution of the problem must fulfil the condition that the yield of the single well is a free function of time governed by the actual consumption of water.

A Well in Extended Area

We assume that a single well, interpreted as a line source, is located in an extended area where the phreatic surface is horizontal and that the well is yielding the constant discharge Q_0 from moment $t = 0$; before this moment we have $Q_0 = 0$. The flow created in this way is radial in vertical plans through the axis of the well according to the well-know differential equations

$$Q = -2\pi RT \frac{\partial \eta}{\partial R} \tag{7}$$

$$\frac{\partial Q}{\partial R} + 2\pi RS \frac{\partial \eta}{\partial t} \equiv 0 \tag{8}$$

where Q is the discharge at the time t through a cylinder with radius R ; η is the corresponding depression of the phreatic surface. S is the specific yield (or storage coefficient) and T the transmissivity of the aquifer; both are assumed to be constant.

The dimensionless function $\phi(r,\tau)$ and the quantities r and τ defined by

$$\eta(\ ) = A\phi(r,\tau)\ , \qquad R = a\,r\ , \qquad t = \frac{Sa^2}{T}\tau \tag{9}$$

are now introduced; A and a are constants to which we shall revert a bit later. Insertion of Eq. (9) and elimination of Q now gives

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{\partial\phi}{\partial\tau} = 0 \tag{10}$$

which equation we recognize to be a dimensionless form of the equation of conduction of heat in solids.

Taking into account the initial condition $\phi = 0$ for $\tau \Rightarrow 0$ and the boundary condition $\phi \equiv 0$ for $r \rightsquigarrow \infty$ together with the condition attached to Q_0 we find – using the theory of Laplace transformation – the following solution

$$\phi(u) = \begin{cases} \dfrac{1}{2}\displaystyle\int_u^\infty \frac{e^{-t}}{t}\,dt = \frac{1}{2}E_1(u) & \text{for } \tau > 0 \\ 0 & \text{for } \tau < 0 \end{cases} \tag{11}$$

where the argument u is defined by

$$u = \frac{r^2}{4\tau} \tag{12}$$

and where $E_1\ (u)$ is the exponential integral discussed in Abramowitz and Stegun (1968); for later use we quote the series

$$E_1(u) = -\gamma - \ln u - \sum_{n=1}^{\infty} \frac{(-1)^n u^n}{n\times n!} \tag{13}$$

where $\gamma = 0.57721$ ... is Euler's constant.

By differentiation of Eq. (11) and insertion in Eq. (9a) we now get

$$\frac{\partial\eta}{\partial R} = \frac{A}{a}\frac{\partial\phi}{\partial r} = -\frac{A}{ar}\,e^{-r^2/4\tau} = -\frac{A}{R}\,e^{-r^2/4\tau} \tag{14}$$

which inserted in Eq. (7) gives

$$Q = 2\pi AT\ e^{-r^2/4\tau} \tag{15}$$

The condition $Q \rightsquigarrow Q_0$ for $r \rightsquigarrow 0$ is evidently fulfilled if we insert

$$A = \frac{Q_0}{2\pi T} \tag{16}$$

for the free constant A in Eq. (9a) and the complete solution is now written in the form

$$\eta(\ ) = \frac{Q_o}{T}\phi_1(r,\tau) \qquad (17)$$

$$Q = Q_o\,\psi_1(r,\tau) \qquad (18)$$

where we have introduced the functions

$$\phi_1\left(\frac{r^2}{4\tau}\right) = \begin{cases} \dfrac{1}{4\pi}\displaystyle\int_{r^2/4\tau}^{\infty} \frac{e^{-t}}{t}\,dt = \frac{1}{4\pi}E_1\left(\frac{r^2}{4\tau}\right) & \text{for } \tau > 0 \\ 0 & \text{for } \tau < 0 \end{cases} \qquad (19)$$

$$\psi_1\left(\frac{r^2}{4\tau}\right) = \begin{cases} e^{-r^2/4\tau} & \text{for } \tau > 0 \\ 0 & \text{for } \tau < 0 \end{cases} \qquad (20)$$

A comparison now shows that this solution is identical with the original Theis - solution except for one important thing, namely that $\phi_1(\ )$ and $\psi_1(\ )$ are equal to zero if $\tau < 0$. We shall make use of this property later on.

