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Sequential Generation of Short Time-Interval Rainfall Data

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Hourly and six-minute interval models were developed to stochastically generate sequences of rainfall depths. The models were applied to data recorded at twelve rainfall stations in Australia, including several from the arid zone. At each station, several replicates of rainfall sequences of length equal to the historical record were generated. Based on a range of parameters covering short rainfall bursts up to annual totals, the models were shown to be performing satisfactorily.

Introduction

Rainfall sequences are necessary in many hydrological applications, such as reservoir operation studies, streamflow generation, and planning and design of urban drainage systems. Short time-interval streamflow data, such as peakflow rates and hourly average flows cannot be easily treated with current stochastic streamflow models. As a result, in the design of urban drainage systems, empirical procedures have been used to estimate peak discharges, and generally the statistical properties of runoff are assumed to be identical to those of the input rainfall. In recent times, rainfall-runoff models have been used with design storms that have been obtained from intensity-duration-frequency curves. Normally, the rainfall depth is disaggregated by a synthetic pattern. There are difficulties in choosing a suitable pattern, as is confirmed by the large number of synthetic patterns that have been

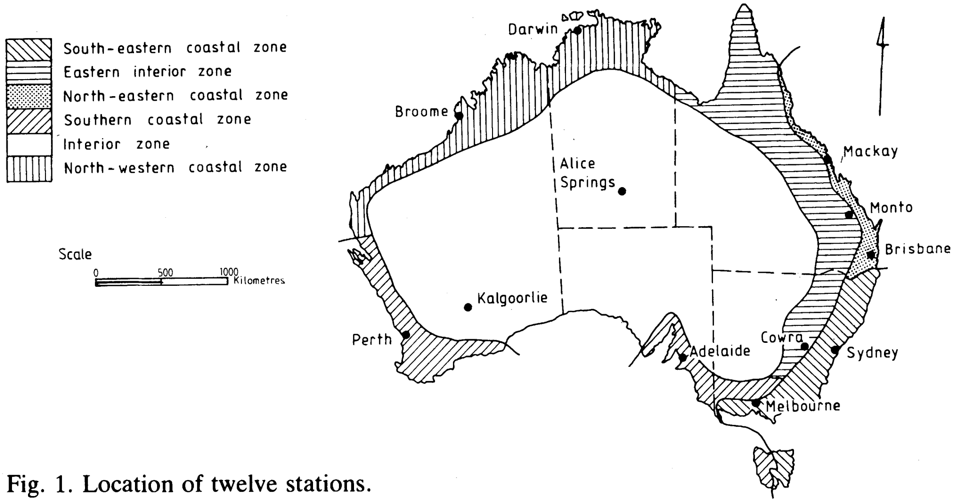

Fig. 1. Location of twelve stations.

proposed (Keifer and Chu 1957; Pilgrim et al. 1969; Chien and Sarikelle 1976; Hall 1977), and by the various approaches that have been advanced.

Because a generally acceptable procedure for the derivation of a synthetic temporal pattern has not been achieved, we provide in this paper a means of generating rainfall sequentially at short time intervals. Two time intervals were chosen for simulation. One was an hourly interval, which was adopted so that the generating model could be used to provide data as input into many rainfall-runoff models. The second interval – one tenth of an hour or six minutes – is the time unit adopted for digitization of Australian rainfall data and is also the basic time interval of operation of the Australian Representative Basins rainfall-runoff model (Chapman 1968).

The models were applied to twelve rainfall stations throughout Australia, as shown in Fig. 1. Details about the data are given in Table 1.

Literature Review

Hourly Models

Chow and Ramaseshan (1965) represented the annual storms for the French Broad River Basin at Brent Creek, North Carolina, by a first order non-stationary Markov chain model with log-normally distributed random components. The annual storms are those which produced the maximum peak discharge in a water year. Since the time distribution patterns of the annual storms were different, Chow and Ramaseshan carried out a "storm shifting" procedure to obtain the best storm orientation, so that the mean, standard deviation, trend and random com-

Table 1 – Rainfall data used in this study.

Station name	Reference number	Location				Data period	Number of years of data
		Latitude		Latitude			
		° S	'	° E	'		
Melbourne	86 071	37	49	144	58	1951-80	30
Sydney	66 062	33	52	151	12	1935-80	46
Monto	39 104	24	51	151	01	1963-80	17
Cowra	63 023	33	49	148	42	1941-80	32
Mackay	33 119	21	07	149	13	1959-81	22
Brisbane	40 214	27	28	153	02	1930-80	51
Darwin	14 016	12	24	130	48	1953-81	28
Broome	03 003	17	57	122	15	1948-79	31
Perth	09 034	31	57	115	51	1946-80	34
Adelaide	23 000	34	56	138	35	1930-79	49
Alice Springs	15 590	23	24	133	32	1951-81	30
Kalgoorlie	12 038	30	47	121	27	1939-79	40

ponents of the hourly rainfall became regular and consistent. The storms were assumed to have the same duration which was taken approximately equal to the longest duration of the storms under consideration. They successfully applied a first order autoregressive model to generate sequentially the hourly rainfall depths within the annual storms having constant duration.

Pattison (1965) represented the hourly rainfall process by a sixth-order Markov chain model (Model I). The model sometimes assumed the characteristics of first-order dependence and sometimes those of sixth-order dependence between observations of hourly rainfall. If the state of the hourly rainfall process during hour t was wet, the model used first-order dependence to determine the state of the process during hour $(t+1)$. If the state of the rainfall process during hour t was dry, the model adopted sixth-order dependence characteristics to determine the state of the process during hour $(t+1)$. Pattison divided the rainfall amounts into 20 states and generated hourly rainfall using transition probabilities. The transition probabilities were assumed to vary from month to month, but remain constant during a day. Results from the model application at one station showed that it adequately described the hourly rainfall process during storm periods. However, dry periods between storms were generally longer than those observed in nature.

Pattison also proposed a second model (Model II), which distributed the observed daily rainfall over the 24 hours preceding the hour of observation. A number of different procedures were investigated and, of these, only a linear regression type model was found to produce acceptable results.

Franz (1971) divided a year into four seasons, namely, dry, wet, and the transistions form one to the other. A multivariate normal distribution was the basic component of his model. The hourly data were normalized by using a generalized power transformation and then a first order Markov model was used to generate the rainfall depth. A variety of distributions were fitted to the lengths of the dry periods, but no analytical expressions were found to be suitable. As a consequence, empirically defined distributions were used to generate the length of the dry period between storms as a function of time of year.

Croley et al. (1978) used six divisions of the year to account for seasonal non-stationarity. Intervals of rainfall corresponding to storm events were scheduled by an exponentially distributed interarrival time model. Intrastorm structure was described in terms of storm segments which were in turn modelled by independent random variables. The intensity and distribution of precipitation within the storm segments were modelled by fitting log-normal intensity probability distributions and by cataloguing sample storm segment shapes.

All the models described above did not consider the variation in the probability of rain occurring throughout a day.

Nguyen and Rouselle (1981) proposed a stochastic model to determine the probability distributions of rainfall accumulated at the end of each hour within a total storm duration. The hourly rainfall depth was assumed to be an exponentially distributed random variable. A second-order Markov model was found to describe more adequately the sequence of wet hours than a first-order model. However, the distribution functions of accumulated amounts of consecutive hourly rainfalls obtained by using the second-order model were not found to be significantly different from those given by using the first-order model. Hence, Nguyen and Rouselle concluded that the use of the first-order Markov model in their study was equally acceptable to using the second-order model. Even though this model considered the variation in probability of occurrence of rain throughout a day, hourly rainfall depth was assumed to be an exponentially distributed random variable. Consequently, the persistence between hourly rainfall depths was ignored.

