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Random Phase Approximation in Relativistic Approach*

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Abstract: Some special issues of the random phase approximation (RPA) in the relativistic approach are reviewed. A full consistency and proper treatment of coupling to the continuum are responsible for the successful application of the RPA in the description of dynamical properties of finite nuclei. The fully consistent relativistic RPA (RRPA) requires that the relativistic mean filed (RMF) wave function of the nucleus and the RRPA correlations are calculated in a same effective Lagrangian and the consistent treatment of the Dirac sea of negative energy states. The proper treatment of the single particle continuum with scattering asymptotic conditions in the RMF and RRPA is discussed. The full continuum spectrum can be described by the single particle Green's function and the relativistic continuum RPA is established. A separable form of the paring force is introduced in the relativistic quasi-particle RPA.

Key words: fully consistent RRPA; coupling to the continuum; pairing correlation

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1 Introduction

The relativistic random phase approximation (RRPA) is a relativistic extension of the random phase approximation (RPA) for studying nuclear excitations and giant resonances in a microscopic way. In the relativistic approach, it is well known that the consistency is extremely important, because the nucleon potentials are determined by a large cancellation between attractive scalar and repulsive vector self-energies. A fully consistent treatment of RRPA within the relativistic mean field (RMF) approximation requires two aspects[1−3]. Firstly, the particle-hole residual interaction must be determined from the same Lagrangian used in the RMF ground states. Secondly, the RRPA configuration space includes not only the usual particle-hole states (ph), but also the pairs formed from occupied states in the Fermi sea (h) and empty negative-energy states in the Dirac sea (α). The inclusion of these configurations is essential for the conservation of the vector current and the decoupling of the spurious states[2]. This coherent effects of αh pairs have been studied in the isoscalar giant monopole resonance (ISGMR). A large effect from the coherent αh pairs has been observed in the ISGMR. The origin of the contributions from Dirac states has been studied and it is found that the coherent effects of αh pairs are mainly through the isoscalar meson[1].

In recent years, the physics of exotic nuclei at-

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tracts more attentions both experimentally and theoretically. Due to the closeness of the Fermi surface to the particle continuum in exotic nuclei the coupling between bound states and the particle continuum become important[4, 5]. The continuum often exhibits resonances with a pronounced single-particle character, which are single-particle resonance states. In microscopic studies nuclear collective giant resonances are explained by the particle-hole excitations in a coherent way, where the particle continuum plays an important role[1, 6, 7]. The effect of the resonant continuum upon pairing correlations was studied in the non-relativistic Hartree-Fock (HF) approximation, where proper boundary conditions are introduced[8]. It was found that the resonant continuum HF-BCS results are generally close to those of the HF-Bogoliubov even in neutron-rich nuclei. The contribution from the resonant states in the giant resonances has been studied in the RRPA[9]. It was found that the contribution of the particle continuum is mainly from the single-particle resonances in the continuum. Actually the particle continuum can be exactly treated by solving the single particle Green's function numerically. A relativistic continuum RPA (RCRPA) will be discussed in this paper.

As regards the relativistic Hartree Bogoliubov (RHB) in finite nuclei, only phenomenological forces have been used in the spin-singlet, isospin-triplet pairing channel so far. A simple monopole or zero range pairing force are commonly adopted[10]. Although successful in describing the low-energy nuclear structure over the known mass table, the two phenomenological pairing interactions lack a link to the bare nucleon-nucleon(NN) interaction. In addition, it is well known that a zero range pairing force has to be regularized with a cut-off in the gap equation to avoid the divergence. Those pairing forces were directly fitted to finite nucleus data, and may thus renormalize beyond-mean-field effects. In addition, their fittings could be only performed where experimental data are available. Extrapolating the use of these interactions towards the drip lines is not reliable. One example of a finite-range pairing force is Gogny effective interaction in the $^1S_0$ channel[11], which has also a clear link with the bare NN interaction. Unfortunately such pairing forces with finite range usually are technically difficult to deal with. It is even more critical when going beyond the mean field calculations. A separable form of pairing interactions in the $^1S_0$ channel for the Gogny effective interaction is introduced. With such a simple separable form pairing properties provided by the Gogny force in nuclear matter can be reproduced. The relativistic quasiparticle RPA (RQRPA) with the Gogny pairing force as well as its separable form could well describe the nuclear excitation energies of the lowest 2^+ states and the BE2 decay rates.

The paper is arranged as follows. The formalism of the RRPA is briefly presented in Sec. II. The effects of the consistency in the RRPA, i. e. the contribution from the αh pairs is depicted in the isoscalar giant monopole resonance for ^{208}Pb. The particle continuum and the giant resonances with those single particle resonances in the continuum are studied in Sec. III. The Gogny pairing interaction with a separable form is adopted in the study of the ground states and collective excitation states, which are presented in Sec. IV.