We close this section by emphasizing that a definition of the constant a entering in Eq. (9) has not been necessary here which fact we conclude from, compare Eqs. (9) and (12),

$$u = \frac{r^2}{4\tau} = \frac{R^2}{a^2}\frac{Sa^2}{4Tt} = \frac{SR^2}{4Tt} \qquad (21)$$

This degree of freedom will become useful in the next section.

A Well between Two Rivers

The actual region is bounded by the lines $x = 0$ and $x = 1$ and the well A is located at a distance a_1 from river I which gives the coordinates $(x,y) = (x_1,0)$ for the axis of the well, compare Fig. 2. It is assumed that no seepage is taking place over the region. A reflection of the well A in the line $x = 0$ now leads to the function.

$$\phi_{2,1}(\ ) = \phi_1\left(\frac{r_1^2}{4\tau}\right) - \phi_1\left(\frac{r_2^2}{4\tau}\right) \qquad (22)$$

where

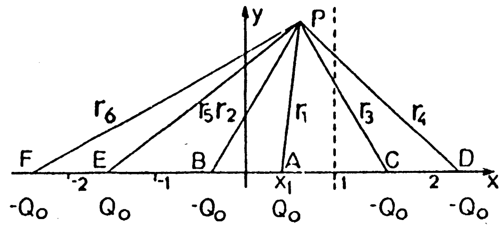

Fig. 3. Reflections at river I and II.

$$r_1^2 = (x-x_1)^2 + y^2 \tag{23}$$

$$r_2^2 = (x+x_1)^2 + y^2 \tag{24}$$

and we note that if we identify Eq. (22) with $\phi(\ )$ in Eq. (9a) we get the solution given by Theis (1941).

The wells A and B, Fig. 3, are now reflected in the line $x = 1$ giving the images C and D; these images are reflected in the line $x = 0$ giving the images E and F. A continuation of this process leads to the series

$$\phi_2(\ ) = \phi_1\left(\frac{r_1^2}{4\tau}\right)-\phi_1\left(\frac{r_2^2}{4\tau}\right)-\sum_{n=1}^{\infty}\left[\left(\phi_1\left(\frac{r_a^2}{4\tau}\right)-\phi_1\left(\frac{r_b^2}{4\tau}\right)\right) - \left(\phi_1\left(\frac{r_c^2}{4\tau}\right)-\phi_1\left(\frac{r_d^2}{4\tau}\right)\right)\right] \tag{25}$$

where the definitions Eqs. (23) and (24) still hold true and where the radii r_a, r_b, ... are defined by

$$r_a^2 = (2n-x_1-x)^2 + y^2 = (x+x_1-2n)^2 + y^2 \tag{26}$$

$$r_b^2 = (2n+x_1-x)^2 + y^2 = (x-x_1-2n)^2 + y^2 \tag{27}$$

$$r_c^2 = (-2n+x_1-x)^2 + y^2 = (x-x_1+2n)^2 + y^2 \tag{28}$$

$$r_d^2 = (-2n-x_1-x)^2 + y^2 = (x+x_1+2n)^2 + y^2 \tag{29}$$

It is possible to prove that the series Eq. (25) is convergent having a positive limit everywhere in the interior of the region and that $\phi_2(\ ) = 0$ along the boundaries $x = 0$ and $x = 1$. Moreover, we have $\phi_2(\ ) \rightarrow 0$ for $\tau \rightarrow 0$ since $\phi_1(\ ) \rightarrow 0$ for $\tau \rightarrow 0$. The special condition $\phi_1(\ ) = 0$ for $\tau < 0$ is, of course, transferred to $\phi_2(\ )$.