Six-Minute Models

No publications dealing with the generation of six-minute rainfall data were found in the literature. However, the following three papers consider events of ten-minute durations.

Grace and Eagleson (1966) separated the series of rainfall depths into storms by establishing a length of time τ_L minutes, such that on average a rainfall event in any ten-minute period was not influenced by any event which occurred at least τ_L minutes before it. Correlation relating the time between storms to the storm durations was found to be very small; a Weibull distribution was used to generate these two variables. On the other hand, storm duration and storm depth were found to be highly correlated. Historical storms were divided into three types, namely, trace storms, moderate storms, and peaked storms. For each type, a regression line was fitted to the storm depth and storm duration. During data generation, no random component was added to the trace storm amounts, but for the other two types, random components were added, using a Beta distribution. A modified Bose-Einstein urn model (Grace and Eagleson 1966) was used to distribute the total storm depth among the time intervals. The procedure was one of trial and error and "... it is essentially impossible to obtain perfect agreement between the characteristics of the simulated and actual storm interiors" (Grace and Eagleson 1966, p. 87). The authors claim that the method adequately generates synthetic rainfall from a moderate sample of historical data, but no results dealing with the parameters of the generated ten-minute rainfall are given in their report.

Raudkivi and Lawgun (1972, 1974) generated the length of rainfall durations that were serially correlated and non-normally distributed, using an autoregressive scheme with a Pearson type III distribution. Each year of the data was separated into months to remove seasonal effects. Data for the same calendar months of different years were assumed homogeneous. Rainfall depths within a given duration were generated by using transition probability matrices. The time intervals between rainfalls were assumed to be independent and sampled from an empirical cumulative distribution function because almost all of the theoretical distributions fitted to the data failed to describe the lower tail (short intervals). Here again, the authors did not give any results of the ten-minute rainfall, other than the cumulative distribution of rainfall yield.

Hourly Rainfall

Generating Model

Hourly rainfall data are generated in two stages. In the first stage, a daily transition probability matrix (TPM) is used to determine the state of a day (wet or dry). The number of states varies with station and month, up to a maximum of seven. The state limits and the number of states for the rainfall stations used to test the models are given in Tables 2 and 3 respectively. State 1 is dry, and the other states

Table 2 – State limits used in daily TPM.

State	Upper state limit
1	0
2	1
3	3
4	7
5	15
6	31
7	∞

Table 3 – Number of states used for various stations in daily TPM.

Station	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Melbourne	6	6	6	6	6	6	6	6	6	6	6	6
Sydney	7	7	7	7	7	7	7	7	7	7	7	7
Monto	6	6	6	6	6	6	6	6	6	6	6	6
Cowra	6	6	6	6	6	6	6	6	6	6	6	6
Mackay	6	6	6	6	6	6	6	6	6	6	6	6
Brisbane	7	7	7	7	7	7	7	7	7	7	7	7
Darwin	7	7	7	7	3	2	2	2	3	7	7	7
Broome	7	7	7	3	3	3	3	3	3	3	3	5
Perth	6	6	6	6	6	6	6	6	6	6	6	6
Adelaide	6	6	6	6	6	6	6	6	6	6	6	6
Alice Springs	4	4	4	4	4	4	4	4	4	4	4	4
Kalgoorlie	5	5	5	5	5	5	5	5	5	5	5	5

are wet. The daily TPM procedure has been used to successfully generate daily rainfalls (Srikanthan and McMahon 1983a). If the day is wet, rainfall depths are generated at hourly intervals in the second stage.

Several models, including many variations, were successively tested for their ability to generate hourly rainfall depths on wet days. These models included:

(i) Hourly TPM method – Hourly rainfall depths on wet days are generated using an hourly TPM.

(ii) Two-state second-order Markov chain with hourly TPM – Wet hours on a wet day are first determined using a two-state second-order Markov chain and rainfall depths during the wet hours are generated from an hourly TPM.

(iii) Spell-distributions and hourly TPM – Wet and dry spells are obtained from fitted distributions and an hourly TPM is used to generate rainfall depths during wet spells.

The results from these models were found to be unsatisfactory; monthly and annual rainfalls were generally too large. (Details can be found in Srikanthan and McMahon 1983b).

To overcome this inadequacy, wet days were divided into two types – those of low and of high rainfall – as follows:

Type 1 – rainfall depth $< RF$
Type 2 – rainfall depth $\geqq RF$

where RF is the dividing rainfall depth.

The following models were tried with this modification:

(iv) Two sets of hourly TPM based on the type of wet day are used to generate hourly rainfall.

(v) Two-state second-order Markov chain with an hourly TPM corresponding to each type of wet day.

Results from models (iv) and (v) indicated that dividing a wet day into two types produces satisfactory annual rainfall parameters. The main drawback with model (iv) was that it did not take into account the variations in probability of rainfall throughout a day. One way of doing this was to use a time dependent Markov chain, and this is possible with model (v). Consequently, model (v) was adopted to generate hourly rainfall; it is described below.

In order to preserve the monthly variations, each month was considered separately. Because of this and the need to use two types of wet days, it was necessary, due to data limitations, to group the hours in a day into six units, each of 4 hours duration. Markov chain probabilities were assumed to vary from one unit to another, but to remain constant within a 4-hour unit. The occurrence of rainfall in any hour was determined from this time dependent second-order Markov chain and then the hourly TPM was used to generate rainfall depths.

The number of states used for the hourly TPM is given in Table 4. State 1 is dry and the other states are wet. The upper state limits in mm for the states are

$$0, \quad 0.4, \quad 1.0, \quad 2.0, \quad 4.0, \quad 8.0, \quad \infty$$

If the number of states is k, it should be noted that in Table 4 the upper state limit for the k^{th} state will be infinity. A linear distribution was used for intermediate states (see Appendix I for details) and the Box-Cox transformation for the largest state; thus

$$y = (x - \Delta)^{\lambda}$$

in which Δ is the lower state limit of the largest state, λ is a parameter to be estimated, and y is the normalized variate corresponding to a value x in the largest state.

The major steps involved in the hourly generation process are given below.

Step 1: Generate a uniformly distribution random number $U_d(0,1)$. Using the daily TPM corresponding to the month, determine whether the day is dry or wet. If it is dry, repeat this procedure; otherwise go to Step 2.

Step 2: Based on U_d and using daily TPM, determine the type of wet day (1 or 2). Generate another uniformly distributed random number $U_m(0,1)$. Using hourly Markov chain probabilities corresponding to the time unit of the day, type of wet day and month, determine whether the hour is dry or wet. If it is dry, repeat this procedure. If it is wet, generate the hourly rainfall depth using the corresponding hourly TPM. When 24 values of hourly rainfall are generated, go to Step 1.

Steps 1 and 2 are repeated until the required length of data is generated.

Table 4 – Number of states for various stations in hourly TPM.

