Nordic Hydrology, 13, 1982, 39-48

Lag Time for Diverging Overland Flow

V. P. Singh
Lousiana State University, Baton Rouge, LA 70803, USA

N. Agiralioglu
Mississippi State University, MS 39762, USA

This paper presents kinematic solutions, both analytically and nomographically, of surface water lag time for diverging overland flow. These solutions can be utilized to estimate lag time from physically measurable watershed characteristics.

Introduction

There exists a multitude of concepts of surface water lag time. This study utilizes the concept proposed by Overton (1970). He defined the lag time t_L as the time difference between 50% of the volume of the effective rainfall q and 50% of the volume of discharge Q. If MI and MV denote respectively the times of 50% of the volumes of q and Q then for a long steady effective rainfall as shown in Fig. 1,

$$t_L = MV - MI \tag{1}$$

From the volume continuity, 50% of the discharge volume must equal 50% of the effective rainfall volume. Referring to Fig. 1, the volumes,

$$I + II = II + III \tag{2}$$

and, therefore, the volume,

$$I = III \tag{3}$$

The volume I equals the watershed surface storage S_e at equilibrium condition, and will remain unchanged as long as q remains steady. The volume III is

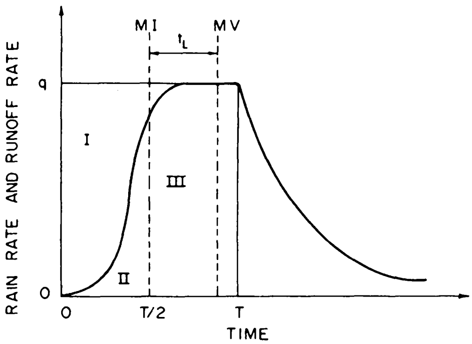

Fig. 1. Derivation of lag time from schematic equilibrium hydrograph (after Overton 1970).

$$S_e = q(MV - MI) = q\, t_L \tag{4}$$

Therefore,

$$t_L = \frac{S_e}{q} \tag{5}$$

It is evident that t_L will remain unchanged for the duration of the equilibrium condition.

Diverging Overland Flow

For a diverging surface as shown in Fig. 2 the basic equations of flow, based on kinematic wave theory as derived by Singh and Agiralioglu (1980), can be written on a unit width basis as

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial r} = q - \frac{Q}{r} \tag{6}$$

$$Q = uh = \alpha\, h^n \tag{7}$$

where

h – local depth,
Q – discharge per unit width,
u – local velocity,
q – effective rainfall intensity,
r – space coordinate,
t – time coordinate,
α – friction parameter,
n – exponent indicative of flow regime.

At equilibrium condition the transient term in Eq. (6) will vanish, i.e.,

$$\frac{\partial Q}{\partial r} = q - \frac{Q}{r} \tag{8}$$

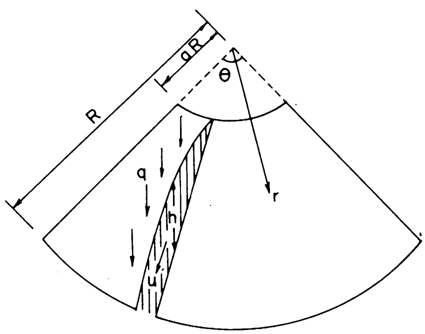

Fig. 2. Diverging geometry.

The solution of Eq. (8) subject to the condition, $Q = 0$ at $r = aR$, is

$$Q = q\left(\frac{r^2 - a^2R^2}{2r}\right) \tag{9}$$

where a is a parameter relating to the degree of divergence. In many watersheds a is vanishingly small. Therefore,

$$Q = \frac{qr}{2} \tag{10}$$

By eliminating Q from Eqs. (9) and (7) the solution for the depth at equilibrium can be written as

$$h = \left[\frac{q(r^2 - a^2R^2)}{2r\alpha}\right]^{\frac{1}{n}} \tag{11}$$

For $a = 0$,

$$h = \left(\frac{qr}{2\alpha}\right)^{\frac{1}{n}} \tag{12}$$

For a diverging surface the lenght of flow is $R(1\text{-}a)$ where R = the length of the section. The average depth at equilibrium S_e then follows

$$S_e = \frac{1}{R(1-a)} \int_{aR}^{R} h(r)dr \tag{13}$$

where $h(r)$ is the depth of flow as a function of space coordinate r. Upon substituting Eq. (11) into Eq. (13),

$$S_e = \frac{1}{R(1-a)} \int_{aR}^{R} \left[\frac{q(r^2 - a^2R^2)}{2r\alpha}\right]^{\frac{1}{n}} dr$$