2 Relativistic Random Phase Approximation

In the RRPA one starts with a self-consistent solution for the ground state and obtains the static mean field. The unperturbed polarization operator on the Hartree ground state can be expressed as[1]

$$\Pi_0(P, Q; x_1, x_2) = \mathrm{i}\mathrm{Tr}[PG_\mathrm{H}(x_1, x_2) \times QG_\mathrm{H}(x_2, x_1)] , \quad (1)$$

where G_H is the single-particle Hartree propagator. In the RMF theory no-sea approximation is imposed. This approximation in the RMF prescription might be implemented by replacing G_{H} with G_{RMF}, i. e. by shifting the negative-energy poles to the lower-half plane[1, 2]. Substituting G_{RMF} into Eq. (1) the unperturbed polarization operator in a spectral representation has the following retarded form,

$$\Pi_0^R(P,Q;\boldsymbol{r},\boldsymbol{r}';E)=\sum_{\mathrm{h},\,\mathrm{a}=\mathrm{p},\alpha}\left[\frac{\bar{\Psi}_{\mathrm{h}}(\boldsymbol{r})P\Psi_{\mathrm{a}}(\boldsymbol{r})\bar{\Psi}_{\mathrm{a}}(\boldsymbol{r}')Q\Psi_{\mathrm{h}}(\boldsymbol{r}')}{E-(E_{\mathrm{a}}-E_{\mathrm{h}})+\mathrm{i}\eta}-\frac{\bar{\Psi}_{\mathrm{a}}(\boldsymbol{r})P\Psi_{\mathrm{h}}(\boldsymbol{r})\bar{\Psi}_{\mathrm{h}}(\boldsymbol{r}')Q\Psi_{\mathrm{a}}(\boldsymbol{r}')}{E+(E_{\mathrm{a}}-E_{\mathrm{h}})+\mathrm{i}\eta}\right]. \quad (2)$$

It shows that the unperturbed polarization includes not only the particle-hole pairs but also pairs formed from the Dirac sea and Fermi sea states.

The response function of a quantum system to an external field is given by the imaginary part of the polarization operator,

$$R(P,P;E)=\frac{1}{\pi}\,\mathrm{Im}\,\Pi^R(P,P;\boldsymbol{k},\boldsymbol{k};E)\,|_{\boldsymbol{k}=\boldsymbol{k}'=0}, \quad (3)$$

where P is an external field operator. The RRPA polarization operator is obtained by solving the linearized Bethe-Salpeter equation,

$$\Pi(P,Q;\boldsymbol{k},\boldsymbol{k}',E)=\Pi_0(P,Q;\boldsymbol{k},\boldsymbol{k}',E)-\sum_i g_i^2\int \mathrm{d}^3k_1\mathrm{d}^3k_2\,\Pi_0(P,\Gamma^i;\boldsymbol{k},\boldsymbol{k}_1,E)\times D_i(\boldsymbol{k}_1-\boldsymbol{k}_2,E)\Pi(\Gamma_i,Q;\boldsymbol{k}_2,\boldsymbol{k}',E). \quad (4)$$

In the relativistic approach, the residual particle-hole interactions are just meson propagators. Therefore, in the above equation the sum i runs over σ, ~ ω and ρ mesons with g_i and D_i being the corresponding coupling constants and meson propagators, $\Gamma^i=1$ for the σ-meson, $\Gamma^i=\gamma^\mu$ and $\gamma^\mu\tau_3$ for the σ and ρ mesons, respectively. The meson propagators for non-linear models are non-local in momentum space, and therefore have to be calculated numerically[1].

In order to show the importance of the consistency in the RRPA, we calculate the ISGMR in ^{208}Pb with and without the contributions of the αh pairs, which are plotted in Fig. 1. The solid and long-dashed curves are the RRPA strengths with and without Dirac states, respectively, which are markedly separated. The ISGMR strength is pushed strongly down without the Dirac state contributions. The short-dashed curve is the RRPA strength with the contributions of αh pairs coming from the vector meson only, which is more or less the same as the case without Dirac states. The dash-dotted curve is calculated with only the scalar meson contributions to the αh pairs. It is clearly seen that the contributions of the scalar meson in the αh pairs dominate whereas those due to the vector meson are largely suppressed. Due to the relativistic structure of the RPA equation Matrix elements of vector fields ω, coupling αh and ph are largely reduced. Cancellations between scalar and vector fields are not taken place.

