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The Step-Drawdown Test and Non-Darcian Flow: A Critical Review of Theory, Methods and Practice

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Step-drawdown tests are usually carried out with succeeding steps of increased pumping yield, each step of equal duration of 1 hour. The background for this procedure is reviewed, and it is shown, that very often much shorter duration of steps is adequate.

The analysis of step-tests is usually based on the formulas of Jacob or Rorabaugh. The physical background for these formulas is reviewed, and the latter is shown to be unsatisfactory. However, the formula is used in cases where the data do not fit the Jacob formula, some typical deviations are shown. Reasons for these problems, trivial and non-trivial, are discussed, and some reasonably sound pshysical explanations for the various systematic deviations from the Jacob formula are given.

Introduction

Principles and Practice

Step-drawdown tests are basically made with the purpose to establish a relation between the yield and the corresponding drawdown in a well. This relation may be used purely empirically to choose a reasonable optimum yield for the well or it may be analysed by more or less physically based methods to yield information about well condition, efficiency or the like.

In short term pumping tests in general, two periods after start (or stop) of pumping are important to separate (Fig. 1):

1) *The period immediately after start/stop of the pump, where the effect of well-storage is important.* During this period the flow pattern in the well and its nearest surroundings in the aquifer is established. The drawdown-expiration in the well in this period is determined by pumping yield/well storage, flowconditions in the well, screens, gravel pack, skin zone and in the reservoir to a certain distance, R, from the well, R also depending on the mentioned parameters.

2) *The period after the well-storage has become negligible.* In this period the drawdown in the well is depending upon the reservoir characteristics outside the distance R. These characteristics may be described by some relevant reservoir-formula; in short term tests in artesian aquifers the simple Theis'-formula will often be satisfactory.

As the flow in the reservoir normally may be supposed to be Darcian, the drawdown during period 2 may be supposed to be proportional to the pumping yield, a trivial $s_w - Q$ relation. In period 1 also, part of the drawdown will be proportional to Q. However, in the well vicinity where the flow velocity and drawdowns increase rapidly with decreasing distance to the well, several types of non-darcian flow may occur, producing a non-linear relation between total well drawdown, s_w, and Q. The purpose of the step-test is to study these non-linear, close-to-the-well-effects, and use them as a guide for development work, maintenance etc. As these effects are established and become stationary during period 1, it is important, that the step duration is longer than period 1. However, there is no reason to make it considerably longer; no additional information of any importance will be obtained this way. Information about relevant reservoir parameters should be obtained by constant-rate tests of longer duration.

The duration of period 1 is often only a few minutes. Therefore a step duration of 5-10 minutes will normally be sufficient in groundwater wells. The necessary information for a closer determination of the needed step duration is contained in a short term recovery-test, Fig. 1.

To save time, step-tests are carried out with no pause between the individual steps. Therefore the measured drawdown during each step not only represents this step, but also contains the residual drawdowns of the preceding steps, Fig. 2. The data should be corrected for this effect, if they are to be analysed. As the residual drawdowns of preceding steps contain only period-2-contributions, the correction can be based simply on superposition, using the appropriate reservoir-formula, often Theis'. The information necessary is contained in a short term recovery-test. The reason for the use of recovery-tests instead of pumping tests is, that the amount of potential sources of error thereby is reduced considerably.

With short step-duration and succeeding steps, the step-test becomes a very quick procedure to accomplish.

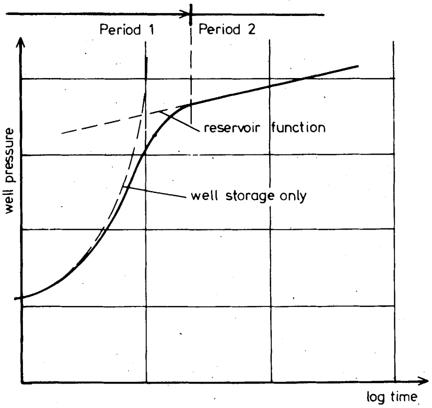

Fig. 1. Well storage effect in recovery test.