Station	RF mm	Type	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Melbourne	15	1	5	5	5	5	5	5	5	5	5	5	5	5
		2	6	6	5	6	6	4	5	5	5	6	6	6
Sydney	31	1	7	7	7	6	6	6	6	6	6	7	7	7
		2	7	7	7	7	7	6	6	6	6	6	6	6
Monto	15	1	5	5	5	4	4	4	4	4	4	5	5	5
		2	6	6	4	4	4	5	4	2	3	3	6	6
Cowra	15	1	5	5	5	5	5	5	5	5	5	5	5	5
		2	6	6	6	5	6	5	5	5	5	6	6	6
Mackay	15	1	5	5	5	5	5	4	4	4	3	4	5	5
		2	7	7	7	7	6	6	3	3	2	3	3	7
Brisbane	31	1	7	7	7	6	6	6	6	6	6	7	7	7
		2	7	7	7	6	6	6	6	6	5	7	7	7
Darwin	31	1	7	7	7	6	6	2	2	2	3	6	6	6
		2	7	7	7	6	–*	–	–	–	–	3	5	7
Broome	15	1	5	5	5	5	5	4	3	2	2	2	3	4
		2	7	7	6	–	–	–	–	–	–	–	–	4
Perth	15	1	5	5	5	5	6	6	6	6	5	5	5	5
		2	–	–	–	5	6	6	6	6	5	5	4	–
Adelaide	15	1	5	5	5	6	6	6	6	6	6	5	5	5
		2	6	6	5	6	6	6	6	5	5	6	6	6
Alice Springs	7	1	4	4	3	3	4	3	4	3	4	4	4	4
		2	5	5	5	5	5	5	5	5	5	5	5	5
Kalgoorlie	7	1	3	4	4	4	4	4	4	4	4	4	4	4
		2	4	5	4	4	5	5	5	4	4	–	5	–

* Because of the small number of wet days, months marked by a dash are not divided into two types.

Application of Hourly Model

The hourly model was applied to the twelve rainfall stations listed in Table 1. Daily TPMs were first calculated using the state limits and number of states given in Tables 2 and 3 respectively. If the number of states is k, it should be noted that in Table 2 the upper state boundary for the k^{th} class will be infinity. Thus the whole of Table 2 applies only to a month with seven states. The wet days were divided into two types using the dividing rainfall depth given in Table 4. For each type of wet day, Markov chain probabilities and hourly TPMs were calculated. The number of states used for each month is also given in Table 4. The parameter λ in the Box-Cox transformation for the largest state was calculated by trial and error, such that the skewness of the transformed values was close to zero. Eight replicates, each of length equal to the historical record, were generated for each of the twelve stations.

Model Testing

The following hourly parameters were used to compare the generated parameters based on the averages of the eight replicates with the historical parameters for each calendar month:

(i) maximum hourly rainfall:
(ii) mean, standard deviation and coefficient of skewness of hourly rainfall;
(iii) mean, standard deviation and coefficient of skewness of hours of wet spell;
(iv) longest wet spell;
(v) correlation between rainfall depth and duration; and
(vi) correlation between successive hourly rainfall depths.

The hourly rainfall depths were aggregated into daily, monthly and annual rainfalls, and the following daily, monthly, and annual parameters were compared with historical values:

(i) average number of wet days for each month;
(ii) maximum daily rainfall for each month;
(iii) mean, standard deviation and coefficient of skewness of daily rainfall depths;
(iv) mean and standard deviation of monthly and annual rainfall depths; and
(v) maximum and minimum of monthly and annual rainfall depths.

Finally, frequency distributions of hourly, 6-hour, daily and 3-day annual maximum rainfalls, calculated from the generated data, were compared with historical ones.

Hourly Results

Data synthesis using stochastic models is considered satisfactory if the values of the historical parameters fall within an appropriate confidence band of the parameters based on the generated sequences, and if the averages of the generated parameters are generally close to the historical values. In this study, because the number of replicates was small (eight for the hourly generation), these criteria need to be relaxed.

Because of the restriction of space, only four months of monthly data are presented for each station. Also, only detailed results for Melbourne are given, as these are typical of the results for the other stations. A complete set of results can be found in Srikanthan and McMahon (1983b). Furthermore, no statistical tests are employed to assess the level of modelling. Rather, the parameters of the historical and generated sequences are tabulated for visual inspection, which is common practice in this field of study (Phien and Vithana 1983).

In Table 5, maximum rainfalls are shown to be satisfactorily preserved, except for two months at Brisbane and Darwin; maximum rainfall was large in December for both stations. Even though the average longest wet spell lengths are smaller than the corresponding historical values, the latter lie in the range of values obtained from various replicates for most cases (Table 6). The correlation between rainfall depth and duration is satisfactorily preserved, as shown in Table 7, while that between hourly rainfall depths is somewhat smaller than the corresponding historical values (Table 8).

The means, standard deviations and coefficients of skewness of hourly rainfall are satisfactorily preserved for all the stations and all months. Results for only one station, namely Melbourne, are given as Table 9. Except for a few cases, the mean wet spell lengths and the coefficients of skewness of wet spell lengths are also satisfactorily reproduced (Table 10). The standard deviation of wet spell lengths is found to be smaller than the corresponding historical values. The frequency distributions of the generated and historical sequences of hourly and 6-hour maximum rainfall are similar for all stations (see, as an example, Fig. 2 for Melbourne).

Except for the dry months of Darwin (May to September) and Broome (June to November), means, standard deviations and coefficients of skewness of daily rainfall are satisfactorily preserved (see Table 11 for Melbourne and Darwin). Table 12 indicates that the number of wet days is well reproduced, and Table 13 indicates satisfactory generation of maximum daily rainfalls for most months. The generated and historical frequency distributions of daily annual maximum rainfall are similar for all stations except Mackay and Kalgoorlie, while those of 3-day annual maximum rainfall are similar for all stations except Mackay, Brisbane, Broome, Alice Springs and Kalgoorlie. Figs. 2 and 3 show the frequency distributions for Melbourne (satisfactory fit) and Mackay (poor fit) respectively.

Except for the dry months of Darwin and Broome (mentioned above), monthly and annual means are satisfactorily preserved for all months. However, the standard deviation generally is found to be smaller than the corresponding historical value for most months, although this is not so for the data tabulated for Melbourne (Table 14). Monthly and annual maximum and minimum rainfalls are satisfactorily reproduced.

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* The values in brackets give the range of the estimates in Tables 5 to 14 and 16 to 18.

Table 5 – Comparison of historical and generated maximum hourly rainfall (mm).

Station		January	April	July	October	Annual
	Hist	23	16	10	18	65
Melbourne	Gen	20	19	11	19	45
		(27-15)*	(32-14)	(14-9)	(25-14)	(61-34)
	Hist	62	47	50	30	68
Sydney	Gen	50	55	37	30	80
		(70-29)	(80-35)	(58-20)	(39-24)	(101-54)
	Hist	31	23	13	26	59
Monto	Gen	42	28	19	44	64
		(58-20)	(45-22)	(26-15)	(55-26)	(143-44)
	Hist	24	13	10	11	24
Cowra	Gen	44	19	11	12	49
		(105-19)	(31-14)	(14-8)	(15-10)	(105-26)
	Hist	60	38	20	54	69
Mackay	Gen	64	33	28	48	93
		(87-51)	(51-26)	(45-18)	(75-29)	(123-78)
	Hist	77	33	16	43	77
Brisbane	Gen	70	49	22	42	110
		(129-44)	(65-34)	(28-16)	(59-33)	(168-70)
	Hist	67	47	8	88	88
Darwin	Gen	72	66	11	71	123
		(91-48)	(75-54)	(15-4)	(90-45)	(134-91)
	Hist	112	47	12	10	112
Broome	Gen	107	43	11	16	118
		(153-69)	(81-21)	(17-9)	(21-12)	(169-89)
	Hist	10	21	26	20	26
Perth	Gen	8	31	24	22	33
		(18-4)	(67-15)	(34-20)	(28-16)	(44-23)
	Hist	18	19	13	19	36
Adelaide	Gen	16	23	18	16	41
		(24-12)	(46-16)	(30-11)	(26-11)	(64-29)
	Hist	53	11	13	17	73
Alice	Gen	46	20	13	17	64
Springs		(69-23)	(38-11)	(19-10)	(29-10)	(100-46)
	Hist	19	12	8	10	31
Kalgoorlie	Gen	34	20	11	14	41
		(45-27)	(30-14)	(17-3)	(21-11)	(47-33)

Table 6 – Comparison of historical and generated longest wet spell lengths (hours).

