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Infiltration in Unsaturated Frozen Soil

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A one-dimensional mathematical model is used to simulate the process of snow-melt infiltration in unsaturated frozen silt. Hydraulic and thermal parameters are mainly based on data given in the literature.

Field observations in a watershed (of area 1.8 km^2) are compared with simulated data and consequences on snow melt run-off are discussed.

Introduction

Infiltration and run-off from surfaces covered with grass and gravel in urban areas, during the period of snow melt are of great importance for the inflow in stormwater drainage systems. The objective of this study is the simulation of one-dimensional vertical water movements in the subsurface frozen zone, and the prediction of run-off during the period of snow melt.

The process of infiltration in frozen soil was studied in the field. Hydrologic observations were made and soil-samples were taken from surfaces representing grass, gravel and forest in a watershed of area 1.8 km^2. The simulations have been concentrated on the soil profile of the grass surface containing Silt Loams. The heat and moisture transfer parameters used in the model are mainly based on data of soil material with similar properties given in the literature.

A Mathematical Description

In the soil-ice-water system there are three principal interacting processes; moisture transfer, heat transfer and phase change. Vertical moisture transfer in the frozen unsaturated subsurface soil is formulated here in terms of the phase change of water and ice

$$\frac{\partial \theta}{\partial t} + \frac{\rho_I}{\rho}\frac{\partial \theta_I}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z}\right) - \frac{\partial K}{\partial z} \qquad (1)$$

where

θ – unfrozen water content, parts by volume water of the total soil volume
θ_I – ice content, parts by volume ice of soil
ϱ_I – density of ice (917 kg/m^3)
ϱ – density of water (1,000 kg/m^3)
K – hydraulic conductivity (m/s)
D – diffusivity coefficient (m^2/s)
t – time (s)
z – depth vertically from the surface, positive downward (m)

The dependence of the coefficient of diffusivity, D, on the hydraulic conductivity and the gradient of pore water pressure ($\partial\Psi/\partial\theta$) in the soil is expressed by the equation

$$D = K\frac{\partial \Psi}{\partial \theta} \qquad (2)$$

where Ψ is the pore water pressure in a water column one metre in length.

The heat transfer in the frozen unsaturated subsurface soil has a strong influence of the latent heat at the phase change ice/water, and can be expressed in the following way (Johansen 1977)

$$C_m\frac{\partial T}{\partial t} - L\frac{\rho_I}{\rho}\frac{\partial \theta_I}{\partial t} = \frac{\partial}{\partial z}\left(K_T\frac{\partial T}{\partial z}\right) \qquad (3)$$

where

C_m – volumetric heat capacity in the soil-ice-water system (J/m^3, ^{0}C)
K_T – thermal conductivity in the soil-ice-water system (J/s, m, ^{0}C)
L – latent heat, effective heat of fusion/ice formation (J/m^3, H_2O)
T – temperature (^{0}C)

The thermal conductivity in the soil-ice-water system is calculated by using the method adopted by De Vries (1966) with

$$K_T = K_w\,\theta + K_I\,\theta_I + K_s\,\theta_s \qquad (4)$$

where K_w, K_I, K_s are thermal conductivities of the water, ice and soil respectively. θ_s is the volumetric soil content with specific gravity ϱ_S. The volumetric heat capacity of the system is obtained from the following equation

$$C_m \equiv c_w \rho \theta + c_I \rho_I \theta_I + c_s \rho_s \theta_s \tag{5}$$

where c_w, c_I, c_s are the heat capacities (J/kg, ^{0}C) of the water, ice and soil respectively.

Only partial freezing of the water content of the soil occurs when the temperature drops below 0^0C. It is standard practice to relate the unfrozen volumetric water content to the temperature by an equation of the form

$$\theta = \theta(T) \quad , \quad T < 0^{\circ}\mathrm{C} \tag{6}$$

The function $\theta(T)$ decays rapidly with $|T|$, in general, but its particular form will depend on the type of soil considered. The behaviour of the unfrozen volumetric water content function, $\theta(T)$ for an unsaturated soil, is assumed to be the same as $\theta(T)$ for a saturated soil, if there is enough water available.

