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Longitudinal Dispersion in a Stream Calculated by One Dimensional Numerical Model

Torulv Tjomsland

Norwegian Institute for Water Research,
Oslo, Norway

A one-dimensional numerical model was used to simulate water stage and dispersion of matter in a stream. The calculated results show quite good agreement with field measurements.

Introduction

Studies concerning the longitudinal dispersion is an important process relevant to pollution in natural water courses. In order to control contamination and predict levels of pollution in a stream the stream's capacity to transport and disperse must be known.

Steady flow in pipelines is described by the one-dimensional Fickian-type diffusion equation derived by Taylor (1954). This equation has been applied to natural streams, and in several cases the observations suggest that the theory is not applicable (Day 1974). The most important mechanisms acting in streams are most often the trapping and release of particles of fluid by peripheral dead zones (Valentine and Wood 1977). This trapping may occur on the riverbottom and on the riverbanks.

Here a one-dimensional numerical model of dispersion is developed. The model considers the dead zone effect. The results are tested using field measurements in a stream.

The Model

The flow condition and the dispersion of material along the river is described by the following one-dimensional equations

$$\frac{\partial z}{\partial t} = -\frac{1}{B}\frac{\partial Q}{\partial x} + q \tag{1}$$

$$\frac{\partial Q}{\partial t} = \frac{QB}{A}\frac{\partial z}{\partial t} - \frac{Q}{A}\frac{\partial Q}{\partial x} + \frac{Q^2}{A^2}\frac{\partial A}{\partial x} - gA\frac{\partial z}{\partial x} + g\frac{Q/Q/}{AM^2R^{\frac{4}{3}}} \tag{2}$$

$$\frac{\partial C}{\partial t} = -\frac{1}{A}\frac{\partial (QC)}{\partial x} + \frac{1}{A}\frac{\partial}{\partial x}(AD\frac{\partial C}{\partial x}) + \frac{1}{BL}u\,KL(CL-C) + S + P \tag{3}$$

$$\frac{\partial CL}{\partial t} = \frac{KL\,u}{BL}(C-CL) \tag{4}$$

where

x – distance along the river (m)
z – water level over a fixed level (m)
t – time (s)
Q – water flow in the river (m^3/s)
q – lateral water inflow (m^3/s)
u – mean velocity (m/s)
B – width (m)
A – cross-sectional area
BL – width of the deadzone (m)
C – concentration in the stream ($-/m^3$)
g – gravitational acceleration (m/s^2)
D – longitudinal dispersion coefficient (m^2/s)
KL – exchange coefficient between the dead zone and the main stream
S – sources (-/s)
M – Manning's coefficient of friction ($m^{1/3}/s$) ($u = M\,R^{2/3}\,S_f^{1/2}$)
P – process of the material (-/s)
S_f – friction slope

The equation of continuity Eq. (1) and the equation of momentum Eq. (2) describe the unsteady flow of water in the river. In natural streams there are pools and stagnant areas (dead zones) caused by unevenness of the banks and bottom. The equations of dispersion Eqs. (3) and (4) describe the effects of temporary storage in such dead zones and the effects of mixing caused by turbulence and velocity gradients (Hays 1966).

The equations of flow are solved by the Preismann's (SOGREAH) "double sweep" method, (Mahmood and Yevjevich 1975) and the equations of dispersion by explicit finite difference schemes.

Application

The model was tested using results from tracer studies in a 2.5-km reach of the river Lena, southeastern Norway. The mean gradient of the stream was about 2 %. The bottom sediments consisted mainly of cobbles and gravel. The specific discharge was about 1 l/s km^2. The stream is comprised of irregular sides and many small ponds.

^{82}Br was used as a tracer. This isotope decays with a half-life of 35.4 hours.

By knowing representative cross sectional profiles, slopes and water discharges from measurements we simulated water stage, mean velocity and isotope concentration along the river (Tabel 1 and Fig. 1).

The waterflow was constant over time, and we could use a simplified version of Eq. (2). In five of the nine control-profiles the deviation between observed and simulated water stages was less than 1 cm. The greatest deviation (6.9 cm) occur-

Table 1 – Simulated and observed water depth.

