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Experimental Estimation of the Current Field of a Complex Lake

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The purpose of this study is to construct an experimental model to determine the dependence of lake-current velocity on the preceding wind velocity. The parameters of the model are determined by linear programming, using measured wind velocity data. The model is usable in studies, in which current field of a complex lake is estimated, for example in water quality studies. With the method it may be possible to compute current velocity in sounds directly from wind velocity, and then use numerical methods for each open area of the lake.

Introduction

In ecological studies, there very often exists a need for computation of a current field of a lake. Numerical finite difference or finite element methods are available, but in general these methods are limited to lakes which are rather regular in shape. The current field of a complex lake, which include sounds, islands and open water, is more difficult to compute. For these kinds of lakes it would be very useful to be able to estimate currents directly from wind measurements at least in sounds. Experimental regression analysis based on wind velocity/lake current studies has been carried out in Finland by Sarkkula (1979).

In the following equations it is assumed that the wind/lake current process is linear. Of course, this is a simplification, but some effects of this assumption are taken into account in the calibration of the model. This assumption is also made in most numerical models. A linear system may be described by the equation

$$v(t) = \int_{-\infty}^{t} h(t-\tau)w(\tau)\,d\tau \tag{1}$$

where $w(\tau)$ is the input of the system, $h(t-\tau)$ the response function and $v(t)$ the output. In this case, w is the wind velocity and v the lake current velocity.

In hydrology, Eq. (1) has been applied to such processes as rainfall/runoff and rainfall minus evaporation/ground-water level. Several methods have been used to describe the runoff process, the newest methods which use linear programming (Neuman and de Marsily 1976) are also applicable to the present problem. Ground-water problems have been also described by using linear programming (Dreiss 1982) and with the ARMAV type model (Gottschalk and Nordberg 1977).

In order to proceed, Eq. (1) is written in discrete form

$$v_d(n) = \sum_{n-M+1}^{n} H_d(D_j, n-j+1) f_d(D_j) w(j) + \varepsilon_n \tag{2}$$

where

$v_d(n)$ – the component of current velocity of the period n along the direction d

$H_d(D_j,i)$ – the response function corresponding to the period i, current direction d and wind direction D_j of period j.

$w(j)$ – the wind velocity of period j

$f_d(D_j)$ – the wind direction correction function

M – the memory of the system, or the maximum lag

ε_n – the error term

It may be possible to combine f_d with H_d. However, this is not practical, because D_j is divided into only 4 classes when it is the argument of H_d. Function $f(D_j)$ may be determined by using a higher number of D_j classes (8).

Measuring

This study concerns Lake Jääsjärvi (φ=61° 37' N, λ=26° 7' E) the map of which is presented in Fig. 1. The 10 m depth contour is shown. This depth corresponds approximately to the average depth of the thermocline. The area of the lake is 65 km^2, mean depth 4 m and greatest depth 22 m. The catchment area of the lake is 1,425 km^2. Due to the high lake percentage (26,4%) of the catchment, the yearly variation of throughflow is rather small.

Current velocity was measured during the period 1979-1981 with AANDERAA

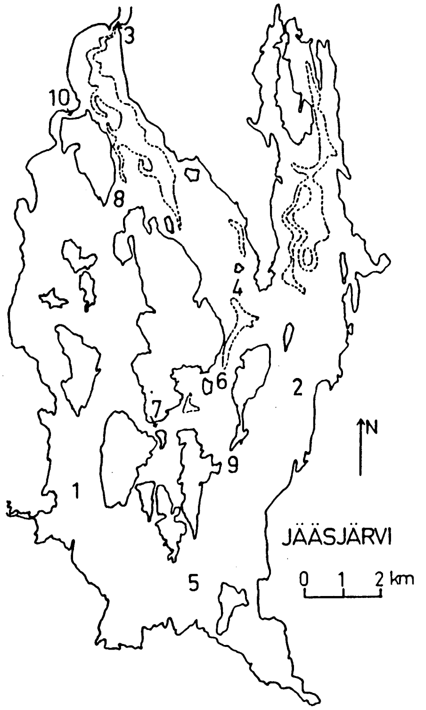

Fig. 1.
Map of Lake Jääsjärvi. Sites of current velocity measurements (numbers) and the 10 m depth contour are shown.

recording current meters. The measurement sites are presented in Fig. 1. The interval between consecutive measurements was 10 minutes and the duration of one measuring period 30-40 days. The depth of measuring was such that the deepest measuring was carried about 1 m from the bottom, and the highest about 1 m below the surface. The wind velocity was recorded on an island in the middle of the lake at a height of 3 m. From these measurements, the hourly mean wind velocity and hourly mean direction were determined.