Station		January	April	July	October	Annual
	Hist	29	40	43	24	43
Melbourne	Gen	21	27	31	20	35
		(29-14)	(33-19)	(43-22)	(27-15)	(49-28)
	Hist	62	40	39	55	69
Sydney	Gen	40	38	35	33	53
		(60-28)	(48-26)	(55-23)	(45-25)	(65-46)
	Hist	37	31	18	11	37
Monto	Gen	21	18	30	15	33
		(29-16)	(22-13)	(42-19)	(25-10)	(42-23)
	Hist	41	26	29	20	18
Cowra	Gen	21	21	27	25	36
		(26-17)	(26-17)	(37-18)	(35-19)	(49-26)
	Hist	32	24	17	18	41
Mackay	Gen	26	16	18	15	37
		(41-20)	(19-13)	(25-11)	(20-12)	(50-29)
	Hist	47	31	45	52	56
Brisbane	Gen	29	26	37	26	47
		(33-24)	(39-21)	(52-26)	(40-19)	(59-37)
	Hist	26	30	3	9	35
Darwin	Gen	25	17	3	8	27
		(29-22)	(33-12)	(4-2)	(11-6)	(33-21)
	Hist	25	35	18	4	31
Broome	Gen	20	12	16	5	25
		(28-13)	(15-10)	(20-12)	(6-5)	(31-19)
	Hist	13	15	24	40	42
Perth	Gen	12	20	33	22	37
		(16-11)	(27-16)	(49-27)	(29-16)	(49-32)
	Hist	26	21	25	20	33
Adelaide	Gen	21	24	21	22	29
		(25-16)	(30-19)	(27-18)	(26-19)	(34-27)
	Hist	41	31	29	16	55
Alice	Gen	20	30	24	16	34
Springs		(33-19)	(24-12)	(35-17)	(18-11)	(43-24)
	Hist	27	18	23	9	36
Kalgoorlie	Gen	24	15	21	11	26
		(36-18)	(16-13)	(27-18)	(13-9)	(36-20)

Table 7 – Comparison of historical and generated correlation between rainfall depth and duration.

Station		January	April	July	October
	Hist	0.75	0.75	0.79	0.78
Melbourne	Gen	0.72	0.74	0.76	0.71
		(0.75-0.63)	(0.76-0.71)	(0.79-0.71)	(0.74-0.68)
	Hist	0.81	0.78	0.78	0.82
Sydney	Gen	0.75	0.77	0.68	0.71
		(0.77-0.72)	(0.82-0.68)	(0.72-0.64)	(0.76-0.68)
	Hist	0.71	0.70	0.71	0.56
Monto	Gen	0.70	0.69	0.89	0.57
		(0.78-0.63)	(0.76-0.61)	(0.92-0.87)	(0.71-0.44)
	Hist	0.81	0.77	0.82	0.78
Cowra	Gen	0.67	0.77	0.77	0.81
		(0.79-0.57)	(0.80-0.73)	(0.79-0.74)	(0.84-0.79)
	Hist	0.83	0.72	0.64	0.75
Mackay	Gen	0.77	0.62	0.57	0.51
		(0.80-0.72)	(0.64-0.61)	(0.69-0.50)	(0.66-0.40)
	Hist	0.77	0.74	0.80	0.75
Brisbane	Gen	0.73	0.64	0.78	0.62
		(0.74-0.65)	(0.67-0.62)	(0.84-0.71)	(0.73-0.55)
	Hist	0.68	0.73	–	0.55
Darwin	Gen	0.59	0.71	–	0.55
		(0.66-0.55)	(0.80-0.56)	–	(0.67-0.41)
	Hist	0.75	0.84	0.76	0.68
Broome	Gen	0.69	0.48	0.79	0.45
		(0.77-0.60)	(0.57-0.41)	(0.83-0.74)	(0.48-0.42)
	Hist	0.51	0.64	0.73	0.77
Perth	Gen	0.57	0.72	0.76	0.72
		(0.59-0.53)	(0.75-0.69)	(0.77-0.74)	(0.74-0.70)
	Hist	0.77	0.71	0.71	0.71
Adelaide	Gen	0.76	0.70	0.70	0.70
		(0.81-0.70)	(0.72-0.68)	(0.73-0.67)	(0.72-0.69)
	Hist	0.71	0.80	0.81	0.71
Alice	Gen	0.70	0.77	0.81	0.69
Springs		(0.76-0.63)	(0.83-0.70)	(0.85-0.78)	(0.75-0.63)
	Hist	0.68	0.52	0.85	0.68
Kalgoorlie	Gen	0.77	0.60	0.76	0.53
		(0.79-0.75)	(0.65-0.56)	(0.80-0.70)	(0.57-0.49)

Table 8 – Comparison of historical and generated correlation between hourly rainfall depths.

Station		January	April	July	October
Melbourne	Hist	0.61	0.63	0.60	0.55
	Gen	0.49	0.49	0.42	0.43
		(0.52-0.47)	(0.54-0.46)	(0.48-0.36)	(0.47-0.39)
Sydney	Hist	0.71	0.74	0.69	0.56
	Gen	0.54	0.46	0.44	0.39
		(0.62-0.52)	(0.55-0.40)	(0.50-0.38)	(0.50-0.30)
Monto	Hist	0.41	0.41	0.77	0.17
	Gen	0.19	0.13	0.29	0.20
		(0.25-0.15)	(0.20-0.10)	(0.40-0.20)	(0.28-0.09)
Cowra	Hist	0.22	0.48	0.62	0.44
	Gen	0.19	0.28	0.39	0.30
		(0.27-0.15)	(0.30-0.26)	(0.50-0.31)	(0.38-0.22)
Mackay	Hist	0.57	0.38	0.67	0.30
	Gen	0.37	0.25	0.34	0.19
		(0.40-0.35)	(0.29-0.21)	(0.40-0.26)	(0.24-0.13)
Brisbane	Hist	0.48	0.62	0.83	0.44
	Gen	0.39	0.40	0.59	0.37
		(0.41-0.37)	(0.47-0.35)	(0.64-0.55)	(0.40-0.31)
Darwin	Hist	0.33	0.30	–	0.09
	Gen	0.26	0.27	–	0.04
		(0.30-0.20)	(0.30-0.24)	–	(0.08-0.01)
Broome	Hist	0.44	0.42	0.29	–
	Gen	0.25	0.16	0.20	–
		(0.30-0.16)	(0.20-0.10)	(0.33-0.07)	–
Perth	Hist	0.26	0.39	0.45	0.37
	Gen	0.15	0.32	0.33	0.34
		(0.26-0.09)	(0.42-0.26)	(0.35-0.31)	(0.41-0.27)
Adelaide	Hist	0.60	0.41	0.52	0.50
	Gen	0.50	0.34	0.34	0.40
		(0.60-0.45)	(0.40-0.28)	(0.40-0.30)	(0.48-0.30)
Alice	Hist	0.23	0.38	0.78	0.35
Springs	Gen	0.14	0.40	0.50	0.23
		(0.20-0.10)	(0.49-0.35)	(0.58-0.41)	(0.30-0.10)
Kalgoorlie	Hist	0.66	0.44	0.51	0.44
	Gen	0.32	0.26	0.34	0.19
		(0.36-0.31)	(0.35-0.18)	(0.48-0.24)	(0.28-0.09)

Table 9 – Comparison of mean, standard deviation and coefficient of skewness of hourly rainfall for Melbourne obtained from the historical and generated sequences.