On simplifying and using Binomial theorem,

$$S_e = \frac{1}{R(1-a)} \left(\frac{q}{2\alpha}\right)^m \int_{aR}^{R} \sum_{j=0}^{\infty} \binom{m}{j} (-1)^j (a^2R^2)^j r^{m-2j} dr$$

where $m = 1/n$. Upon integration,

$$S_e = \frac{1}{R(1-a)} \left(\frac{q}{2\alpha}\right)^m \sum_{j=0}^{\infty} \binom{m}{j} (-1)^j (aR)^{2j} \left[\frac{(R)^{m-2j+1} - (aR)^{m-2j+1}}{(m-2j+1)}\right] \tag{14}$$

However, if $a = 0$,

$$S_e = \left(\frac{q}{2\alpha}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) R^{\frac{1}{n}} \tag{15}$$

Therefore, the solution of lag time follows directly from Eqs. (14) – (15) for two different hydrologic situations. If a is not negligible,

$$t_L = \frac{(q)^{m-1}}{R(1-a)} \left(\frac{1}{2\alpha}\right)^m \sum_{j=0}^{\infty} \binom{m}{j} (-1)^j (aR)^{2j} \left[\frac{(R)^{m-2j+1} - (aR)^{m-2j+1}}{m-2j+1}\right] \tag{16}$$

For $a = 0$,

$$t_L = \left(\frac{1}{2\alpha}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) q^{-(n-1)/n} R^{\frac{1}{n}} \tag{17}$$

Eqs. (16) – (17) can simply be written as

$$t_L = \mu\, q^{-(n-1)/n} \tag{18}$$

where

$$\mu = \frac{1}{R(1-a)} \left(\frac{1}{2\alpha}\right)^m \sum_{j=0}^{\infty} \binom{m}{j} (-1)^j (aR)^{2j} \left[\frac{(R)^{m-2j+1} - (aR)^{m-2j+1}}{m-2j+1}\right]$$

and when $a = 0$,

$$\mu = \left(\frac{n}{n+1}\right) R^{\frac{1}{n}} \left(\frac{1}{2\alpha}\right)^{\frac{1}{n}} \tag{19}$$

The quantity μ is what Overton (1971) termed as lag modulus which depends on watershed physiography, and can be assumed to be constant for a given watershed. The variation of μ with a is shown in Fig. 3. Eq. (18) points out that the lag time changes as the effective rainfall intensity changes. This has been observed by investigators before (Overton 1971; Singh 1975). Eq. (18) may be useful in hydrologic regionilization studies. For example, hydrologically similar basins will manifest their lag time behavior in a similar manner. Therefore, a similarity factor Φ can be defined as

$$\Phi = \frac{M_1}{M_2} \tag{20}$$

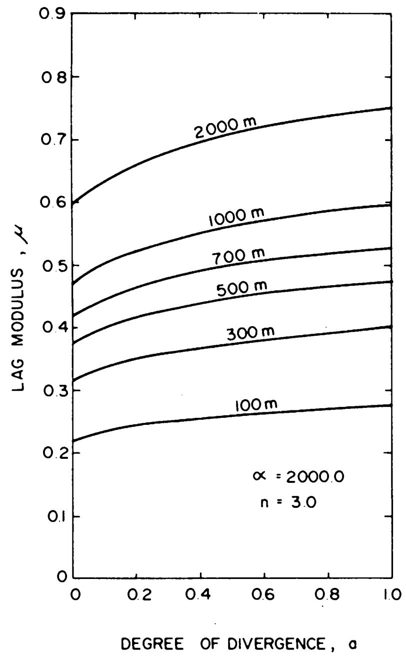

Fig. 3. Variation of lag modulus with degree of divergence.

where subscripts 1 and 2 denote basins 1 and 2. Hence,

$$t_{L_1} = \Phi M_1 q^{-(n-1)/n} \tag{21}$$

That is, the lag time of one basin can be obtained from the lag modulus of another basin if the basins are hydrologically similar.