Estimation of Parameters

In the following computations, the duration of the averaging period in Eq. (2) has been selected to be two hours. Also a one-hour averaging period was tried out, but due to the long memory of the system it seemed better to use this longer period.

Function f_d of Eq. (2) was determined by computing regression equations between six hours current velocity average and six hours wind velocity average for eight direction classes D_n^k ($k=1,2,...,8$), each class representing the angle $\pi/4$. From the regression coefficients f_d was computed in the form

$$f_d(D) = K_1 \cos(D-d) + K_2 \cos 2(D-d) + K_3 \sin(D-d) + K_4 \sin 2(D-d) + K_5 \qquad (3)$$

The estimation of time lag, M, was carried out using trial computations, it corresponds to 50 hours.

In the determination of numbers $H_d(D_j,i)$, the argument D_j was divided into four classes D_j^m (m=1,2,3,4) such that winds coming from $\pi(2m-3)/4+d$ to $\pi(2m-1)/4+d$ are classified as m.

Numbers H_d were estimated using a BURROUGHS 7800 LP-programme TEMPO for each direction class. The method is similar to that used by Neuman and de Marsily (1976) in the calibration of a rainfall/runoff model. In this method, parameters H_d are computed from the condition

$$\sum |\varepsilon_n| = \text{minimum} \qquad (4)$$

In accordance with Neuman and de Marsily (1976), the plausibility criterion

$$\sum |H_d(D,i+1) - 3H_d(D,i) + 3H_d(D,i-1) - H_d(D,i-2)| < C \qquad (5)$$

was used in order to ascertain the stability of numbers H_d. This criterion also increases the degree of freedom of the computation. C in Eq. (5) is a parameter the value of which is found by parametric programming.

Computation Results

Figs. 2 and 3 present H_d functions for sites 7 and 10. Values of H_d for different direction class D^m may be compared, but it must be remembered that the function $f_d(D)$ is also dependent on the direction class. In Figs. 2 and 3 a slight oscillation at greater time values can be seen. The reason for this is not quite clear, but it may be due to the combined oscillation of different open parts of the lake. Furthermore, the internal seiche in the deep part of the lake may have an effect on the oscillation of H_d, at least at site 10.

Fig. 4 gives an example of measured (solid line) and computed (broken line) components of velocity at Hopeasaaren Salmi (site 7) along the direction of the sound positive to 270°. During this time (Oct 3 to Oct 10, 1981), the wind was blowing from SW-SE 2-8.2 m/s. In this case, the current direction is in opposition to the wind velocity. This shows that the meter was measuring the return current.

To get an idea how well the model gives the current velocity, see Table 1 which gives some results from the calibration period, and Table 2 which gives some results from the test periods. It may be seen that the error seems to be about equal, both during the test and calibration periods. This shows, for example that

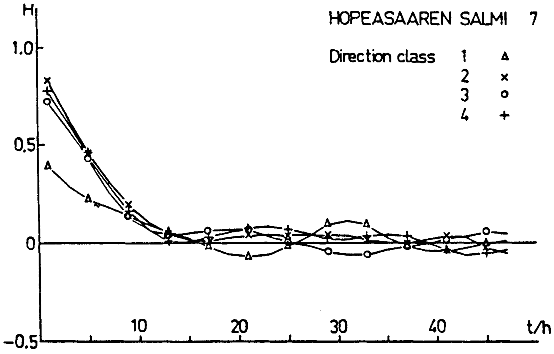

Fig. 2.
H_d function for Hopeasaaren Salmi (site 7) for different direction classes D^m of wind velocity. Meter depth 3.6 m, bottom depth: 5.4 m.

