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α-decay Branching Ratios to Rotational Band of Heavy Even-even and Odd-*A* Nuclei*

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Abstract: By using a simple barrier penetration approach, we predict the α-decay branching ratios to members of ground-state rotational band of heavy even-even No isotopes. We also extend our approach to calculate the α-decay branching ratios to the rotational band of heavy odd-*A* nuclei. The theoretical branching ratios of α-decays are found in good agreement with the available experimental data.

Key words: α-decay; branching ratio; heavy nuclei

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1 Introduction

The synthesis of long-lived superheavy elements beyond the actinides is a hot topic of nuclear physics in recent years[1,2]. Experimentally, the production of superheavy elements is very difficult because the cross-section of these nuclei decreases rapidly with the increasing of proton number. Up to now, a number of superheavy nuclei have been reported in experiments with proton number up to $Z=118$[1,2]. These newly synthesized superheavy nuclei mainly decay by α-particle emission. Various theoretical approaches have been used to study the α-decay of heavy and superheavy nuclei with the experimental data, such as ground-state half-lives and ground-state decay energies[3—19].

For the superhavy nuclei, the detailed information on the excited states is unavailable because only a few atoms per month can be produced with the very small cross-sections. Recently, it has become possible to perform spectroscopic studies on nuclei beyond fermium due to the development of highly efficient detector equipment. For instance, the ground-state rotational bands of ^{252}No and ^{254}No have been identified up to spin 20—22 $\hbar$, which is the heaviest element with observed high-spin data[20—22]. They provide a good opportunity to testify the validity of current theoretical models for nuclei approaching to the superheavy mass region.

The first purpose of this work is to investigate the fine structures of ^{252}No and ^{254}No by a simple barrier penetration approach of α-decay. As we know, the ground-state of actinides can decay to the excited states of the daughter nucleus, i.e. the α-decay to members of the ground-state rotational

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band. We have carried out systematic calculations on the α-decay chains of nuclei with proton number $Z \leqslant 100$[23]. Because of the success of the barrier penetration approach for these α-emitters, the exploration to heavier mass region is necessary and useful for experiments. The second purpose is to extend our present approach to the α-decays of odd-A nuclei. We note that the situation of α-decay of deformed odd-A nuclei is much more complex as compared with that of the even-even nuclei.

The outline of this paper is as follows. In Section 2, we present the detailed formula of the calculations. The numerical results and corresponding discussions are given in Section 3. Section 4 is a brief summary.

2 Formalism

Our calculations start with the radial Schrödinger equation[23]

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}r^2}+\left[U(r)+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]\psi(r)=E\psi(r)\ , \quad (1)$$

where the centrifugal potential $\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}$ is included in the Schrödinger equation and $U(r)$ is the standard square well potential

$$U(r)=\begin{cases}-U_0 & (r<R_0)\\ \dfrac{Z_1 Z_2 e^2}{r} & (r\geqslant R_0)\ .\end{cases} \quad (2)$$

Using the well known WKB technique, one can obtain the penetration probability of the α-particle[23].

$$P_\alpha(Q_\alpha, E_l^*, l) \propto \left|\frac{\psi_{\text{out}}}{\psi_{\text{in}}}\right|^2 = \exp\left[-2\int_{R_0}^{R_{\text{out}}} k(r)\mathrm{d}r\right]\ , \quad (3)$$

with

$$k(r)=\sqrt{\frac{2\mu}{\hbar^2}}\times\left[\frac{Z_1 Z_2 e^2}{r}+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}-(Q_\alpha-E_l^*)\right]^{\frac{1}{2}}, \quad (4)$$

where Z_1, Z_2 are the charge numbers of α-particle and the daughter nucleus, respectively. μ is the reduced mass of α-core system and l is the angular momentum carried by α-particle. Q_α is the decay energy of the ground-state transition and E_l^* is the excitation energy of state l. R_0 is the radius of the daughter nucleus ($R_0=1.2A_2^{1/3}$) and R_{out} is the outer classic turning point[23]. Usually the height of the centrifugal barrier at $r=R_0$ is very small compared with the Coulomb barrier[24]

$$\varepsilon=\left(\frac{\hbar^2}{2\mu}\frac{l(l+1)}{R_0^2}\right)\Big/\left(\frac{Z_1 Z_2 e^2}{R_0}\right)\ . \quad (5)$$