We are now entitled to identify $\phi(\ )$ in Eq. (9a) with $\phi_2(\ )$ which gives

$$\eta(\ ) = \frac{Q_o}{L}\phi_2(\ ) \tag{30}$$

and we conclude that this solution fulfils the initial condition $\eta \rightarrow 0$ for $\tau \rightarrow 0$ and the boundary conditions $\eta = 0$ along both rivers. For the special case $\tau \to \infty$ we get, using Eq. (13)

$$\eta(\ ) \to \frac{Q_o}{L}\frac{1}{4\pi}\{\ln(\frac{r_2}{r_1})^2 - \sum_{n=1}^{\infty} \ln(\frac{r_b}{r_a}\frac{r_c}{r_d})^2\} \tag{31}$$

which shows that Eq. (30) is describing the transition of the phreatic surface from the original horizontal state to a new steady state if the yield Q_0 is kept constant.

The fact that a steady flow will be established in the region where no seepage exists implies that an inflow from both rivers will begin. The velocity vectors of these inflows are perpendicular to the rivers and therefore the inflow q_1 ( ) (per unit of length) is found from

$$q_1(\ ) \equiv T\frac{\partial\eta}{\partial s}\bigg|_{s=0} \equiv \frac{Q_o}{L}\frac{\partial\phi_2}{\partial x}\bigg|_{x=0} \tag{32}$$

By differentiation of Eq. (25) we get after reduction

$$\begin{aligned}\frac{\partial\phi_2}{\partial x}\bigg|_{x=0} = \frac{1}{\pi}\Bigg\{ & \frac{x_1}{x_1^2+y^2}\, e^{-(x_1^2+y^2)/4\tau} \\ & - \sum_{n=1}^{\infty}\Bigg[\frac{2n-x_1}{(2n-x_1)^2+y^2}\, e^{-((2n-x_1)^2+y^2)/4\tau} \\ & - \frac{2n-x_1}{(2n+x_1)^2+y^2}\, e^{-((2n+x_1)^2+y^2)/4\tau}\Bigg]\Bigg\}\end{aligned} \tag{33}$$

and the total inflow Q_1 from river I is now found from

$$Q_1 \equiv \int_{-\infty}^{\infty} q_1\,dl \equiv Q_o\int_{-\infty}^{\infty}\frac{\partial\phi_2}{\partial x}\bigg|_{x=0} dy = Q_o\psi_2(\ ) \tag{34}$$

where we have introduced the function

$$\psi_2(\ ) = \int_{-\infty}^{\infty}\frac{\partial\phi_2}{\partial x}\bigg|_{x=0} dy \tag{35}$$

which integral has to be found by numerical methods.

For the special case $\tau \to \infty$ it is, however, possible to find the limit value of ψ_2 ( ); integration over the finite interval $(-y|y)$ gives

$$\int_{-y}^{y} \frac{\partial \phi_2}{\partial y}\bigg|_{x=0} dy = \frac{2}{\pi}\{\arctan \frac{y}{x_1} - \sum_{n=1}^{\infty} \arctan \frac{2x_1 y}{4n^2 - x_1^2 + y^2}\} \quad (36)$$

and for $|y| \to \infty$ we get

$$\psi_2(\ )\big|_{|y| \to \infty} = \{1 - \frac{2}{\pi}\sum_{n=1}^{\infty} \arctan \frac{2x_1 y}{4n^2 - x_1^2 + y^2}\bigg|_{y \to \infty}\} = 1 - x_1 \quad (37)$$

which inserted in Eq. (34) gives the very simple result

$$Q_1 = Q_0(1 - x_1) \qquad \text{for } \tau \to \infty \quad (38)$$

It should be added that the inflows q_1 and Q_1 from river I, respectively q_2 and Q_2 from river II, involves a kind of symmetry since it is only by chance that the l-axis is placed along river I and not along river II; for this reason the inflows q_2 and Q_2 are found from Eq. (32) and Eq. (34) by exchanging x_1 for 1-x_1 in Eqs. (33) and (35).