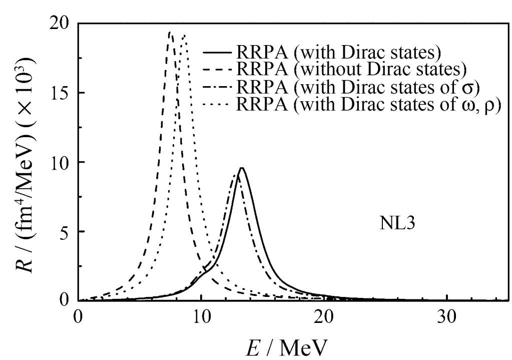

Fig. 1 ISGMR strength distribution in ^{208}Pb calculated with NL3 parameterization.

3 Single Particle Resonances in the Continuum

The single particle resonances are meta-stable states captured by the centrifugal and Coulomb barrier, which wave functions have large probabilities inside the nucleus. They could be calculated in the Dirac equation with imposing proper scattering asymptotic conditions. The scattering asymptotic conditions are composed of the Dirac Coulomb wave functions, because at a large distance, nuclear potentials generated by exchanging mesons vanish, and only Coulomb and centrifugal potential remains. In the relativistic approach a spin dependent interaction even with only Coulomb interaction is built into the theory automatically, which is different from that in the non-relativistic approach. The resonant-states are characterized by the phase shift crossing to $\pi/2$, where the scattering cross section of the corresponding partial wave reaches its maximum. The decay widths of those resonant-states can be roughly explained by considering the penetrability through the Coulomb and centrifugal barriers in view of the quantum mechanics.

The effect of resonant states on pairing correlations is studied in the RMF-BCS approximation. A quantity of $E_{\mathrm{BCS}} = E_{\mathrm{RMF}} - E_{\mathrm{RMF+BCS}}$, which characterizes the pairing correlations energy. The E_{BCS} in Ni-isotopes calculated in the RMF-BCS with discretized states or resonant states and including the widths are plotted in Fig. 2. It is shown that the width effect gets more and more pronounced as the neutron number increases, especially near the drip line. Therefore a proper treatment of the resonant continuum including its width might be necessary for the nucleus near the drip line.

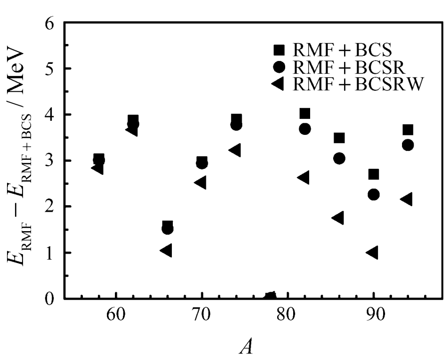

Fig. 2 The pairing correlation energies E_{BCS} in the RMF-BCS with discretized states (RMF+BCS) or resonant states (RMF+BCSR) and including the widths (RMF+ BCSRW) in Ni-isotopes.

It is important to take account of the pairing correlations in the study of multipole collective excitations for open shell nuclei. The relativistic quasi-particle RPA (RQRPA) in the response function formalism, which has provided a convenient and useful method to describe collective excited states of nuclear many-body systems is performed. For the detailed description of RQRPA based on the RMF-BCS ground state can be found in Ref. [12]. We apply the RQRPA to investigate the evolution of isovector giant dipole resonance (IVGDR) in the neutron rich Ni-isotopes. It is found that in addition to the normal GDR strength around the energy at 16 MeV, the low-lying dipole strength appears at the excitation energy below 10 MeV. Differing from the normal GDR response, the low-lying resonance can be interpreted as the vibration of the excess neutrons against the core formed with equal number of protons and neutrons out of phase[13]. The calculated centroid energies of low-lying dipole strengths as a function of the differences of the neutron and proton rms radii in Ni-isotopes is plotted in Fig. 3. We notice that the evolution of centroid energies in the low energy region has a strong dependence on the thickness of neutron skin. The experimental measurements on the low-lying dipole excitation in neutron-rich nuclei might provide information on the neutron skin.

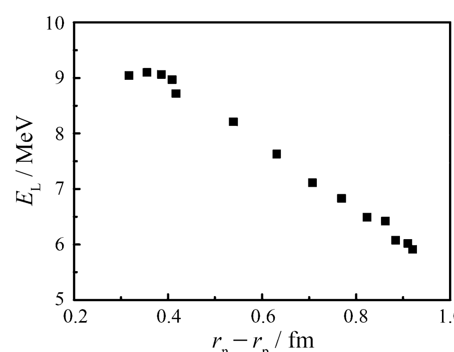

Fig. 3 The centroid energies of the isovector dipole strengths in the low-lying region below 10 MeV versus the differences of the neutron and proton rms radii in Ni-isotopes.

The single particle continuum in the RPA could be exactly treated by employing the Green's function technique. The single particle Green's function characterizes all information of occupied and unoccupied single particle states. It can be calculated numerically as proper combinations of the regular and irregular solutions of the Dirac equation. Based on the Green's function method we construct the relativistic continuum random phase approximation (RCRPA) to describe nuclear collective excitations in finite nuclei. We apply the fully consistent RCRPA to study the ISGDR and ISGMR in ^{208}Pb and compare with those obtained in the RRPA, which are shown in Fig. 4.