Parameter		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean (mm)	Hist	12.1	14.2	9.3	10.0	8.6	6.2	6.8	7.1	8.1	9.0	9.8	10.3
	Gen	10.8	13.9	8.9	9.1	8.3	6.6	7.2	7.3	7.7	9.5	9.5	10.6
		(12.3-9.7)	(16.8-11.6)	(9.3-8.5)	(9.7-8.2)	(8.6-7.8)	(7.0-6.3)	(7.8-6.9)	(7.8-6.7)	(8.4-7.0)	(10.2-8.8)	(10.0-9.1)	(11.4-8.7)
Standard deviation (mm)	Hist	21.9	31.2	14.1	16.0	13.7	8.4	9.9	9.6	11.4	14.4	15.0	20.1
	Gen	19.6	27.8	13.0	14.4	12.4	8.5	9.9	9.2	10.6	14.8	14.0	20.9
		(22.1-17.1)	(32.6-21.5)	(13.9-11.9)	(17.5-12.7)	(13.4-10.6)	(9.2-8.0)	(10.5-9.5)	(9.9-8.4)	(12.1-9.7)	(16.7-13.8)	(14.9-12.4)	(23.7-14.4)
Coefficient of skewness	Hist	4.02	9.83	3.95	3.76	4.02	3.06	3.38	2.94	3.30	4.19	3.59	5.73
	Gen	4.53	6.13	4.02	4.63	4.42	3.47	3.47	3.15	4.23	4.68	3.68	7.50
		(5.8-3.7)	(9.1-5.1)	(4.8-3.2)	(6.4-3.8)	(6.2-2.9)	(4.9-3.0)	(4.1-3.1)	(3.7-2.5)	(6.7-3.2)	(5.3-4.1)	(4.1-2.9)	(11.1-4.7)

Table 10 – Comparison of mean, standard deviation and coefficient of skewness of wet spells for Melbourne obtained from the historical and generated sequences.

Parameter		January	April	July	October
Mean	Hist	3.4	3.5	3.3	3.0
	Gen	3.2	3.3	3.3	3.1
		(3.4-3.0)	(3.4-3.1)	(3.5-3.1)	(3.1-3.0)
Standard deviation (hours)	Hist	3.1	3.9	3.6	2.8
	Gen	2.8	3.0	2.9	2.5
		(3.2-2.4)	(3.5-2.8)	(3.3-2.7)	(2.8-2.4)
Coefficient of skewness	Hist	2.98	4.26	5.63	3.03
	Gen	2.53	3.12	3.04	2.22
		(3.1-1.8)	(4.2-2.4)	(4.8-2.2)	(2.8-1.7)

Table 11 - Comparison of mean, standard deviation and coefficient of skewness of daily rainfall obtained from the historical and generated sequences for Melbourne and Darwin.

Station	Parameter		January	March	May	July	September	November
Melbourne	Mean (mm)	Hist	5.9	4.6	4.7	3.2	4.3	5.0
		Gen	5.0	4.0	4.8	4.0	4.1	4.9
			(5.8-4.2)	(4.2-3.7)	(5.3-4.4)	(4.4-3.8)	(4.7-3.7)	(5.4-4.3)
	Standard deviation (mm)	Hist	11.1	7.2	6.9	5.3	6.2	8.1
		Gen	9.4	5.1	7.4	5.9	5.5	8.0
			(11.2-8.1)	(5.6-4.4)	(8.2-6.3)	(6.7-5.4)	(6.8-4.0)	(8.8-6.9)
	Coefficient of skewness	Hist	4.74	3.08	2.97	4.42	3.69	4.43
		Gen	4.27	2.83	3.93	4.55	3.42	4.13
			(5.4-3.3)	(4.1-2.2)	(4.8-3.3)	(5.8-3.8)	(4.1-2.7)	(5.1-3.4)
Darwin	Mean (mm)	Hist	21.0	16.5	8.3	1.5	8.8	10.6
		Gen	18.6	15.8	13.5	4.2	13.6	12.4
			(20.1-16.8)	(17.5-14.4)	(17.7-14.0)	(5.4-2.9)	(15.1-12.3)	(13.1-11.1)
	Standard deviation (mm)	Hist	27.4	21.5	13.5	1.7	10.3	12.3
		Gen	26.6	23.8	18.6	5.3	14.8	21.3
			(29.3-22.9)	(27.3-21.3)	(22.6-12.8)	(6.8-4.5)	(16.3-12.2)	(23.2-19.7)
	Coefficient of skewness	Hist	2.35	3.10	2.47	3.29	1.72	1.76
		Gen	2.87	3.59	1.31	1.98	1.30	4.62
			(3.18-2.64)	(3.7-3.5)	(1.8-0.5)	(2.3-1.7)	(1.9-0.7)	(6.6-3.4)

Table 12 – Comparison of historical generated number of wet days.

Station		January	April	July	October	Annual
	Hist	7.8	11.4	15.7	14.6	149
Melbourne	Gen	7.7	11.5	15.8	14.5	150
		(8.9-6.9)	(12.3-10.7)	(16.8-15.4)	(14.9-13.6)	(152-148)
	Hist	12.1	12.7	10.0	11.8	143
Sydney	Gen	12.1	13.4	10.6	11.9	146
		(12.7-11.3)	(14.3-12.2)	(11.2-9.9)	(12.2-11.5)	(148-142)
	Hist	9.8	5.8	4.8	6.3	80
Monto	Gen	9.8	6.2	5.5	6.5	82
		(10.7-8.7)	(7.6-5.5)	(6.0-4.8)	(7.1-5.9)	(83-80)
	Hist	5.9	6.0	10.3	9.5	95
Cowra	Gen	5.6	6.2	10.6	9.6	95
		(6.5-5.1)	(6.9-5.6)	(11.4-9.3)	(10.2-9.0)	(97-90)
	Hist	16.9	15.6	10.2	7.3	143
Mackay	Gen	17.2	16.2	11.3	7.6	146
		(19.2-15.8)	(18.1-15.0)	(12.9-10.2)	(8.1-6.9)	(149-143)
	Hist	12.8	10.3	6.8	9.8	118
Brisbane	Gen	12.9	10.6	6.7	9.8	119
		(13.4-11.7)	(11.4-9.8)	(7.3-5.9)	(10.3-9.4)	(120-117)
	Hist	19.4	6.1	0.3	5.2	94
Darwin	Gen	18.7	7.4	0.3	5.5	96
		(19.5-17.6)	(8.0-7.0)	(0.4-0.2)	(5.6-5.3)	(97-93)
	Hist	11.3	3.0	2.2	1.2	53
Broome	Gen	11.6	3.6	2.3	1.3	52
		(12.6-10.3)	(9.3-2.4)	(2.6-2.0)	(1.5-0.9)	(54-50)
	Hist	3.0	8.7	19.7	10.4	120
Perth	Gen	3.0	8.8	19.9	10.2	119
		(3.4-2.2)	(9.6-8.1)	(21.2-18.8)	(10.9-9.8)	(121-117)
	Hist	4.9	10.4	16.4	11.6	124
Adelaide	Gen	4.8	10.8	16.9	11.7	126
		(5.1-4.2)	(12.0-9.1)	(17.3-15.9)	(12.8-11.1)	(130-122)
	Hist	4.9	2.2	3.3	4.9	44
Alice	Gen	4.3	2.1	3.6	5.0	42
Springs		(5.1-3.8)	(3.0-1.4)	(4.3-2.7)	(5.2-4.2)	(45-39)
	Hist	3.3	5.7	11.0	4.3	72
Kalgoorlie	Gen	3.5	5.8	11.5	4.3	74
		(3.8-2.5)	(6.4-5.1)	(12.4-10.7)	(4.9-3.6)	(76-71)

Table 13 – Comparison of historical and generated maximum daily rainfall (mm).

