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$Z^+(4430)$ and Analogous Heavy Flavor States*

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Abstract: We have studied $Z^+(4430)$ as a $D^* D_1$ molecule from the quark model, state mixing effect is considered by solving the coupled channel Schrödinger equation numerically. More precise measurements of $Z^+(4430)$ mass and width, partial wave analysis are helpful to understand its structure. If it lies below the $D^* D_1$ threshold, molecule interpretation with $J^P=1^-$ is favored, and $J^P=0^-$ can not be ruled out. Otherwise $Z^+(4430)$ may be a virtual state with $J^P=2^-$. The analogous heavy flavor mesons Z_{bb}^+ and Z_{bc}^{++} are considered as well, and the masses predicted in our model are in agreement with the predictions from the potential model and QCD sum rule.

Key words: nonrelativistic quark model; hadronic molecule; coupled-channel analysis

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1 Introduction

Recently the Belle collaboration has reported a narrow peak in the $\pi^+\psi'$ invariant mass spectrum in $B\to K\pi^\pm\psi'$ with statistical significance greater than 7σ[1]. This structure is denoted as $Z^+(4430)$. The Breit Wigner fit for this resonance yields the peak mass $M=(4\ 433\pm4(\text{stat})\pm1(\text{syst}))$ MeV and the width $\Gamma=44^{+17}_{-13}(\text{stat})^{+30}_{-11}(\text{syst})$ MeV. Since the G-parity of π^+ and ψ' is positive, $Z^+(4430)$ is a isovector with positive G-parity.

Because it is very close to the threshold of $D^* D_1(2420)$, and the width of $Z^+(4430)$ is approximately the same as that of $D_1(2420)$, it is very likely that $Z^+(4430)$ is a $D^* D_1(2420)$ molecular state[2, 3]. In Ref. [2], we have studied $Z^+(4430)$ from the effective field theory, and how to distinguish the molecule from the tetraquark hypothesis was suggested. In this work, we will further study $Z^+(4430)$ and analogous heavy flavor molecules Z_{bb}^+ and Z_{bc}^{++} dynamically from quark model. The effective potential including screened color-Coulomb, screened linear confinement and spin-spin interactions are employed to describe the interactions between the components of the interacting hadrons.

2 Canonical Coordinate System and the Effective Interactions

The coordinate shown in Fig. 1 is taken as the canonical coordinate system, which defines the asymptotic states. The relevant coordinates for this

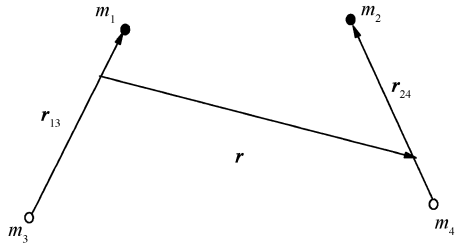

Fig. 1 Canonical coordinate system for the four quark system, where black circle denotes quark and empty circle denotes antiquark.

system can be expressed in terms of $\boldsymbol{r}_{13}$, $\boldsymbol{r}_{24}$ and $\boldsymbol{r}$ as

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$$\boldsymbol{r}_{ij} = f_{\mathrm{A}}(ij)\boldsymbol{r}_{13} + f_{\mathrm{B}}(ij)\boldsymbol{r}_{24} - \boldsymbol{r}, \quad i \in \mathrm{A} \text{ and } j \in \mathrm{B} \tag{1}$$

The parameters $f_{\mathrm{A}}(ij)$ and $f_{\mathrm{B}}(ij)$ are listed in Table 1, where m_1, m_2, m_3 and m_4 respectively are the masses of constituents 1, 2, 3 and 4.