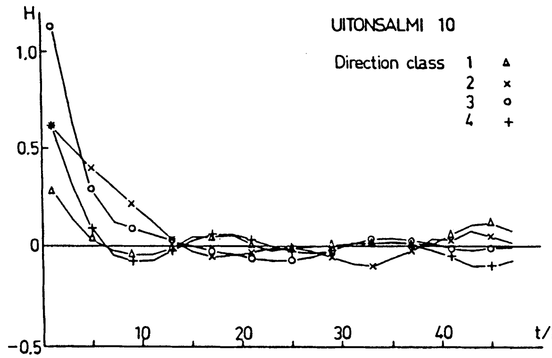

Fig. 3.
H_d function for Uitonsalmi (site 10) for different direction classes D^m of wind velocity. Meter depth: 1.3 m, bottom depth: 2.4 m.

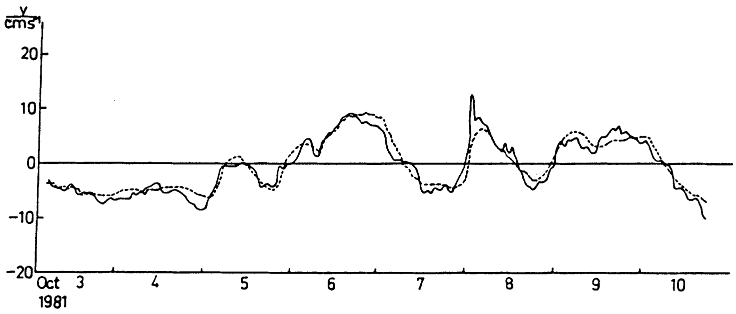

Fig. 4. Measured (solid line) and computed (broken line) current velocity at Hopeasaaren Salmi. Positive current direction is to West. Direction of oncoming wind varied between SW and SE.

Table 1 – Calibration periods. *n* is the number of two-hour periods. $|\bar{\varepsilon}|$ is the average deviation of measured and computed velocities and $|\bar{\sigma}|$ is the average deviation of the measured velocity from the measured velocity mean.

	Period	$\lvert\bar{\varepsilon}\rvert$ cm/s	$\lvert\bar{\sigma}\rvert$ cm/s	*n*	Depth m	Meter depth m
Uitonsalmi	11 Jul – 31 Jul	1,7	2,8	240		
1981	21 Jul – 10 Aug	1,6	3,7	240	2,4	1,3
	30 Jul – 20 Aug	1,6	3,4	240		
Hopeasaaren						
Salmi	9 Jul – 29 Jul	0,9	2,7	240		
1981	19 Jul – 8 Aug	1,0	2,5	240	5,4	3,6
	29 Jul – 18 Aug	0,9	2,0	240		
	8 Aug – 29 Aug	0,8	2,2	240		

the response function is rather stable in time and does not depend on temperature stratification of the lake. Greater errors at site 10 may be caused by the fact that the meter was located at a shallow depth, and it is possible that when the wind started the wind drift current was measured instead of the return current. This meter was measuring the return current more frequently than the wind drift current. Larger error of the first period at Hopeasaaren Salmi may be due to the fact that the meter was at a different place in the sound during the first period than during the other periods.

Table 2 – Calibration (*c*) and test (*t*) periods. *n* is the number of hours. Other symbols as in Table 1.

	Period	$\lvert\bar{\varepsilon}\rvert$ cm/s	$\lvert\bar{\sigma}\rvert$ cm/s	*n*	Depth m	Meter depth m
Uitonsalmi	13 Jul – 25 Aug c	2,5	4,2	1049	2,4	1,3
1981	5 Sep – 10 Oct t	3,0	5,8	844	2,2	1,2
Hopeasaaren						
Salmi	30 May – 8 Jul t	2,6	3,8	933	6,1	4,3
1981	11 Jul – 1 Sep c	1,4	2,7	1248	5,4	3,6
	6 Sep – 10 Oct t	1,7	3,1	826	5,4	3,6

Summary and Conclusions

A model for computing current velocity in sounds was formulated. The formulation needs two-hour wind velocity averages for 50 hours. Parameters of the model were determined by linear programming. This kind of experimental computation may be valuable when it is combined with numerical current field estimations. Before this method is fully applicable, the relation between wind drift current and return current must be studied.

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