Station		January	April	July	October	Annual
	Hist	108	80	48	61	108
Melbourne	Gen	78	67	59	66	98
		(108-45)	(75-57)	(76-49)	(78-52)	(125-72)
	Hist	169	114	127	141	281
Sydney	Gen	162	155	130	112	219
		(187-134)	(283-122	(163-99)	(143-91)	(283-174)
	Hist	141	59	62	80	152
Monto	Gen	89	59	87	86	126
		(119-76)	(63-50)	(105-56)	(123-68)	(196-101)
	Hist	113	55	32	65	113
Cowra	Gen	86	48	45	54	103
		(154-54)	(64-34)	(59-32)	(82-41)	(154-80)
	Hist	247	106	72	103	389
Mackay	Gen	189	84	75	99	210
		(250-147)	(99-70)	(93-59)	(140-61)	(250-178)
	Hist	314	178	193	136	314
Brisbane	Gen	185	129	128	147	233
		(313-174)	(168-79)	(152-101)	(174-103)	(313-193)
	Hist	174	168	5	91	182
Darwin	Gen	202	184	17	163	227
		(264-174)	(221-100)	(24-2)	(174-86)	(242-209)
	Hist	351	107	27	15	351
Broome	Gen	175	66	25	27	202
		(268-133)	(96-43)	(31-19)	(37-17)	(268-159)
	Hist	22	54	68	55	87
Perth	Gen	15	62	77	58	89
		(25-11)	(79-45)	(93-61)	(75-46)	(121-78)
	Hist	50	50	39	43	81
Adelaide	Gen	64	61	49	53	101
		(83-43)	(80-41)	(60-35)	(74-46)	(132-80)
	Hist	85	44	54	36	136
Alice	Gen	85	46	59	40	106
Springs		(113-49)	(57-25)	(83-37)	(52-33)	(134-88)
	Hist	154	50	28	26	178
Kalgoorlie	Gen	79	47	36	25	79
		(115-61)	(61-35)	(43-28)	(32-20)	(115-61)

Table 14 - Comparison of monthly and annual parameters for Melbourne.

Parameter		January	March	May	July	September	November	Annual
	Hist	46	42	74	51	62	63	684
Mean	Gen	39	35	77	62	59	62	688
(mm)		(47-29)	(39-32)	(86-66)	(70-60)	(66-47)	(73-52)	(711-653)
Standard	Hist	37	32	29	24	26	44	131
deviation	Gen	31	21	39	31	28	38	116
(mm)		(40-22)	(26-16)	(53-26)	(35-25)	(39-20)	(47-25)	(165-84)
	Hist	176	137	136	126	113	206	876
Maximum	Gen	117	95	190	153	130	172	910
(mm)		(153-92)	(140-69)	(262-130)	(221-111)	(168-104)	(246-143)	(1033-834)
	Hist	3	5	21	10	17	11	335
Minimum	Gen	1	4	21	17	17	13	465
(mm)		(4-1)	(7-2)	(32-9)	(22-12)	(25-8)	(24-6)	(547-390)

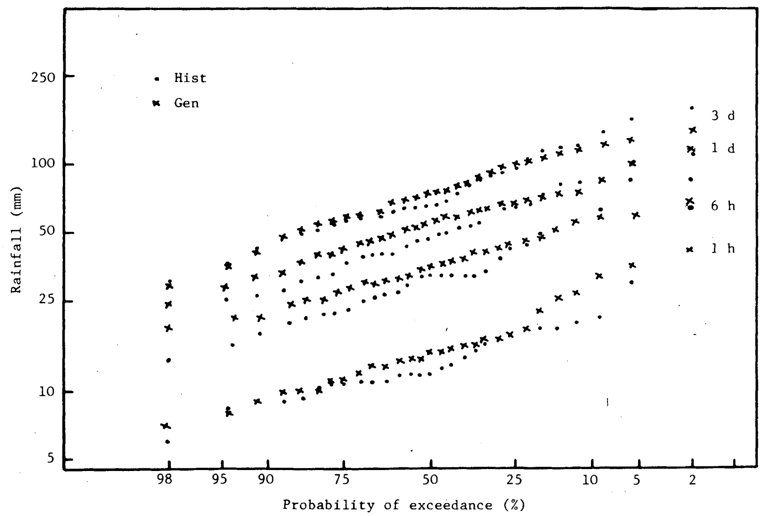

Fig. 2. Depth probability of exceedance curves based on historical and generated data for Melbourne.

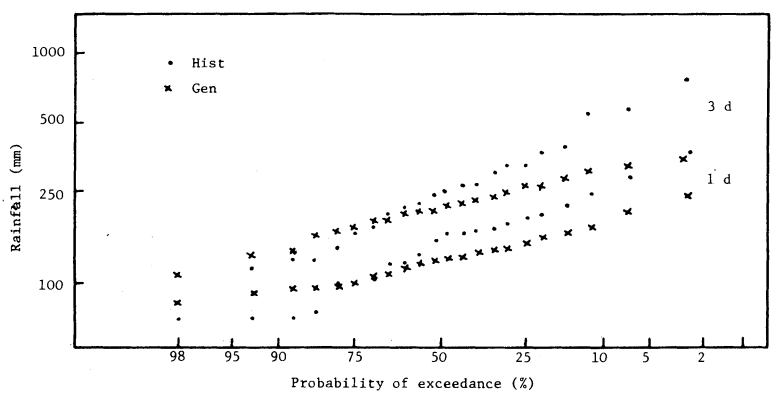

Fig. 3. Depth probability of exceedance curves based on historical and generated data for Mackay.

Six-Minute Rainfall

Generating Model

The general approach to generating six-minute interval rainfall data was to use a daily transition probability matrix to decide on the state of a day – wet or dry. On wet days, a second order non-stationary Markov chain dependent on the type of wet day was used to determine wet hours. Finally, rainfall depths were generated at six-minute intervals by an appropriate model.

However, before a six-minute model was adopted, a number of models were examined, as follows:

(i) a six-minute TPM dependent on the state of the previous two hours;
(ii) a six-minute TPM dependent on the state of the previous three hours;
(iii) a two-state second order Markov chain and six-minute TPM based on the state of the previous three hours;
(iv) a six-minute TPM dependent on the magnitude of the hourly rainfall;
(v) a six-minute TPM and a two-state Markov chain dependent on the magnitude of the hourly rainfall; and
(vi) a two-state second order Markov chain and a Gamma distribution of rainfall depth.

Based on the application of these models and several variations to several data sets, the following model was adopted to generate six-minute rainfall data (Srikanthan and McMahon 1983c). It consists essentially of three submodels, namely, a daily, an hourly, and a six-minute model.