By expanding the wave number $k(r)$ in powers of the small quantity ε, the penetration probability can be written in a simple form[24]

$$P_\alpha(Q_\alpha, E_l^*, l)=\exp\left[-\sqrt{\frac{2\mu}{\hbar^2}}\frac{Z_1 Z_2 e^2 \pi}{(Q_\alpha-E_l^*)^{\frac{1}{2}}}\right]\times \exp\left[-\sqrt{\frac{\hbar^2}{2\mu}}\frac{2\,l(l+1)}{(Z_1 Z_2 e^2 R_0)^{\frac{1}{2}}}\right], \quad (6)$$

where the first term represents the influence of the excitation energy E_l^* on the penetration factor and the second term denotes the influence of the nonzero angular momentum l. We assume that the probability of the residual daughter nucleus to stay in its excited states ($I^+=2^+, 4^+, 6^+, \ldots$) obeys the Boltzmann distribution

$$w_l(E_l^*)=\exp[-cE_l^*]\ , \quad (7)$$

where E_l^* is the excitation energy of state l and c is a free parameter. The value of parameter c is fixed to 1.5 in calculation. The inclusion of the excitation probability is reasonable in physics and it can lead to a good agreement between experiment and theory. Here we define I_{l^+} as the product of the penetration factor and the excitation probability[23]

$$I_{l^+}=w_l(E_l^*)\,P_\alpha(Q_\alpha, E_l^*, l)\ , \quad (8)$$

which denotes the total probability of α-transition from the ground-state of the parent nucleus to the excited state l^+ of the daughter nucleus. It is very convenient to estimate the influence of these factors on the hindered α-transitions from I_{l^+}. With the help of I_{l^+}, the branching ratios of α-decay to each state of the rotational band of the daughter nucleus can be written as[23]

$$b_{gs}^{0^+}\% = I_{0^+}/(I_{0^+}+I_{2^+}+I_{4^+}+I_{6^+}+\cdots)\times 100\%,$$
$$b_{es}^{2^+}\% = I_{2^+}/(I_{0^+}+I_{2^+}+I_{4^+}+I_{6^+}+\cdots)\times 100\%,$$
$$b_{es}^{4^+}\% = I_{4^+}/(I_{0^+}+I_{2^+}+I_{4^+}+I_{6^+}+\cdots)\times 100\%,$$
$$\cdots.$$

3 Numerical Results and Discussions

The ground-state rotational band of ^{252}No was observed up to spin 20 via the ^{206}Pb(^{48}Ca, 2n) reaction[22]. The excitation energies of 2^+ and 4^+ states of ^{252}No were extrapolated from a Harris fit[22]. For ^{254}No, it is also a well deformed nucleus. The excitation energies of the ground-state band of ^{254}No were measured from the ^{208}Pb(^{48}Ca, 2n) reaction. Experimentally, the ground states of ^{252}No and ^{254}No mainly decay to the 0^+ states of their daughter nuclei[25]. The ground-state of ^{252}No and ^{254}No can also decay to the excited states of the daughter nucleus. Such α-transitions belong to the unfavored case, which are hindered by the non-zero angular momentum and excitation of nucleons. From the experimental side, the measurement of α-decay branching ratios to high-lying states is very difficult. As far as we know, the experimental branching ratios of high-lying states for ^{252}No and ^{254}No are still unknown[25].

Theoretically, the calculations of unfavored α-transitions are more difficult as compared with the favored cases. We systematically calculate the α-decay branching ratios of ^{252}No and ^{254}No to the members of ground-state rotational band of their daughter nuclei. We take into account the influences of α-decay energy, the angular momentum of α-particle, and the excitation probability of the daughter nucleus in our calculations. In Tables 1 and 2, we give the detailed results for α-decay fine structures of ^{252}No and ^{254}No. The first column of Table 1 denotes the α-emitter. Column 2 is the ground-state to ground-state α-decay energy. The spin and excitation energy of the rotational band members are given in column 3 and 4. The experimental and theoretical α-decay branching ratios are listed in the last 2 columns, respectively.