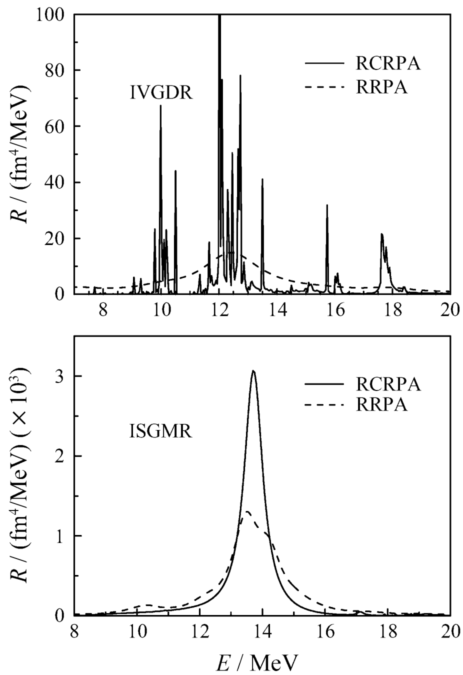

Fig. 4 The isovector giant dipole and isoscal monopole resonances in ^{208}Pb.

4 Gogny Pairing Interaction with a Separable Form

We adopt a method, which has been proposed by Duguet[4] to derive a separable form of the Gogny pairing interaction by recasting the gap equation in nuclear matter. In the center of mass frame the matrix element is approximated by a separable form, $\langle \boldsymbol{k} | V^{^1S_0}_{\text{sep}} | \boldsymbol{k}' \rangle = \lambda v(k) v(k')$. A simple Gaussian form for the separable force is assumed, $v(k) = e^{-\alpha^2 k^2}$, where λ and α are two parameters of the separable force. The gap equation in the plane wave basis, which is the usual BCS equation in the $^1S_0$ channel, reads as

$$\Delta_k = -\int_0^\infty \frac{dk'}{2\pi^2} k'^2 \langle \boldsymbol{k} \mid V^{^1S_0} \mid \boldsymbol{k}' \rangle \frac{\Delta_{k'}}{2E_{k'}}, \quad (5)$$

where $E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$ is a quasi-particle energy with ε_k being the in medium on-shell single particle energy associated with the chemical potential μ. One solves the gap equation Eq. (5) in $^1S_0$ channel at various densities in nuclear matter with the Gogny force: D1 and D1S. We obtain sets of parameters $\alpha^2 = 0.405$ fm^2, $\lambda = -738$ MeV fm^3 for D1 and $\alpha^2 = 0.415$ fm^2, $\lambda = -728$MeV fm^3 for D1S. The separable forces can reproduce the gap closure almost perfectly, especially in the case of Gogny force with D1S parametrization. This new separable force has a very simple form, and makes mean field plus Bogoliubov calculations in the coordinate space tractable.

In order to carry out the RHB calculation with the Gogny separable pairing interaction in finite nuclei a Moshinsky transformation has to be introduced in the calculation of the pairing matrix elements. The wave functions of paired nucleons are expressed in the laboratory coordinate, while the separable pairing interaction is obtained in the center of mass frame of two paired particles. We calculate the pairing energies in isotope chains ^{100}Sn—^{160}Sn and ^{164}Pb—^{264}Pb in the RHB approach. Good agreement of the pairing energy calculated with Gogny pairing force and its separable form is observed, where the largest discrepancy is less than 10%.

Collective low-lying excited states in weakly bound nuclei are best described by the RQRPA based on the RHB framework. It has been investigated that the experimental data of the lowest $E2$ states as well as the B$E2$ in Sn-isotopes are well described in the RQRPA with NL3 and Gogny D1S pairing interaction. As far as we are aware, no other theoretical calculation has produced better results. The calculation with the Gogny separable pairing force can give almost the same results as the Gogny force. In Fig. 5 we show the lowest $E2$ states in Sn-isotopes, where the squares are the experimental data. The solid circles and triangles are calculated in the RHB-RQRPA with the Gogny pairing and its separable force, respectively.

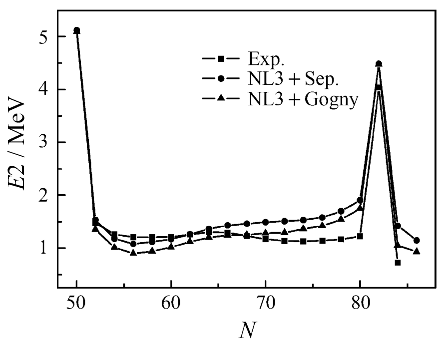

Fig. 5 The collective low-lying excited state $E2$ in Sn-isotopes.

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