Table 1 The parameters $f_{\mathrm{A}}(ij)$ and $f_{\mathrm{B}}(ij)$

	$f_{\mathrm{A}}(ij)$	$f_{\mathrm{B}}(ij)$
$i=1, j=2$	$\frac{m_3}{m_1+m_3}$	$-\frac{m_4}{m_2+m_4}$
$i=1, j=4$	$\frac{m_3}{m_1+m_3}$	$\frac{m_2}{m_2+m_4}$
$i=3, j=2$	$-\frac{m_1}{m_1+m_3}$	$-\frac{m_4}{m_2+m_4}$
$i=3, j=4$	$-\frac{m_1}{m_1+m_3}$	$\frac{m_2}{m_2+m_4}$

In the above canonical coordinate, the Hamiltonian for this system, including the relative motion and the interaction between two mesons, is split into

$$H = H_0(\mathrm{A}(13)) + H_0(\mathrm{B}(24)) - \frac{1}{2\mu_{\mathrm{AB}}}\nabla_r^2 + V_I, \tag{2}$$

where $H_0(\mathrm{A}(13))$ and $H_0(\mathrm{B}(24))$ are respectively the Hamiltonian for two mesons A and B, which contains all interactions within each meson. μ_{AB} is the reduced mass $\mu_{\mathrm{AB}} = M_{\mathrm{B}}/(M_{\mathrm{A}}+M_{\mathrm{B}})$. The third term $-1/(2\mu_{\mathrm{AB}})\nabla_r^2$ is the kinetic energy operator of relative motion. The interaction potential V_I is the sum of two-body interactions between quarks in the mesons A and B,

$$V_I = \sum_{i\in \mathrm{A},\, j\in \mathrm{B}} V_{ij}(r_{ij}) . \tag{3}$$

In this work, the effective interaction $V_{ij}(r_{ij})$ between two quark is taken as[4, 5]

$$V_{ij}(\boldsymbol{r}_{ij}) = C_iC_j\left[\frac{\alpha_{\mathrm{s}}\mathrm{e}^{-\mu r_{ij}}}{r_{ij}} + \frac{3b}{4\mu}\mathrm{e}^{-\mu r_{ij}} - \frac{8\pi\alpha_{\mathrm{s}}}{3m_im_j}\delta^3(\boldsymbol{r}_{ij})\boldsymbol{s}_i\cdot\boldsymbol{s}_j\right], \tag{4}$$

where α_{s} is the strong coupling constant, b is the string tension, μ is the screening mass, and $C_i(C_j)$ is the effective charge with $C_q = \sqrt{(N_{\mathrm{c}}^2-1)/2N_{\mathrm{c}}}$ and $C_{\bar{q}} = -\sqrt{(N_{\mathrm{c}}^2-1)/2N_{\mathrm{c}}}$[4].

As a result of the residual interaction V_I between two mesons, at short distance the mesons may excite as they interact, and they can be virtually whatever the dynamics requires. This means that we need to consider the state mixing effect. It has been shown that states mixing plays an important role in obtaining the phenomenologically required potential, when we study the nucleon-nucleon interaction and nucleon-antinucleon interaction from the chiral soliton model[6, 7]. The eigenvalue equation for the system is

$$(H-E)\,|\,\Psi\rangle = 0 , \tag{5}$$

where E and $|\Psi\rangle$ are the eigenvalue and the corresponding eigenfunction respectively. If there were no residual interaction V_{I} between A and B, the eigenfunction of the total system would simply be product of A meson's wavefunction and B meson's. Consequently it is natural to expand the eigenfunction $|\Psi\rangle$ in terms of the model wavefunctions

$$|\,\Psi\rangle = \sum_\alpha \psi(\boldsymbol{r})_\alpha\,|\,\Phi_\alpha(\mathrm{A},\mathrm{B})\rangle , \tag{6}$$