Elimination of the term $\partial\theta_I/\partial t$ in Eqs. (1) and (3) yields the following expression

$$\frac{C_m}{L}\frac{\partial T}{\partial t} + \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(\frac{K_T}{L}\frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z}\right) - \frac{\partial K}{\partial z} \tag{7}$$

This equation can be rewritten with Eq. (6) and the relations

$$\frac{\partial T}{\partial t} = \left(\frac{\partial \theta}{\partial t}\right)^{-1}\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial t\,\theta^*}$$

$$\frac{\partial T}{\partial z} = \left(\frac{\partial \theta}{\partial T}\right)^{-1}\frac{\partial \theta}{\partial z} = \frac{\partial \theta}{\partial z\,\theta^*}$$

Eq. (7) can then be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[\left(\frac{K_T + D\,L\,\theta^*}{C_m + L\,\theta^*}\right)\frac{\partial \theta}{\partial z}\right] - \frac{\partial}{\partial z}\left[\left(\frac{L\,\theta^*}{C_m + L\,\theta^*}\right)K\right] \tag{8}$$

The relationship Eq. (8) implies that the coupled heat and soil-moisture transfer problem is expressed in terms of only one equation. Hromadka et al. (1981) used this for the numerical solution when they discussed a horizontal, freezing soil column, in which gravitational forces could be ignored.

Numerical algorithms of solution are formulated here for Eqs. (1), (3), (6) and (8) with an explicit finite difference scheme, in which θ, T and θ_I are determined at every time step.

Field Measurements and Model Applications

Field Measurements

The field observations were conducted in the Bensby area, approximately five kilometers north of Luleå. The run-off from the area was registered by a 90^0 V-notch weir, while precipitation and air temperature were measured manually once daily. The watershed is composed of approximately 70% forest land and of 30% arable and pasture land.

The infiltration observations were made on three sites, representing grass, gravel and forest. Soil-samples were taken at the depths 0.10, 0.30 and 0.55 m under the surface. In Table 1 material data on the soil profile of the grass site is shown.

The soil types for the soil profile of the grass site were found to be Sandy Loams, Silt Loams and Clay Loams at the depths of 0.10, 0.30 and 0.55 m respectively. These determinations were based on mechanical wet screening and sedimentation analysis for fractions finer than 0.053 mm. The average value of the specific gravity was 2,700 kg/m^3.

The temperatures in the soil profile were measured with a thermocouple (copper constantan) every 15 cm to a depth of 90 cm. Fig. 1 shows the temperature variation in the profile of the soil of the grass site during the period of snow melt, April 1983.

The snow depth and snow density were determined from data taken from snow surveys. The snow depth was an average value, taken from 25 measuring points in line, on a field at the observation sites. The snow density was an average value taken from five determinations at each snow survey. Observed snow and frost depths at the grass site are shown in Fig. 2. Air temperatures during April 1983 are shown in Fig. 3.

The effective snow melt (snow plus rain) during the period of March 31-April 25 1983 was measured to 181 mm.

Moisture measurements were made using a depth moisture probe fitted with a neutron source. Measurements were made every 15 cm to a depth of 90 cm. Reliable values could not be obtained at every depth owing to calibration problems.

Table 1 – Material data for the soil profile of the grass site at Bensbyn, November 1982.

Depth (m)	Dry density (kg/m^3)	Porosity (% by volume)	Water content (% by volume)
0.10	1350	49	45.2
0.30	1360	51	40.5
0.55	1360	50	40.2

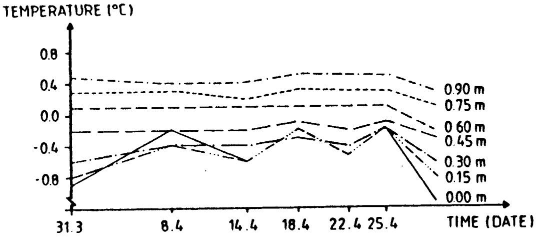

Fig. 1.
Measured temperatures in the soil of the grass site at Bensbyn (March 31st-April 29th, 1983)

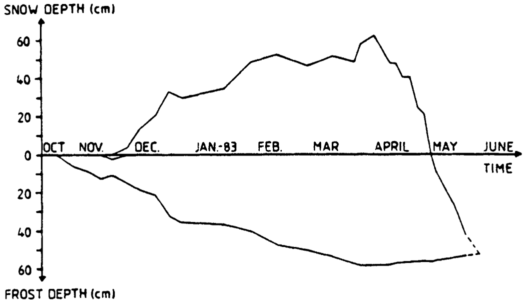

Fig. 2.
Snow and frost depths for the grass site at Bensbyn. (October 14th, 1982-June 9th, 1983).

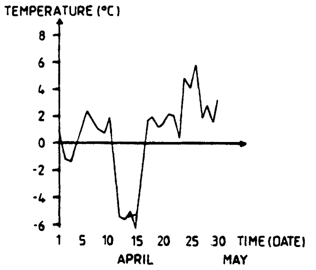

Fig. 3. Air temperature at Bensbyn at 8.00 a.m. during April 1983.

Boundary and Initial Conditions

At the upper boundary, a horizontal surface, measured values were assigned. The temperature was assigned once every twenty-four hours, and was interpolated linearly between the measuring times. The moisture content at the surface was set equal to its value at saturation, which is the most critical value from the infiltration point of view. The porosity was taken to be fifty percent (by volume) through the whole profile (Table 1).