The daily model is a set of transition probability matrices, one for each month. A two-state second order Markov chain (probabilities of which vary with the time of a day) and set of transition probability matrices make up the hourly model. Markov chain probabilities and the transition probability matrices are estimated for each month and each type of day separately. The six-minute model consists of four sets of transition probability matrices, one set for each type of wet hour, as defined below:

Type 1, wet hour – hourly rainfall depth $\leqq RH1$
Type 2, wet hour – $RH1 <$ hourly rainfall depth $\leqq RH2$
Type 3, wet hour – $RH2 <$ hourly rainfall depth $\leqq RH3$
Type 4, wet hour – hourly rainfall depth $> RH3$

where $RH1$, $RH2$, $RH3$ are the dividing rainfall depths.

For the first three types of wet hour, all the states have closed boundaries, while for type 4 wet hour, the largest state is unbounded and the Box-Cox transformation is used. The dividing hourly rainfall depths and the number of states used for various stations are given in Table 15. These were found by trial and error.

Six-minute rainfall data are generated by following the steps given below:

Table 15 – Types of wet hours and the number of states for six-minute TPM.

Station	Hourly rainfall depth (mm)	Type	J	F	M	A	M	J	J	A	S	O	N	D
Melbourne	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	4	4	4	4	4	4	4	4	4	4	4	4
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	5	5	5	5	5	5	5	5	5	5	5	5
Sydney	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	4	4	4	4	4	4	4	4	4	4	4	4
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	7	7	7	7	7	7	7	7	7	7	7	7
Monto	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	3	3	3	3	3	3	3	3	3	3	3	3
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	6	5	5	5	5	5	5	5	5	5	5	5
Cowra	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	3	3	3	3	3	3	3	3	3	3	3	3
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	5	5	5	5	5	5	5	5	5	5	5	5
Mackay	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	4	4	4	4	4	3	3	3	3	3	3	3
	1.0-2.0	3	4	4	4	4	4	4	4	3	3	4	4	4
	2.0-∞	4	7	7	7	6	6	6	5	5	5	6	6	7
Brisbane	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	4	4	4	4	4	4	4	4	4	4	4	4
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	7	7	7	7	6	6	6	6	7	7	7	7
Darwin	0-0.5	1	3	3	3	3	3	–	–	–	2	3	3	3
	0.5-1.0	2	4	4	4	3	2	–	–	–	2	3	4	4
	1.0-2.0	3	5	5	5	4	3	–	–	–	2	3	4	4
	2.0-∞	4	7	7	7	6	4	3	3	3	5	6	7	7
Broome	0-0.5	1	3	3	3	3	3	3	3	2	2	2	2	3
	0.5-1.0	2	3	3	3	3	3	3	2	2	2	2	2	3
	1.0-2.0	3	4	4	4	3	3	3	3	2	2	2	3	4
	2.0-∞	4	7	7	6	5	5	4	4	3	3	4	4	6
Perth	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	3	3	3	4	4	4	4	4	4	4	4	3
	1.0-2.0	3	3	3	4	5	5	5	5	5	5	5	5	3
	2.0-∞	4	2	4	5	6	6	6	6	6	6	6	6	4

cont.

Table 15 cont.

Adelaide	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	3	3	3	4	4	4	4	4	4	3	3	3
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	5	5	5	5	5	5	5	5	5	5	5	5
Alice Springs	0-0.5	1	2	2	2	2	2	2	2	2	2	2	2	2
	0.5-1.0	2	3	3	3	3	3	3	3	3	3	3	3	3
	1.0-2.0	3	3	3	3	3	3	3	3	3	3	3	3	3
	2.0-∞	4	5	5	5	5	5	4	5	5	4	5	5	5
Kalgoorlie	0-0.5	1	3	3	3	3	3	3	3	3	3	3	3	3
	0.5-1.0	2	3	3	3	3	3	3	3	3	3	3	3	3
	1.0-2.0	3	4	4	4	4	4	4	4	4	4	4	4	4
	2.0-∞	4	5	5	5	5	5	5	5	5	5	5	5	5

Step 1: Determine the state of the present day. If dry, proceed to the following day. If the state is wet, determine the type of day (1 or 2).

Step 2: Determine the state of the present hour on a wet day. If dry, proceed to the following hour. If wet, generate the hourly rainfall. Go to step 1 when 24 hours are completed.

Step 3: Determine the type of wet hour and generate the depth of rainfall at six-minute intervals during the wet hour, using the appropriate six-minute transition probability matrix. If the hourly rainfall depth exceeds 5 mm, rainfall is assumed to be continuous over the hour. The generated six-minute interval rainfalls are then linearly adjusted to equal the hourly rainfall depth obtained in step 2. After ten six-minute rainfalls have been generated, return to step 2.

Steps 1 to 3 are repeated until the required length of data is generated.

Model Testing

Three replicates of six-minute rainfall data, each of length equal to the historical record, were generated for all twelve stations. The following six-minute parameters were used to compare the generated parameters, based on the average of three replicates, with historical parameters for each calendar month:

(i) maximum six-minute rainfall depth;
(ii) mean, standard deviation and coefficient of skewness of six-minute interval rainfall;
(iii) longest wet spell; and
(iv) mean, standard deviation and coefficient of skewness of wet spells.

As the six-minute rainfall depths were linearly adjusted to equal the appropriate hourly rainfall depths generated by the hourly model, hourly, daily, monthly and annual parameters need not be examined.

Table 16 = Comparison of historical and generated maximum six-minute interval rainfall (mm)

Station		January	April	July	October
Melbourne	Hist	5.8	6.2	3.6	6.4
	Gen	(8.5-6.2)*	(13-4.6)	(5.3-3.8)	(5.8-4.8)
Sydney	Hist	8.3	10.7	9.7	10.2
	Gen	(23-15)	(13-12)	(19-14)	(7.9-5.3)
Monto	Hist	13.0	5.6	7.8	7.8
	Gen	(13-10)	(8.5-5.8)	(6.8-4.6)	(11-7.5)
Cowra	Hist	12.1	5.2	3.2	6.3
	Gen	(8.9-7.5)	(6.8-4.2)	(6.9-3.1)	(6.5-3.5)
Mackay	Hist	17.8	9.0	5.4	12.2
	Gen	(31-19)	(14-8.5)	(14-8.3)	(18-7.0)
Brisbane	Hist	21.8	12.9	6.5	13.8
	Gen	(24-19)	(14-11)	(10-5.0)	(15-11)
Darwin	Hist	26.3	12.4	3.1	14.5
	Gen	(45-29)	(24-20)	(17-0.7)	(53-30)
Broome	Hist	21.0	9.0	2.6	2.8
	Gen	(39-22)	(16-10)	(9.1-3.9)	(5.2-3.4)
Perth	Hist	4.0	7.2	27.7	5.9
	Gen	(3.2-1.8)	(11-5.6)	(7.9-6.0)	(7.6-5.6)
Adelaide	Hist	6.3	10.0	3.7	5.4
	Gen	(5.3-4.0)	(11.5-6.5)	(4.9-4.1)	(4.7-3.4)
Alice Springs	Hist	12.9	4.7	2.7	5.1
	Gen	(13-10)	(4.5-3.6)	(7.1-3.5)	(8.0-4.5)
Kalgoorlie	Hist	6.6	5.6	3.2	2.8
	Gen	(11-6.5)	(10-4.5)	(4.3-3.1)	(4.1-3.1)

* The range is based on three replicates in Tables 16 to 18.

Six-Minute Results

Here again, because of space limitations, results for only four months are presented and Melbourne is used as a typical station. The complete set of results can be found in Srikanthan and McMahon (1983c). Because of limitations of computer availability, only three replicates were generated. The range of parameter estimates obtained from the generated sequences are given in Tables 16 to 18.