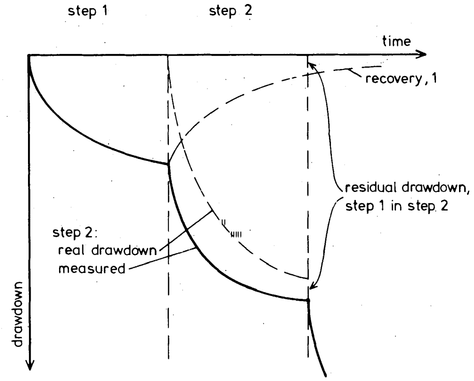

Fig. 2. Superposition of succeeding steps.

Analysis and Theory

The drawdown in the well, s_w, may be regarded as an integration of the hydraulic loses everywhere in the flowfield. In Fig. 3 e.g., s_w includes contributions from the hydraulic losses in the casing, the screens, the gravel pack, the skin-zone, the aquifer and the aquitard above the aquifer. Therefore it is important to know the physics of flow in all these regions to analyse the $s_w - Q$ relation properly. If each region is considered homogeneous, the total system may be regarded as a series combination of hydraulic resistances with different physical and geometrical characteristics. If significant inhomogeneities occur in some of the regions the arrangement will be more complicated.

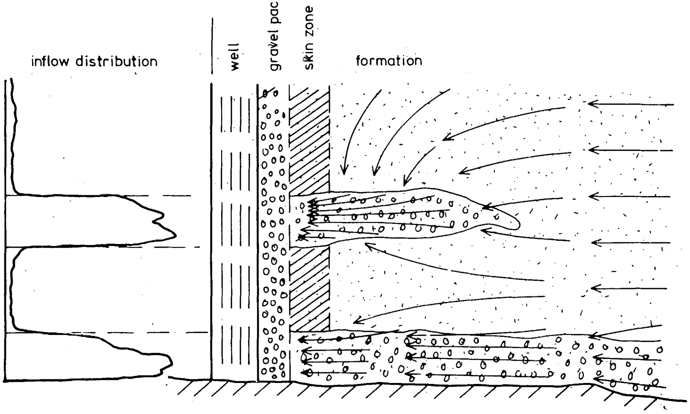

Fig. 3. Flowfield and hydraulic loss components after pumping time t.

If the flow in all regions is Darcian, that is proportionality between hydraulic gradient and flow velocity everywhere in the flowfield, the $s_w - Q$ relation will simply be

$$s_w = BQ \tag{1}$$

where B is a constant representing the integrated hydraulic resistance/conductivity of the system.

However, it is a well-known fact, that close to the well, where the flow velocities are large, the flow is not obeying Darcy's law. Theoretical considerations as well as numerous laboratory tests have shown, that in a homogeneous porous medium, the basic equation of flow is (Bear 1975, Engelund 1953 and Irmay 1958)

$$i = av + bv^2 \tag{2}$$

Which is called Forcheimer's formula with i being the hydraulic gradient, v the flow velocity and a and b are constants, depending on medium and fluid characteristics. The term bv^2 is caused by inertial forces, which are neglected in the physical basis of Darcy's law. If this formula is used in aquifer and gravel pack (which are considered homogeneous) the $s_w - Q$ relation can be shown to be of the form (Jacob's formula)

$$\text{a)}\quad s_w = BQ + CQ^2 \quad \text{or} \quad \text{b)}\quad s_w/Q = B + CQ \tag{3}$$

in which B and C represent the integration of a and b in Eq. (2). Part of CQ^2 may be due to turbulent flow in the well between screens and pump intake. However in most cases this distance is rather short and the well diameter so large, that this contribution to CQ^2 becomes negligible.

Jabob's formula is the most widely used for analysis of step-test data. It is customary to denote B the »formation factor« and C the »well loss coefficient«, based on Jacob's assumption, that the loss BQ takes place in the formation and CQ^2 in the well construction. This has often led to the conclusion, that a well with a large CQ^2 was badly constructed, which is not necessarily correct. In fact the »first order loss« BQ will normally include contributions from formation, skin zone, gravel pack and screens and the »second order loss« from well casing, screens, gravel pack, skin zone and formation close to the well, which makes a straightforward use of Eq. (3) rather dubious in combination with the traditional perception of B and C.