The lower boundary was a horizontal plane at a depth of 0.45 m below the upper boundary. The boundary values were assigned on the basis of measured data of moisture content and temperature.

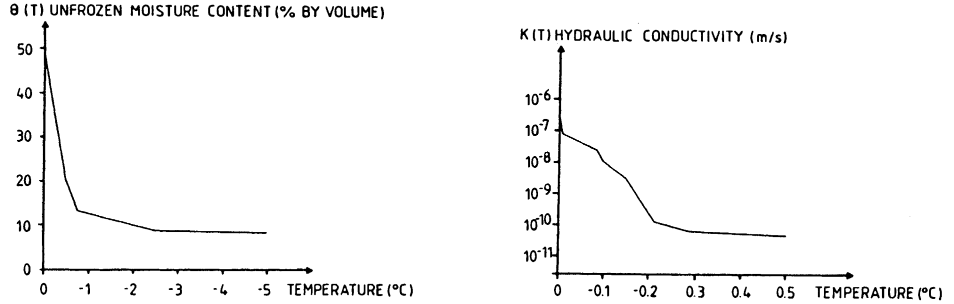

Fig. 4. Unfrozen moisture content and hydraulic conductivity as a function of the temperature below freezing point (0°C) for Bensby Silt. The relationships are estimated from experimental data measured by Burt and Williams (1976).

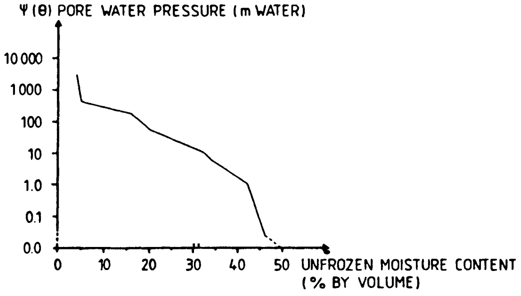

Fig. 5.
The pore water pressure as a function of the unfrozen moisture content for Bensby Silt. The relationship is based upon data from Andersson et al. (1972).

Parameter Choice

Grain size curves of the different soil types beneath the grass site, see Table 1, indicated that the soil profile was not completely homogeneous. In spite of this, however, properties of the soil at a depth of 0.30 m (Silt Loam, "Bensby Silt") was assumed to apply to the entire soil profile.

The water and heat transfer parameters of the soil material were assigned on the basis of measured data reported in the literature for similar materials. Burt and Williams (1976) developed an apparatus for studying the relationship between the hydraulic conductivity and the negative temperature in different soil materials. Based upon their results, the relationships between unfrozen water content/hydraulic conductivity and negative temperature were proposed for Bensby Silt.

Fig. 4 shows variations of θ and K with $T<0^0$C estimated from the literature for Bensby Silt.

$\Psi(\theta)$-relationship for Bensby Silt was estimated from results by Andersson et al. (1972) through comparisons of representive grain fractions (Fig. 5).

In the simulation, the following values were used for the thermal conductivity and heat capacity; K_w = 0.56 W/m,^{0}C, K_I = 2.25 W/m,^{0}C, K_s = 3.8 W/m,^{0}C, c_w = 4.18 kJ/kg, ^{0}C, c_I = 2.04 kJ/kg, ^0C and c_s = 0.82 kJ/kg, ^{0}C. The simulations of moisture and temperature distributions were based on estimated values of characteristic soil parameters and on the assumption that the soil profile was homogeneous. This is of course, a crude approximation to in-situ conditions.

Results of Simulations, Discussion

Results of Simulations

The process of infiltration to a depth of 0.45 m was simulated for the month April 1983. The results showed that the infiltration in the entire period came up to approximately 40-50 mm. This implied that roughly 25% of the effective snow melt was brought into the soil and the remainder ran off on the surface. Furthermore, it appeared that the groundwater storage was filled with approximately the same quantity of water as infiltrated at the surface in this period (the month April 1983).

In Fig. 6 a comparison is made of the measured and the simulated moisture contents at the depth 0.30 m under the grass site, as a function of time.

Simulated time dependence of temperature is compared with measured values at the depths of 0.15 m and 0.30 m in the period March 31-April 29 1983 in Fig. 7.

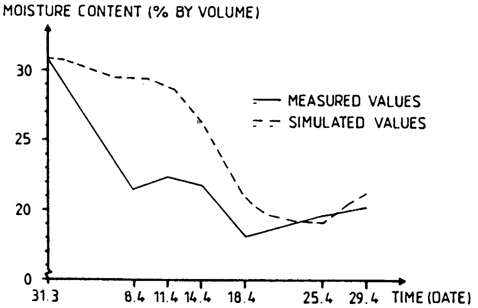

Fig. 6.
Simulated and measured changes of the total moisture content for the period March 31st-April 29th, 1983, 0.30 m under the grass site.