Agiralioglu and Singh (1980) have shown that the geometric parameters *a* and *R* can be determined from the watershed topography. The parameters *n* and α can be determined using open channel flow formulas for turbulent flow. To illustrate we consider the following.

Chezy Equation. This forms a special case of Eq. (7), i.e.,

$$u = C(HR\, S_f)^{0.5} \tag{22}$$

where

C – Chezy friction coefficient,
HR – hydraulic radius,
S_f – friction slope.

In kinematic approximation $S_0 = S_f$ where S_0 = bottom slope. *HR* can be expressed as

$$HR = a_1 h^{b_1} , \quad a_1 > 0 \ , \ b_1 > 0$$

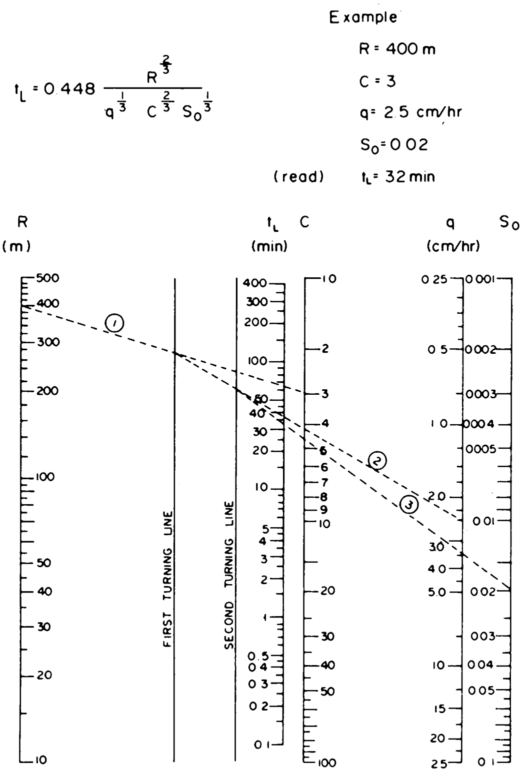

Fig. 4. Nomograph for lag time using Chezy coefficient.

where a_1 = a constant, and b_1 = a constant. Then

$$Q = Ch(a_1 h^{b_1} S_0)^{0.5} = C(a_1 S_0)^{0.5} h^{(b_1+2)/2} \tag{23}$$

Therefore

$$\alpha = C(a_1 S_0)^{0.5} \tag{24}$$

and

$$n = \frac{2+b_1}{2} \tag{25}$$

The nomographic solution of Eq. (17), in conjunction with Eqs. (24) - (25), is shown in Fig. 4. The parameter values used were: $a_1 = b_1 = 1$, which yielded $n = 1.5$ and $\alpha = CS_0^{0.5}$. The equation for lag time then becomes

$$t_L = 0.448 \frac{R^{\frac{2}{3}}}{q^{\frac{1}{3}} C^{\frac{2}{3}} S_0^{\frac{1}{3}}} \tag{26}$$

where R is in meters, q in cm/hour and t_L in minutes.