The formulas Eqs. (30), (32) and (34) now constitute together with functions $\phi_2(\ )$, $\delta\phi_2/\delta x||_{x=0}$, *and* $\psi_2(\ )$ the necessary solution for the following analysis of the four problems mentioned in the introduction. A complete mathematical discussion of the three functions is superfluous here but some properties important for the following have to be rendered. The functions are all asymptotic with the limit zero for $|y| \to \infty$ and with finite (positive) limits for $\tau \to \infty$. The limit for $|y| \to \infty$ is, in fact, reached with a good approximation if $|y| > 1.5$ which limit defines a practical demarcation of the zone of influence of the well. The limit for $\tau \to \infty$ is reached with a good approximation if $\tau > 1$ which defines a practical duration, compare Eq. (9c)

$$t_d = \frac{Sa^2}{T} = \frac{SL^2}{T} \quad (39)$$

of the transition from the original steady flow to the new one, if Q_0 is kept constant.

Non-Steady Gain of Water

We now have to improve the solution deduced in above section so that it holds true when the gain of water is an arbitrary function of time defined by the actual consumption according to the full-drawn curve shown in Fig. 4. This function is approximated by a staircase function with arbitrary intervals, the only demand being that the constant values $Q_{0,k}$ have to be chosen so that the total volume of water gained at the end of each interval must equal the real consumption.

For the arbitrary interval $(\tau_{k-1}|\tau_k)$ we get, using the law of translation of Laplace-transforms and of superposition

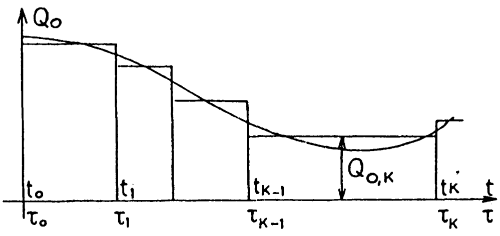

Fig. 4. Consumption of water as a function of time.

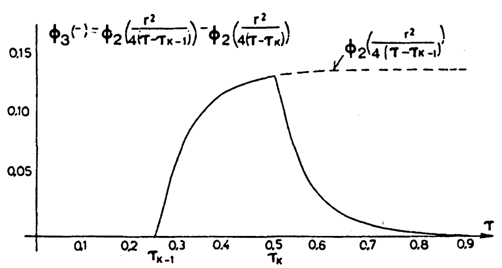

Fig. 5. The function $\phi_3(\ )$.

$$\eta(\ ) \equiv \frac{Q_{o,k}}{T}\ \phi_{3,k}(\ ) \tag{40}$$

where we have introduced the function

$$\phi_{3,k}(\ ) = \phi_2\left(\frac{r^2}{4(\tau-\tau_{k-1})}\right) - \phi_2\left(\frac{r^2}{4(\tau-\tau_k)}\right) \tag{41}$$

in which the law of translation implies that all terms for which the denominator of the argument is negative have to be put equal to zero. Fig. 5 shows, as an example, the course of $\phi_{3,k}$ ( ) for the arbitrary point (x,y) = (0.5,0) and for the parameter values τ_{k-1}=0.25, τ_k=0.5, and x_1=0.25; it should be noted that $\phi_{3.k}(\ )$ is almost negligible for τ-τ_k > 1, compare previous section in fine, which means that the influence of the gaining of water during this interval has faded out.