No.	Distance km	Wetted perimeter m	Cross sectional area m^2	Slope ‰	Discharge l/s	Mean velocity cm/s	Water depth simulated cm	Water depth observed cm	Difference cm
1	0.00	3.8	0.34		23	6.7	15.4		
2	0.06	2.8	0.21	28	23	11.0	12.9		
3	0.12	3.3	0.32	27	24	7.5	15.9	17.5	–1.6
4	0.18	2.4	0.25	17	25	10.0	9.7		
5	0.23	4.5	0.39	24	26	6.7	14.1	14.0	0.1
6	0.31	3.9	0.28	6	26	9.2	13.2		
7	0.38	4.0	0.30	21	27	9.0	13.7	15.5	–1.8
8	0.49	3.9	0.30	20	29	9.6	12.0		
9	0.63	5.8	0.37	19	29	7.8	12.1	13.0	–0.9
10	0.76	4.5	0.31	22	30	9.6	11.5		
11	0.86	4.1	0.35	20	31	8.8	13.9		
12	0.98	4.5	0.50	10	32	6.4	17.3	17.3	0.0
13	1.08	4.7	0.36	12	32	8.8	11.4	14.5	–3.1
14	1.19	5.5	0.73	10	33	4.5	18.6		
15	1.33	3.9	0.35	10	33	9.4	13.9	21.0	–6.9
16	1.45	3.5	0.22	44	34	15.4	10.1		
17	1.55	5.3	0.36	22	35	10.0	11.1		
18	1.65	5.3	0.49	30	35	7.1	20.1	20.0	0.1
19	1.81	9.8	0.40	15	36	9.0	18.0		
20	1.96	5.0	0.34	17	36	10.5	9.7		
21	2.11	2.7	0.21	16	37	17.6	11.8		
22	2.27	4.1	0.28	19	38	13.6	13.2	13.0	0.2
23	2.37	4.0	0.29	18	38	13.1	12.1		
24	2.48	5.5	0.68	9	39	5.7	20.0		

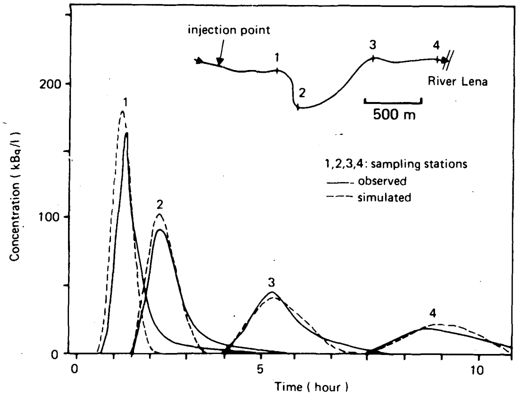

Fig. 1. Observed and simulated transport down the stream.

red on the single reach with rapid runs. On this location the assumption of the model was broken. Values of the Manning friction coefficient (M) of about 10 $m^{1/3}/s$ gave the best results.

The transport of the tracer was measured at four stations placed 0.3, 0.5, 1.2 and 2.5 km downstream from the injection point. The maximum values and time when the maximum concentration of tracer reached the stations were well approximated by the model calculations. The rising part of the curves were quite well simulated. At the "tails" of the curves the calculated values were too low.

Discussion

The results in Fig. 1 are dependent on the choice of coefficients.

For the solution of the flow Eq. (2) we must stipulate the friction coefficient in the Manning formula (M). A value of about 10 $m^{1/3}$ gave the best results.

The accuracy of the dispersion calculations depends on the iteration steps at time (Δt) and space (Δx). For the results in Fig. 1 Δt was set at 1.5 min and Δx at 25 m, respectively. Longer intervals gave smoother curves. The peaks were reduced, and the time required for the tracer to pass by a station was lengthened. The basic mechanism causing dispersion is variation in flow velocity in different parts of the stream cross section. The fraction of the material in the faster-flowing sections of the stream is carried ahead of the fraction in the slower-moving sections. Counteracting this propensity towards greater spreading is the tendency of material in the faster-flowing sections to be carried laterally and vertically by turbulent diffusion into the slower-moving sections and stagnant areas, thus slowing the dispersion process. In one-dimensional models there is no variation in the stream properties with any direction other than longitudinal.

The most common way of modelling this transport is to use the average cross-sectional velocity and a virtual coefficient of diffusion (D), which indicates the overall effect of mixing process (Taylor 1954). The values of D were calculated according to the formula proposed by Liu (1977)

$$D = 0.18(Us/U)^{1.5}\ Q^2/(Us\ R^3)$$

where

Us – shear velocity (m/s)
U – mean velocity (m/s)
Q – discharge (m^3/s)
R – hydraulic radius (m).

By using this method the simulated transport of the tracer was faster than the observed. In this study it does not appear possible to describe the mixing process only by the diffusion coefficient (D)

In natural streams there are local areas where the velocity is essentially zero or even negative. This is because of the existence of pools and stagnant areas along the banks and bottom. The particles of fluid will be temporarily trapped in these dead zones. We had insufficient field measurements to quantify the size of the dead zones along the stream. In the model the size of the zones were stipulated. With a dead zone volume of about 20 % of the main stream the simulated concentrations agreed reasonably well with the observations (Fig. 1). The coefficient (KL), which describes the diffusion between the dead zone area and the main stream was set at 0.02 according to laboratory studies of rectangular zones (Westrich 1976 and Valentine 1977).

The use of the dead zone technique in this stream produced a good result. However, the "tails" of the curves in Fig. 1 show that some of the tracer was trapped more effectively than the model calculations indicated. A better estimate of the size of the dead zones may have improved these results.

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Address:
Torulv Tjomsland,
Norwegian Institute for Water Research (NIVA),
P.O. Box 333, Blindern,
Oslo 3,
Norway.