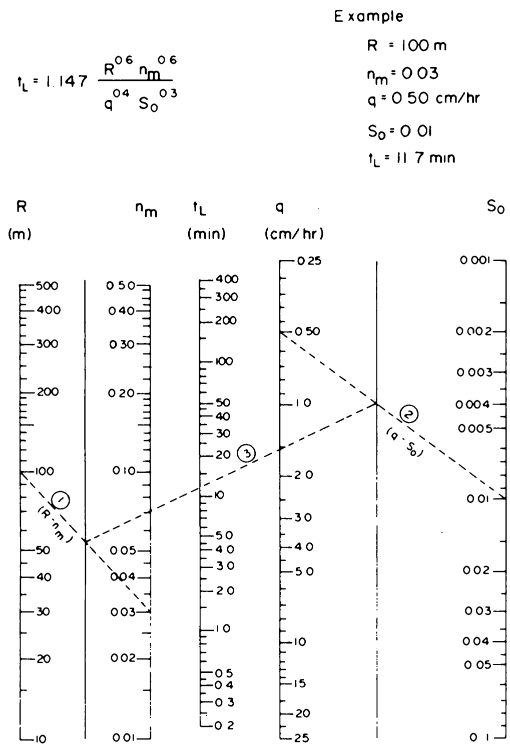

Fig. 5. Nomograph for lag time using Manning coefficient.

Manning Equation. If Manning equation is applied,

$$u = \frac{1}{n_m} (HR)^{\frac{2}{3}} S_f^{\frac{1}{2}}$$

where n_m = Manning frictio

$$n_m = a_2 Q^{b_2} \quad , \quad a_2 > 0 \quad , \quad b_2 > 0$$

where a_2 = a constant and b_2 = *a* constant. Therefore,

$$Q = \left(\frac{a_1^{\frac{2}{3}} S_0^{\frac{1}{2}}}{a_2} \right)^{1/(1+b_2)} h^{(3+3b_1)/3(1+b_2)} \tag{27}$$

Then

$$\alpha = \left(\frac{a_1^{\frac{2}{3}} S_0^{\frac{1}{2}}}{a_2} \right)^{1/(1+b_2)} \tag{28}$$

$$n = \frac{3+2b}{3(1+b_2)} \tag{29}$$

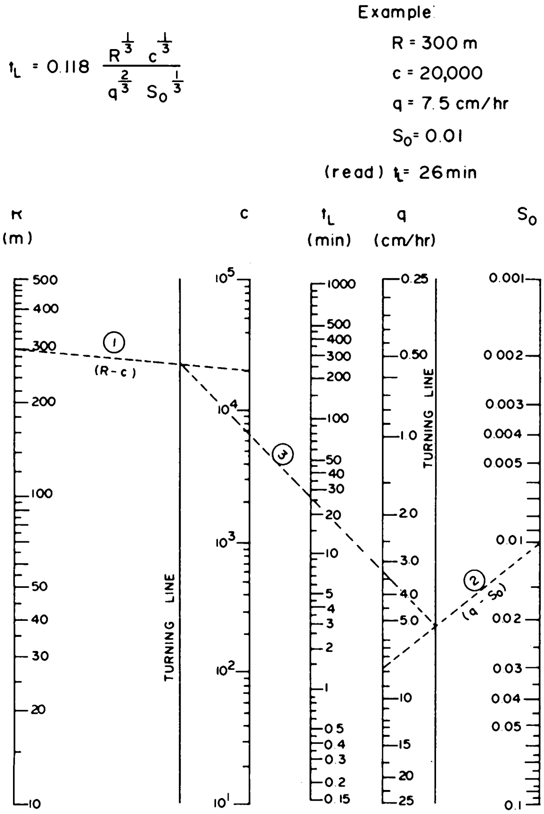

Fig. 6. Nomograph for lag time using Darcy-Weisbach coefficient.

The nomographic solution of Eq. (17) in conjunction with Eqs. (28) – (29) is shown in Fig. 5. Similarly, the parameters used were $a_1 = b_1 = 1$, $a_2 = n_m$ and $b_2 = 0$ which gave $n = 5/3$ and $\alpha = S_0^{0.5}/n_m$. The equation for lag time then becomes

$$t_L = 1.147 \frac{R^{0.6}\, n_m^{0.6}}{q^{0.4}\, S_0^{0.3}} \tag{30}$$

Darcy-Weisbach Relation. If the Darcy-Weisbach friction factor is used,

$$u = \left(\frac{8gHRS_f}{f}\right)^{\frac{1}{2}} \tag{31}$$

where f = friction factor and can be expressed as

$$f = \frac{c}{R_e^b} = \frac{c}{(Q/\nu)^b} \tag{32}$$

where

R_e – Reynolds number, ν – kinematic viscosity, c – a constant, b – constant.