The solution for the approximative staircase function shown in Fig. 4 is now found by a simple superposition. For the η-solution we get, compare Eq. (30).

$$\eta = \frac{1}{T} \sum_{k=1} Q_{o,k}\ \phi_{3,k} \tag{42}$$

while for the inflow q_1 we get, compare Eq. (32)

$$q_1(\ ) \equiv \frac{1}{L} \sum_{k\equiv 1} Q_{o,k}\ \partial\phi_{3,k}/\partial x\Big|_{x=0} \tag{43}$$

from which the total inflow Q_1 has to be found by numerical integration.

Thus far it has been assumed that only one well has been activated. If two or more wells are exploited, it is reasonable to examine whether they will interfere with one another; if this is the case, a simple superposition will solve the problem.

Base Flow and Water Supply

The steady base flow described in the first section and the non-steady flow described in the two previous sections hold true for the same region bounded by two parallel rivers. We are, therefore, entitled to combine them, using the law of superposition.

If we take the solution deduced in the section on non-steady gain of water we get for the level of the phreatic surface

$$
\begin{aligned}
h &= h_b - \eta(\ ) \\
&= h_1 - (h_2 - h_1)x + \frac{PL^2}{2T}(1-x)x - \frac{1}{T}\sum_{k=1} Q_{o,k}\, \phi_{3,k}(\ )
\end{aligned}
\tag{44}
$$

while for the runoff to river I we get

$$
\begin{aligned}
q &= q_{b,1} - q(\ ) \\
&= -\frac{T}{L}(h_1 - h_2) + \frac{1}{2}PL - \frac{1}{T}\sum_{k=0} Q_{o,k}\, \partial\phi_{3,k}/\partial x\Big|_{x=0}
\end{aligned}
\tag{45}
$$

It should be noted that the last term in Eq. (45) represents the decrease of base-flow runoff; the total decrease of runoff to river I is found by numerical integration of this term.

A Numerical Example

It seems appropriate to illustrate the theoretical solution by a numerical example for which we choose the following values of the parameters: $L = 2{,}500$ m, $h_1 = 2$ m, $h_2 = 0$ m, $T = 0.002$ m^2/sec, $S = 0.2$, $P = 300$ mm/year, $a_1 = 1{,}000$ m, and $Q_0 = 120{,}000$ m^3/year.

If this yield is distributed evenly over the year, the duration of the transition is, compare Eq. (39)

$$
t_d = \frac{SL^2}{T} = \frac{0.2\times 2{,}500^2}{0.002} \equiv 6.25\times 10^8 \text{ sec} \sim 240 \text{ months}
$$

which indicate that 1 month (= 1/12 of a year) is an appropriate unit of time; the consequence hereof is that T, P and, Q_0 have to be converted correspondingly. This gives the conversional factor in Eq. (9c)

$$
\tau \equiv \frac{T}{SL^2}t \equiv \frac{0.002\times 31.7\times 10^6}{0.2\times 2{,}500^2\times 12}t \equiv 0.004227\, t
$$

where one month is used as the unit of time.

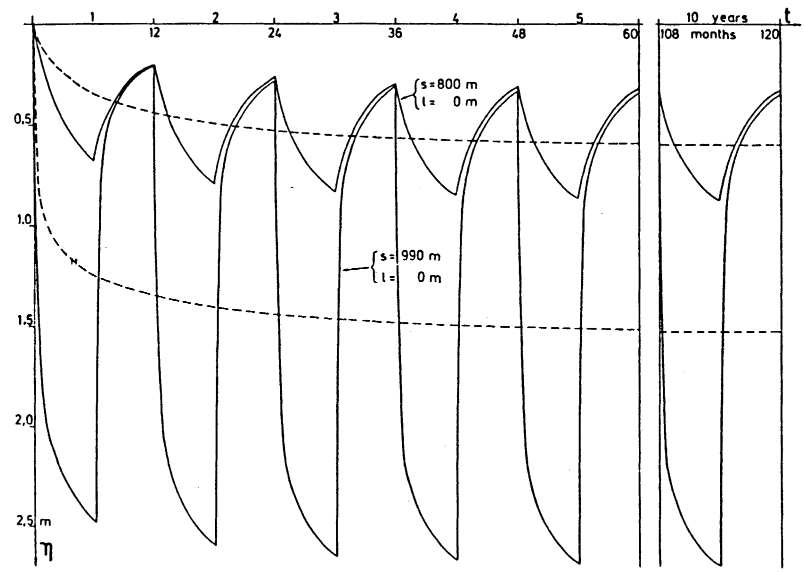

Fig. 6. The transition of the depression η of the phreatic surface.