In Table 16, maximum six-minute rainfalls are shown to be satisfactorily pre-

Table 17 – Comparison of mean, standard deviation and coefficient of skewness of six-minute rainfall for Melbourne.

Parameter		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean (mm)	Hist	2.30	2.73	1.99	1.86	1.63	1.24	1.38	1.45	1.61	1.89	1.93	2.08
	Gen	(2.25-1.96)	(3.20-2.56)	(1.90-1.76)	(1.72-1.64)	(1.59-1.50)	(1.31-1.23)	(1.41-1.32)	(1.46-1.35)	(1.64-1.43)	(1.96-1.76)	(1.95-1.78)	(2.10-1.87)
Standard deviation (mm)	Hist	4.17	5.82	3.81	2.87	2.51	1.68	1.95	1.87	2.23	2.99	3.09	4.30
	Gen	(4.18-3.39)	(6.79-5.10)	(3.18-2.74)	(2.92-2.67)	(2.47-2.38)	(1.98-1.68)	(2.10-1.91)	(1.97-1.84)	(2.34-2.05)	(3.28-2.80)	(2.99-2.84)	(4.22-3.59)
Coefficient of skewness	Hist	5.29	8.09	9.57	5.66	5.59	4.92	4.83	3.68	5.04	5.76	7.72	7.90
	Gen	(6.49-5.88)	(7.15-6.82)	(5.19-4.50)	(11.16-5.33)	(5.19-4.65)	(7.04-4.51)	(5.33-5.26)	(4.47-3.88)	(4.42-4.20)	(5.69-4.78)	(5.56-4.14)	(7.72-5.64)

Table 18 – Comparison of historical and generated longest wet spell (six-minute model) (in units of six minutes).

Station		January	April	July	October
Melbourne	Hist	104	135	141	115
	Gen	(73-41)	(81-62)	(78-34)	(79-53)
Sydney	Hist	183	143	147	163
	Gen	(149-111)	(115-80)	(110-70)	(78-61)
Monto	Hist	59	56	82	46
	Gen	(77-44)	(39-46)	(55-42)	(46-38)
Cowra	Hist	168	66	166	96
	Gen	(58-45)	(62-56)	(82-63)	(67-53)
Mackay	Hist	117	85	71	36
	Gen	(137-80)	(63-50)	(43-30)	(40-30)
Brisbane	Hist	220	157	130	119
	Gen	(111-79)	(83-60)	(108-91)	(103-67)
Darwin	Hist	81	49	10	33
	Gen	(100-73)	(87-44)	(13-4)	(50-29)
Broome	Hist	122	100	63	21
	Gen	(91-62)	(66-34)	(36-27)	(26-24)
Perth	Hist	32	38	63	57
	Gen	(24-17)	(47-35)	(56-46)	(46-36)
Adelaide	Hist	68	116	99	76
	Gen	(75-41)	(91-60)	(48-37)	(54-33)
Alice Springs	Hist	89	67	102	51
	Gen	(71-61)	(106-53)	(134-47)	(43-39)
Kalgoorlie	Hist	120	43	96	50
	Gen	(76-53	(71-35)	(50-41)	(29-27)

served. The means, standard deviations and coefficients of skewness of six-minute rainfall are also satisfactorily preserved for all the stations and months. Results for Melbourne are given in Table 17, and the complete set of results can be found in Srikanthan and McMahon (1983c).

In Table 18, it can be seen that the longest wet spells are for most cases smaller than the corresponding historical values. However, on examining the historical data, it was observed that most of the long wet spells contained one or more small rainfall values of the order of 0.01 mm per six-minute period. These very small amounts (equivalent to 0.1 mm/h) may not be real and may have resulted from recording chart misalignment, or may have been created in the process of digitizing the pluviograph charts. Because of this, historical long spells were considered to be unreliable.

Conclusions

The hourly rainfall model, which consists of a daily transition probability matrix incorporating an hourly time-dependent second-order Markov chain with hourly transition probabilities, satisfactorily generated hourly rainfall for all the twelve stations used in this report. The daily, monthly and annual parameters obtained by aggregating the hourly rainfall are also satisfactorily reproduced. However, the preservation of monthly and annual parameters is not as good as that from a monthly model. This is probably due to the increased complexity of generating rainfalls at small intervals compared with generation on a monthly basis. Moreover, only eight replicates, each of length equal to the historical record, were produced.

Six-minute rainfall generation is an extension of hourly generation. It was found that the six-minute model reproduced most of the characteristics of the six-minute rainfall. Because the six-minute rainfall depths are adjusted to hourly rainfall depths synthesized from the hourly model, hourly, daily, monthly and annual parameters are also preserved.

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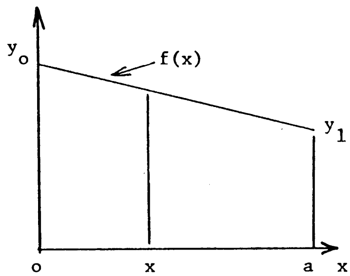
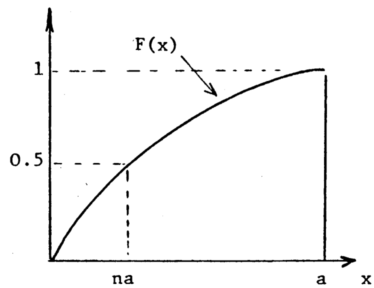
Appendix 1

Distribution for Intermediate States

Since the distribution of rainfall depth is often J-shaped, a linear distribution is used for intermediate states.





Let $f(x)$ be the probability density function, then

$$f(x) = y_1 + \frac{(a-x)}{a}(y_0 - y_1) \qquad 0 \leq x \leq a \tag{1}$$

$$\frac{1}{2}(y_1 + y_0)\,a = 1 \tag{2}$$

Let the mean of the distribution lie at a distance na from the origin. Taking moments about the origin,

$$na\frac{1}{2}(y_1 + y_0)\,a = ay_1\frac{a}{2} + \frac{1}{2}(y_0 - y_1)\,a\frac{a}{3}$$

$$y_0 = \left(\frac{2-3n}{3n-1}\right)y_1$$

$$y_0 = my_1 \tag{3}$$

where

$$m = \frac{2-3n}{3n-1} \tag{4}$$

From Eqs. (2) and (4),

$$y_1 \equiv \frac{2}{(1+m)a} \tag{5}$$

The probability distribution function, $F(x)$, is given by

$$F(x) = \int_0^x f(x)\,dx$$

$$F(x) = my_1 x - \frac{(m-1)}{2a} y_1 x^2$$

$$\frac{(m-1)}{2a} y_1 x^2 - my_1 x + F(x) = 0$$

$$Ax^2 - Bx + F(x) = 0$$

where

$$A = \frac{(m-1)}{2a} y_1 \quad (6)$$

$$B = my_1 \quad (7)$$

Hence,

$$x = \frac{B \pm \sqrt{B^2 - 4A\,F(x)}}{2a}$$

When $F(x) = 0$

$$x \equiv \frac{B \pm B}{2a}$$

$$= 0 \quad \text{or} \quad \frac{B}{A}$$

Since $F(x) = 0$ at $x = 0$,

$$x \equiv \frac{B - \sqrt{B^2 - 4A\,F(x)}}{2a} \quad (8)$$

It can be shown from Eq. (8) that $x = a$ when $F(x) = 1$. Hence for given n, m and y can be obtained from Eqs. (4) and (5) respectively, and then A and B from Eqs. (6) and (7). When $n = 0.45$ (which is the average value from data), $A = 0.3$ and $B = 1.3$. These values were used in the generation procedure.