Therefore,

$$Q = h S_0^{\frac{1}{2}} \left(\frac{8 g a_1 h^{b_1} (Q/\nu)^b}{c} \right)^{\frac{1}{2}} \tag{33}$$

On simplifying it,

$$Q = \left(\frac{8 g S_0 a_1}{c \nu^b} \right)^{1/(2-b)} h^{(2+b_1)/(2-b)} \tag{34}$$

Therefore,

$$\alpha = \left(\frac{8 g S_0 a_1}{c \nu^b} \right)^{1/(2-b)} \tag{35}$$

$$n = \frac{2 + b_1}{2 - b} \tag{36}$$

The nomographic solution of Eq. (17) in conjunction with Eqs. (35) – (36) is shown in Fig. 6. Here the parameters used were $a_1 = b_1 = 1$ which yielded $n = 3$ and $\alpha = 8qS_0/(c\ \nu)$. The equation for lag time then becomes

$$t_L = 0.118 \frac{R^{\frac{2}{3}} c^{\frac{1}{3}}}{q^{\frac{2}{3}} S_0^{\frac{1}{3}}} \tag{37}$$

Application

Determination of lag time of a given watershed will require knowledge of geometric parameters and friction characteristics. The former can be obtained as discussed by Agiralioglu and Singh (1980), and Singh and Agiralioglu (1981a, 1981b). The latter can be obtained as discussed by Lane, Woolhiser and Yevjevich (1975), and Lane and Woolhiser (1977). Once this information is obtained, lag time can be obtained either from mathematical equations or nomographs as shown in Figs. 4-6.

Conclusions

The surface water lag time for diverging overland flow has been derived using kinematic wave theory. Both analytical and nomographic solutions have been presented. The lag time for a given watershed is not constant but changes with rainfall characteristics.

Acknowledgement

The work upon which this study is based was supported in part by funds provided by National Science Foundation under the project, Free Boundary Problems in Water resource Engineering, No. ENG 79-23345.

References

Agiralioglu, N., and Singh, V. P. (1980) A mathematical investigation of diverging overland flow, 2. numerical solutions and applications. Technical Report MSSU-EIRS-CE-80-4, Engineering and Industrial Research Station, Mississippi State University, Mississippi State, Mississippi, 95 pp.

Lane, L. J., and Woolhiser, D. A. (1977) Simplifications of watershed geometry affecting simulation of surface runoff. *Journal of Hydrology 35,* 173-190.

Lane, L. J., Woolhiser, D. A., and Yevjevich, V. (1975). Influence of simplications in watershed geometry in simulation of surface runoff. Colorado State University Hydrology Paper No. 81. 50 pp., Fort Collins, Colorado.

Overton, D. E. (1970) Route or convolute! *Water Resources Research 6(1),* 43-62.

Overton, D. E. (1971) Estimation of surface water lag time from kinematic wave equations. *Water Resources Bulletin 7(3),* 422-440.

Singh, V. P. (1975) Derivation of surface water lag time for converging overland flow. *Water Resources Bulletin 11(3),* 505-513.

Singh, V. P., and Agiralioglu, N. (1980) A mathematical investigation of diverging overland flow, 1. analytical solutions. Technical Report MSSU-EIRS-CE-80-3, Engineering and Industrial Research Station, Mississippi State University, Mississippi State, Mississippi, 175 p.

Singh, V. P., and Agiralioglu, N. (1981a) Diverging overland flow, 1. analytical solutions. *Nordic Hydrology 12(2),* 81-89.

Singh, V. P., and Agiralioglu, N. (1981b) Diverging overland flow, 2. application to natural watersheds. *Nordic Hydrology 12(2),* 99-110.

First received: 15 June, 1981
Revised version received: 17 September, 1981

Address:
V. P. Singh,
Department of Civil Engineering,
Louisiana State University,
Baton Rouge, LA-70803,
USA.

Department of Civil Engineering,
Mississippi State University,
P.O.Box Drawer CE,
Mississippi State, MS 39762,
USA.