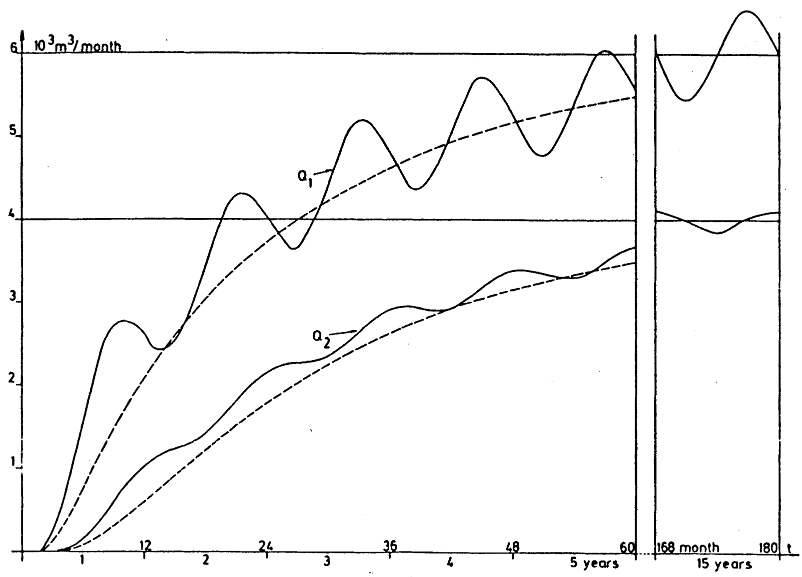

Fig. 7. The transition of the decrease of the runoff Q_1 and Q_2 to the rivers.

The results of the computations are all rendered in form of graphs. The computations are carried through for two cases: 1) Q is constant and equal to 10,000 m^3/month and 2) Q_0 is a periodical function with $Q_0 = 20{,}000$ m^3/month over 6 months followed by $Q_0 = 0$ over the next 6 months; thus the yearly yield of the well is 120,000 m^3 in both cases.

Figs. 6 and 7 show the transition of η and Q_1, respectively Q_2, over the first 5 years and the 10th year, respectively the 15th year; the values of η have been calculated for the two points (s,l) = (800 m, 0) and (990 m, 0), i.e. at a distance of 200 m, respectively 10 m, from the well. If Q_0 is kept constant, we get smooth curves for η as well as for Q_1 and Q_2 which curves asymptotically approach finite limits; these limits can be calculated directly from Eq. (31) and Eq. (38). If Q_0 is a periodical function, η as well as Q_1 and Q_2 tend to become periodical functions oscillating around the said limits; it is noteworthy that the η-oscillations are strictly in phase with the Q_0- oscillations whereas the Q_1- as well as the Q_2-oscillations are displaced in phase; it is also evident that these displacements depend on the distances from the well to the two rivers. Furthermore we note that Q_1 and Q_2 are hardly perceptible before the lapse of 3, respectively 6 months after the start of the well. Finally, we note that Q_1 and Q_2 have to be interpreted as the reduction of the baseflow runoff to the rivers.

Figs. 8 and 9 show how these reductions are distributed along the two rivers; the time t has been used as a parameter.