Various kinds of tests, theoretical considerations etc. have shown, that in wells constructed according to reasonable standards and with inflow distributed uniformly over the inflow area, second order losses are not likely to occur. The fact that they, nevertheless, normally do in practice, may be explained by the circumstance, that uniform inflow distribution to a well probably never occurs. Either inhomogeneities in the aquifer or in the skin zone will inevitably cause a

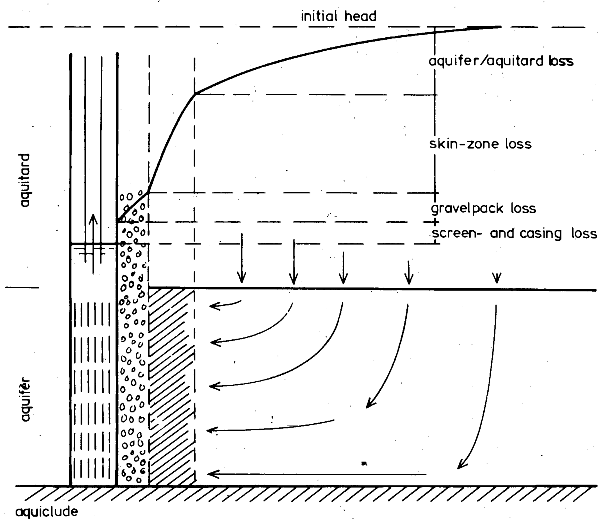

Fig. 4. Inflow distribution and flow field in well vicinity in inhomogeneous formation with skin zone.

significant part of the flow in the well vicinity to take place in a minor, often rather coarse grained, part of the inflow area. As the occurrence of second order losses in every part of the formation is depending on the local value of the Reynold's number

$$R = \frac{vd}{\nu} \qquad (4)$$

in which v is local flow velocity, d local mean grain diameter and υ kinematic fluid viscosity, a concentration of the flow in coarse grained parts of the formation may produce large local Reynold's numbers and thereby significant second order losses, Fig. 4.

In the following considerations it is assumed, that 1) s_w and Q have been measured with reasonable accuracy, 2) Q has been constant in each step, 3) the well has been stabile (the test may be repeated with the same result) and 4) the appropriate corrections for step superposition have been made.

Data from step-tests are often plotted in a linear diagram according to Eq. (3b), which is a linear relation between s_w/Q and Q, Fig. 5a.

The Jacob formula has a sound, well documented physical background in Eq. (2) (which was not recognized by Jacob in 1946), provided the flow regions (Fig. 3) may be considered as porous media and the regions close to the well are reasonably homogeneous. However, data from step-tests often fail to fit the Jacob formula. A number of interesting, principially different examples are shown in Fig. 5, which has been put together on the basis of personal experience and theoretical studies.

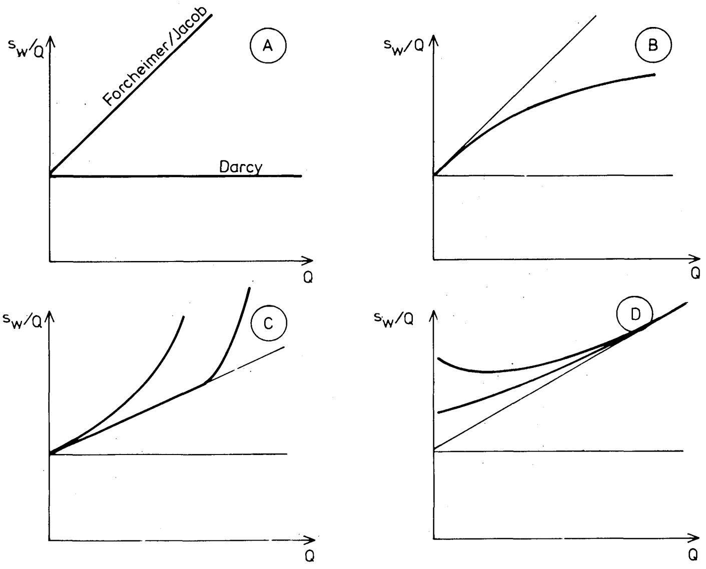

Fig. 5. Typical deviations from the Jacob formula in s_w/Q vs. Q-plots of step test data.