Table 1 Experimental and calculated branching ratios of α-decay to members of the ground-state rotational band of ^{252}No

Nuclei	Q_α /MeV	I^+	$E_{I^+}^*$ /MeV	$b_\alpha^{I^+}$(%) (Exp.)	$b_\alpha^{I^+}$(%) (Cal.)
^{252}No	8.549	0^+	0.000	55	52.5
		2^+	0.046	18	18.8
		4^+	0.154		1.69
		6^+	0.321		0.038 1
		8^+	0.545		0.000 209
		10^+	0.822		2.73×10^{-7}

Table 2 Experimental and calculated branching ratios of α-decay to members of the ground-state rotational band of ^{254}No

Nuclei	Q_α /MeV	I^+	$E_{I^+}^*$ /MeV	$b_\alpha^{I^+}$(%) (Exp.)	$b_\alpha^{I^+}$(%) (Cal.)
^{254}No	8.226	0^+	0.000	77	64.7
		2^+	0.044		23.2
		4^+	0.146		2.10
		6^+	0.305		0.047 5
		8^+	0.519		0.000 260
		10^+	0.786		3.30×10^{-7}

We can see from Tables 1 and 2 that the α-decay branching ratio to 0^+ and 2^+ states of the rotational band have been measured for ^{252}No, but only the branching ratio of 0^+ state has been measured for ^{254}No[25]. The experimental data of high-lying states(4^+, 6^+, 8^+,...) are still unavailable[25]. We can also see from Table 1 that the sum of branching ratios to the 0^+ and 2^+ states is as large as 73% for ^{252}No. The residual daughter nucleus ^{248}Fm has the most probability to stay in its ground state(0^+) because the penetration probability of the α-particle usually reaches maximum value when both the excitation energy E_l^* and the angular momentum l are zero. Although it is difficult to describe quantitatively the unfavored α-transitions, we can see from Table 1 that the theoretical values follow the experiment data well for the 0^+ and 2^+ states. For other states, the theoretical α-decay branching ratios are predicted by our barrier penetration approach. These predictions are very helpful for future experiments.

Besides the even-even deformed nuclei, we also try to extend the present approach to the cases of odd-A nuclei. In Table 3, we list the detailed results for α-decay fine structure of odd-A nucleus ^{243}Bk. It is seen from Table 3 that the calculated branching ratios are very close to the experimental data for the $5/2^-$, $7/2^-$, $9/2^-$, and $11/2^-$ states. For instance, the measured branching ratio of $11/2^-$ state of ^{243}Bk is 0.001 0%, while the theoretical value is 0.000 9%. Note that the α-decay fine structure of odd-A nuclei is very difficult to calculate, thus the agreement between experiment and data is very satisfactory.

Table 3 Experimental and calculated branching ratios of α-decay to excited states of ^{243}Bk

Nuclei	Q_α /MeV	I^+	$E^*_{I^+}$ /MeV	$b^{I^+}_\alpha$(%) (Exp.)	$b^{I^+}_\alpha$(%) (Cal.)
^{243}Bk	6.874	$5/2^-$	0.000	0.023 1	0.025 9
		$7/2^-$	0.041	0.018 8	0.015 9
		$9/2^-$	0.094	0.001 8	0.002 0
		$11/2^-$	0.156	0.001 0	0.000 9

4 Conclusions

In summary, we use a simple barrier penetration approach to calculate α-decay branching ratios of ^{252}No, ^{254}No and ^{243}Bk. The experimetnal branching ratios to the rotational band of ground state of these nuclei are reproduced very well. The theoretical branching ratios of high-lying states are predicted for ^{252}No and ^{254}No. It will be very interesting to compare the present theoretical predictions with the future experimental observations.

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