The new steady state of the phreatic surface obtained for Q_0 being constant and $t \rightarrow \infty$ is illustrated in Figs. 10 and 11. Fig. 10 shows a section through the well from river I to river II; the upper curve shows the levels of the phreatic surface before the activation of the well which gives a natural watershed at $s_w = 1{,}081$ m whereas the lower curve shows the levels of this surface when the yield of Q_0 of the well has been constantly equal to 10,000 m^3/month for a long time, and this curve shows two vertices at $s = 778$ m and 1,242 m. The explanation of these vertices is found in Fig. 11 which shows some contour lines of the phreatic surface (for $t \rightarrow \infty$) for a part of the region. The two points are stagnation points from which it is possible to sketch orthogonal trajectories to the contour lines and these trajectories are joining the natural watershed at $l \sim 800$ m. Taking into account the symmetry around the s-axis the trajectories form a closed curve bounding the catchment area of the well. A planimetric measurement shows that this area is 40 ha and is divided by the natural watershed in a left part of 24 ha and a right part of 16 ha. Remembering that the seepage is 300 mm/year, we get a total seepage per year of 120,000 m^3 over the catchment area divided into 72,000 m^3 to the left and 48,000 m^3 to the right, which figures are in complete accordance with the yearly consumption and yearly volumes withheld from the two rivers, compare Fig. 7.

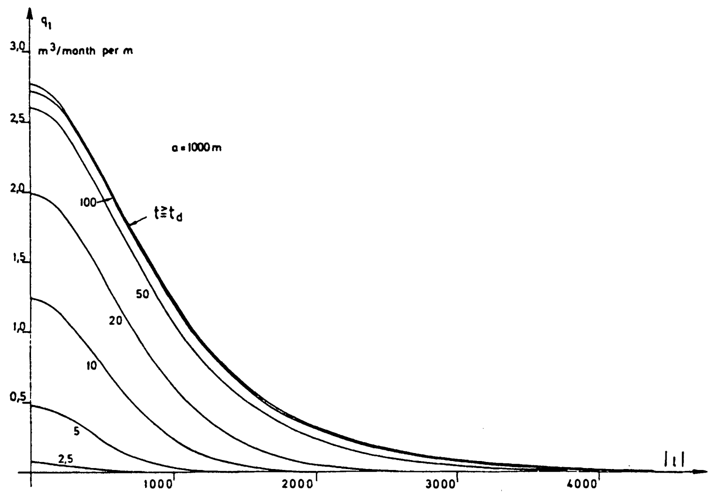

Fig. 8. Distribution and transition of q_1 at river I.

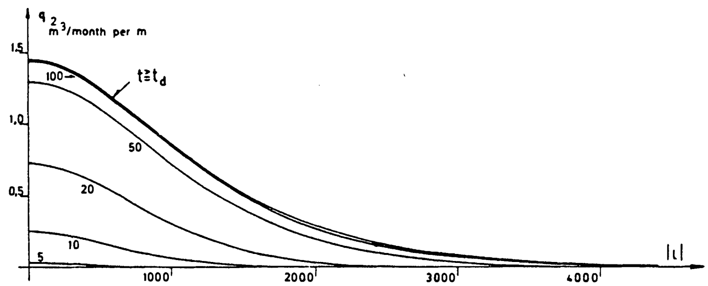

Fig. 9. Distribution and transition of q_2 at river II.

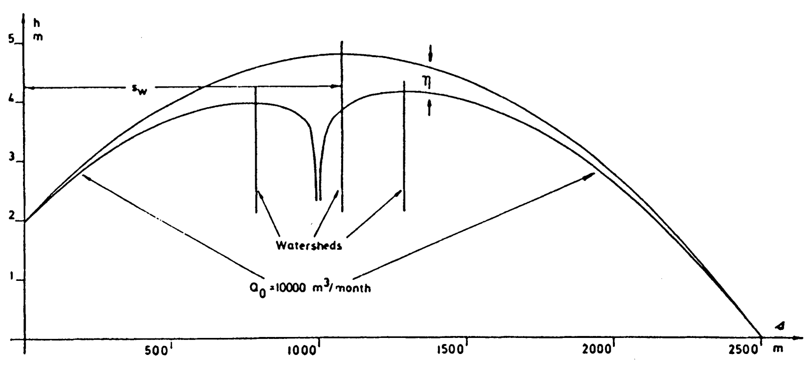

Fig. 10. Cross section of the phreatic surface before and after the transition.