s_w/Q-Q-relations with upward curvature as in Fig. 5c and d are rather common, and have initated creation of the formula (Rorabaugh 1953)

$$s_w \equiv BQ + CQ^n \tag{5}$$

where n is larger than 2; with n smaller than 2 the formula may also fit data of the type in Fig. 5b. The question arises whether or not this formula is reasonable from a physical point of view. Rorabaugh bases his argument on a formula of the form Eq. (2), the terms of which he considers caused by »laminar flow« in the formation and »turbulent flow« in the well vicinity respectively. Assuming a »critical Reynold's number« at which turbulent flow starts, he states, that this value of R moves outward in the formation as Q increases, thereby increasing the size of the zone of turbulence, and thus creating an integrated turbulent loss in the well with a power larger than 2. The defect in this argument stems from the fact, that the critical Reynold's number, which is well known from the hydraulics of tubes, does not exist in porous media flow, where the hydraulic characteristics are different. As earlier mentioned, integration of Eq. (2) will lead to a relation of the form Eq. (3), and thus retain the second power term. Further, the flow generating the second order term is normally not turbulent; second order losses become significant in the interval $1<R<10$, while turbulent flow does not occur until R is larger by 1 og 2 orders of magnitude (Bear 1975 and Irmay 1958). Eq. (5) must therefore mainly be considered as a curve-fitting formula with no physical basis.

However, the deviations from the Jacob formula do exist and if Eq. (5) is unsatisfactory other explanations must be found:

s_w – Q-relations as in Fig. 5b occur when an aquifer is screened in both

A) a rather short, coarse-grained and
B) a rather large, fine-grained interval,

both yielding a significant part of the total abstraction from the well at small values of Q. In such a case the second order loss of B) may be negligible, while in A) it may be important even for moderate values of Q. Therefore the hydraulic resistance of A) will increase rapidly with Q, while the resistance of B) will be nearly constant, and therefore a still larger part of Q will flow in B), resulting in a relation as shown in Fig. 5b. This may therefore be interpreted as an effect of significant formation inhomogeneities near the well.

Plots as showed in Fig. 5c are generally obtained from fractured aquifers and may show as well a gradual bending upward as a sudden jump from the »Jacob-line« at a certain value of Q. This is caused by the fact, that fractured aquifers have a hydraulic behaviour which is basically different from what is normally called »porous media« (of course, transitional forms exist). First the hydraulic characteristic of a fracture is similar to that of tubes, that is: a critical Reynold's number exists. Second, and most important, fractures »squeeze« when the fluid potential in the fracture is lowered, and as the hydraulic resistance of a fracture is inversely proportional to the third power of fracture width, even a small »squeeze« may have a significant effect on formation transmissivity and thus on well drawdown. This is normally a close-to-the-well-effect, as the large drawdowns here generate the squeezing effect (Sørensen unpublished).

Upward bending relations as in Fig. 5c will also occur in cases where the water table in the well is lowered below part of the inflow interval. In such a case the inflow from this part will be slowly decreasing with time, and therefore further increase of Q must be abstracted from lower parts of the inflow interval only, with increased hydraulic resistance as the result.

Another type of upward-bending relations between s_w/Q and Q is shown in Fig. 5d. This one seems to approach the straight line for large values of Q and is peculiar by the fact, that, occasionally, the values of s_w/Q decrease for small values of Q (upper curve). This is equivalent to a decrease in the total hydraulic resistance of the system with increasing, small values of Q, and means, that in part of the system in Fig. 3 the hydraulic resistance actually decreases with increasing Q! It should here be mentioned, that in all the other types of flow discussed, the resistance increases with increasing Q, for Darcian flow the resistance is constant.