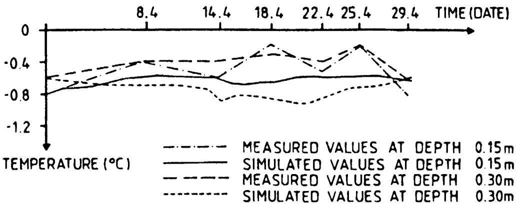

Fig. 7.
Simulated and measured temperatures March 31st-April 29th, 1983, 0.15 and 0.30 m under grass site.

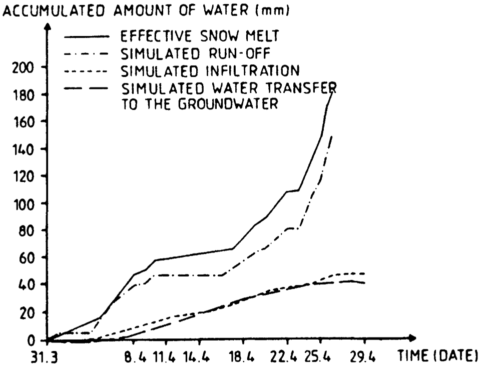

Fig. 8.
Accumulated effective snow melt and results from simulations of accumulated run-off, infiltration and water transfer to the groundwater for the grass site at Bensbyn, April 1983.

Discussion

A sensitivity analysis of the model parameters showed that the moisture distribution (Fig. 6) was significantly influenced by the hydraulic relationships; $\theta(T)$, $\Psi(\theta)$ and $K(T)$. However, the temperature dependence was not so sensitive to these parameters. Effective numerical algorithms are needed to attain sufficient precision in the solutions of the highly non-linear relationships.

The simulated water transfer conditions in the frozen soil are now used together with measured run-off in the Bensby area.

The effective snow melt was estimated on a daily basis from the snow survey with a degree-day method

$$S_{eff} = G\,T^{+} + P \tag{9}$$

where

S_{eff} – effective snow melt per day (mm water)
G – degree-day coefficient (mm water of snow melt/degree, day)
T^{+} – degrees above freezing (^{0}C)
P – precipitation of rain (mm water)

For the snow melt period at Bensbyn, April 1983, G was calculated to be 4.49 mm water/degree, day.

Fig. 8 illustrates calculated accumulated values of effective snow melt, run-off, infiltration and water transfer to the groundwater for the period March 31-April 29 1983.

Run-off recordings from the Bensby area were not started until April 19, due to freezing problems. However, the main snow melt occurred in a ten-day period after this date. Hence our discussion will be focused on this period. The entire

Table 2 – Distribution of surveyed effective snow melt on infiltration and run-off from the soil types; open field (circ.30%) and forest (circ. 70%) at Bensbyn, April 17-26 1983.

	Meadow (Silt)	Forest (Medium Sand)	Total
Effective snow melt (mm)	37	75	112
Infiltration (mm)	7	48	55
Run-off (mm)	30	27	57

area is assumed to contribute to the run-off. The time of concentration for the entire area was estimated to be roughly two days by Bengtsson (1982).

Calculated and measured data were used to estimate the distribution on infiltration and run-off from forest (Medium Sand) and open field (Silt Loams) in the phase of maximum snow melt. The forest ground received roughly 15% less snow than the open field. The calculated distribution is shown in Table 2.

On basis of the results in Table 2 and other data from the area it appeared that the infiltration over the ten-day period was 19% and the run-off was 81% of the effective snow melt on open field. The corresponding values in forest were 64% and 36%.

The effective snow melt was distributed on a daily basis using Eq. (9) in which the daily infiltration increased to a maximum of 43% of the effective snow melt at Bensby Silt. Over a two-day period conditions prevented water infiltration. In the Medium Sand of the forest, 5% infiltrated as a minimum which occurred one day and in a two-day period all melt water in the forest infiltrated.

Conclusions

A rough estimate of the snow melt run-off from arable and pasture land was conducted on the basis of field observations and simulations of the infiltration process in the frozen unsaturated subsurface soil. The run-off during the ten-day long snow melt period was roughly 80% (30 mm) of the effective snow melt while roughly 20% infiltrated to the grass-covered land. In the forest land which consisted mainly of Medium Sand a much larger part of water infiltrated into the soil.

The infiltration process was simulated with a one-dimensional mathematical model which described the processes of moisture and heat transfer, and the phase change ice/water in the soil material.

Parameter data for the simulation was based mainly on data given in the literature for similar soil material. The simulation results were sensitive to the choice of hydraulic parameters.

Future work will concentrate on observations on infiltration process in frozen soil under controlled laboratory conditions.

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