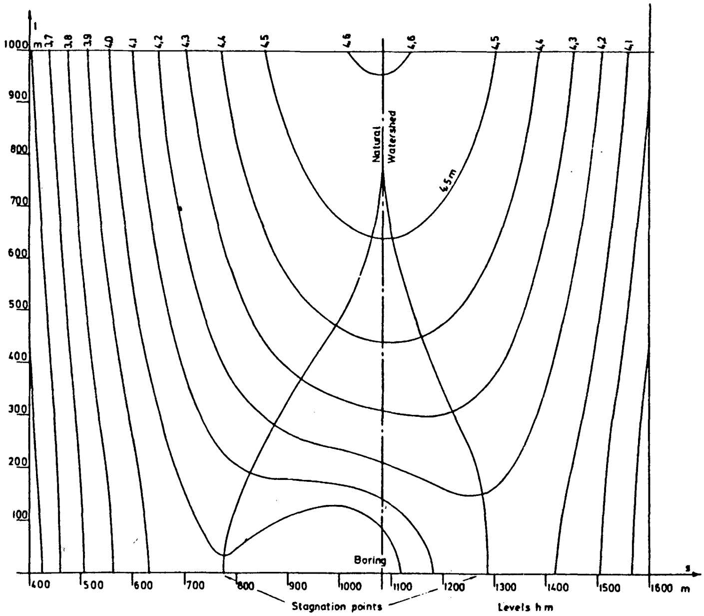

Fig. 11. Contour lines and watersheds of the phreatic surface after the transition.

Conclusions

The four problems mentioned in the introduction have been solved. The actual technical quantities are found as products of dimensionless mathematical functions and combinations of the local geo-hydraulic parameters.

It is reasonable to point out that the definition of the catchment area of the well renders it possible to discuss in a meaningful way the transport of pollutants caused by the groundwater flow towards the well.

It should also be emphasized that the mathematical functions, because of their asymptotical nature, define partly the practical zone of influence of the well, partly the practical duration of the transition. These practical limits are found directly from the actual geo-hydraulic parameters.

Main Symbols

The units of the symbols is noted in a parenthesis; any consistent system of units can be used.

P –	Seepage through the unsaturated zone	$[LT^{-1}]$
h_1 and h_2	Levels of the water surfaces in the two rivers	[L]
L –	Distance between the rivers	[L]
h –	Level of the phreatic surface	[L]

T	–	Transmissivity of the aquifer	$[L^2T^{-1}]$
S	–	Specific yield of the aquifer	[-]
s and l		Coordinates perpendicular to and along river I	[L]
t	–	Time coordinate	[T]
Q_0	–	Discharge of the well	$[L^3T^{-1}]$
Q_1 and Q_2		Discharges withheld from the two rivers	$[L^3T^{-1}]$
q_1 and q_2		Intensity of discharge towards the two rivers	$[L^2t^{-1}]$
η	–	Depression of the phreatic surface	[L]
R	–	Distance from the well	[L]
a_1	–	Distance from the well to river I	[L]
t_d	–	Duration of the transition	[T]

The following symbols are all dimensionless

(x,y,τ)	–	Coordinates related to (s, l, t)
r	–	Distance related to R
ϕ,ϕ_1,ϕ_2, and ϕ_3	–	Functions related to η
$\delta\phi/\delta x\|_{x=0}$ etc.	–	Functions related to q_1
ψ_1, ψ_2, and ψ_3	–	Functions related to Q_1

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