In fact such a type of flow, with a velocity-gradient relationship as sketched in Fig. 6, curve 4, exists in fine-grained sediments at rather small flow gradients (Bear 1975, Kutilek 1969, Lutz and Kemper 1959, Swartzendruber 1968). The reason for this is, that in fine-grained sediments, the pores may be so small, that

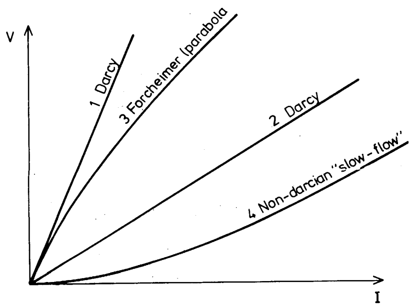

Fig. 6. Velocity-gradient diagram, showing characteristics of various kinds of porous-media-flow.

the movement of the water molecules, which are electric dipoles, may be significantly affected by electrostatic forces from the pore walls. In this case the currently used theories of flow in porous media are not applicable, and therefore the Darcy and Forcheimer equations are not valid. As this type of flow takes mainly place in some types of clay, the aquitard in Fig. 3 is likely to be responsible for s_w/Q-Q relations as shown in Fig. 5d. This is therefore an example of non-darcian flow in the reservoir and not in the well vicinity.

It may be asked if the aquitard influences the drawdowns at all during a short term pumping test in a well. According to Hantush (1960) it does: The drawdowns in the aquitard-aquifer system in Fig. 7 are depending on the permeability of the aquitard, which may again be a function of the flow gradient and thereby of the well abstraction Q. The aquitard influence on the system is not ignorable inside normal parameter-ranges.

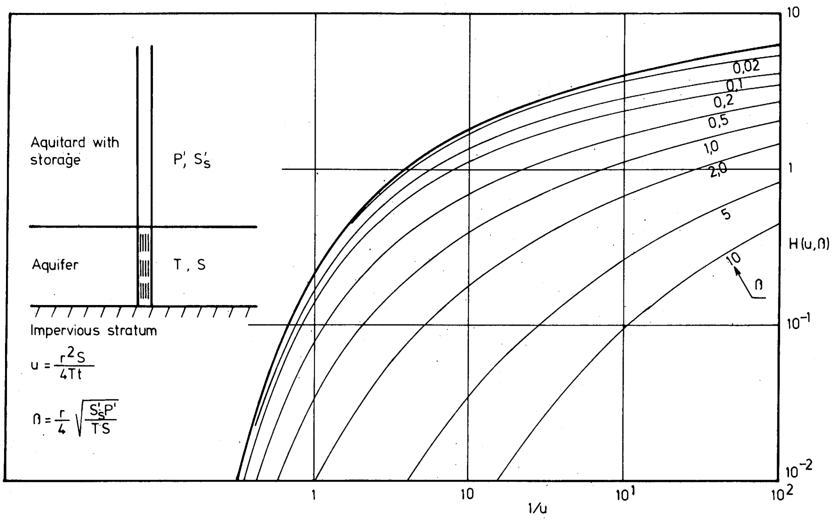

Fig. 7. Hantush' model of influence of aquitard permeability and storage on aquifer tests.

Conclusion

The Theory and principles of the step-drawdown test have been shortly reviewed and discussed. Some unexplained effects have been discussed and reasonable physical explanations suggested. Mathematical treatment has not been made and the use in practice of most of these things is probably rather limited. The motive force has primarily been a wish to understand observed data instead of fitting them into a more or less relevant formula.

One may ask the question: What can a step-test tell about well-condition and improvement? The answer is very shortly: If large second order losses exist where not expected, a very nonuniform inflow distribution is probably present. This may be caused by a skin zone, which may be removed by appropriate development work. Also in the case of strongly upward bending s_w/Q-Q relations in fractured aquifers, development work resulting in widening of the fractures will improve well condition. However, even if such losses of non-darcian kind do not exist, or are small, well condition may still be bad as a skin-loss in a well may be of pure Darcian nature. Therefore a general comparison between the actual specific yield of a well and the yield expected on the basis of known reservoir parameters without accounting for additional close-to-the-well-losses should always be carried out.

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