where $\psi(\boldsymbol{r})_\alpha$ is the relative wavefunction between the mesons A and B, and $|\Phi_\alpha(\mathrm{A},\mathrm{B})\rangle = |\Phi_{\mathrm{A}}\rangle|\Phi_{\mathrm{B}}\rangle$ denotes the intrinsic states of the two mesons, which will be mixed under the interaction V_I. The wavefunction $|\Phi_{\mathrm{A}}\rangle$ satisfies the Schrödinger equation $(H_0(\mathrm{A}(13)) - M_{\mathrm{A}})\,|\Phi_{\mathrm{A}}\rangle = 0$, $|\Phi_{\mathrm{A}}\rangle$ depends on the relative coordinate $\boldsymbol{r}_{13}$, and similarly for $|\Phi_{\mathrm{B}}\rangle$. Inserting wavefunction $|\Psi\rangle$ into the eigenequation Eq. (5), multiplying by $\langle\Phi_\alpha|$ and integrating over internal coordinates, we obtain

$$\left(-\frac{1}{2\mu_{\mathrm{AB}}}\nabla_r^2 + V_{I\alpha\alpha}(\boldsymbol{r}) + E_\alpha - E\right)\psi_\alpha(\boldsymbol{r}) = -\sum_{\alpha'\neq\alpha} V_{I\alpha\alpha'}(\boldsymbol{r})\psi_{\alpha'}(\boldsymbol{r}) , \tag{7}$$

where $E_\alpha = M_{\mathrm{A}} + M_{\mathrm{B}}$ is the energy eigenvalue of channel α. $V_{I\alpha\alpha'}(\boldsymbol{r}) = \langle\Phi_\alpha|V_I|\Phi_{\alpha'}\rangle$ is the matrix element of the interaction potential V_I, it is a function of the relative coordinate $\boldsymbol{r}$, and the intrinsic coordinates $\boldsymbol{r}_{13}$ and $\boldsymbol{r}_{24}$ have been integrated out. There

is clearly one equation for each state α, and they are coupled each other by the terms on the right-hand side. It is important to notice that all of the transitions represented by the right hand of Eq. (7) contribute coherently. If $|E_\alpha - E_{\alpha'}| \gg |V_{I\alpha\alpha'}(\boldsymbol{r})|$ with $\alpha \neq \alpha'$, then the coupled channel Schrödinger equation Eq. (7) is reduced to the single channel Schrödinger equation

$$\left(-\frac{1}{2\mu_{\mathrm{AB}}}\nabla_{\boldsymbol{r}}^2 + V_{I\alpha\alpha'}(\boldsymbol{r}) + E_\alpha - E\right)\psi_\alpha(\boldsymbol{r}) = 0 \ , \tag{8}$$

where $V'_{I\alpha\alpha}(\boldsymbol{r})$ is the effective interaction potential

$$V'_{I\alpha\alpha}(\boldsymbol{r}) = V_{I\alpha\alpha}(\boldsymbol{r}) - \sum_{\alpha' \neq \alpha} \frac{|V_{I\alpha\alpha'}(\boldsymbol{r})|^2}{E_{\alpha'} - E_\alpha} \ . \tag{9}$$

From Eqs. (8) and (9), we can see those are exactly the results of the second order perturbation theory to deal with the state mixing effect. This simplification is widely used[4, 6, 7]. However, if $|E_\alpha - E_{\alpha'}|$ is rather small, we have to solve the coupled-channel Schrödinger equation exactly. Although in principle we should solve the infinite set of equations implied by Eq. (7), in practice we only need to concentrate on the nearly degenerate channels, which is a good approximation.

3 $Z^+(4430)$ and $D^* D_1$ Molecule State

In this section we will study dynamically whether there exist $D^* D_1$ molecule state consistent with $Z^+(4430)$. Since $m_{D_1} \simeq 2.422$ GeV, $m_{D_1'} \simeq (2.441 \pm 0.032)$ GeV and $m_{D_2} \simeq 2.459$ GeV, the masses of $D^* D_1$, $D^* D_1'$ and $D^* D_2$ are close to each other. Under the residual interaction V_I in Eqs. (3) and (4), these three channels would be coupled together. However, the width of D_1' is very large $\Gamma \approx 384$ MeV[8], consequently there should be very small component of $D^* D_1'$ in the molecular state, otherwise it would decay so quickly that a weakly bound molecule can not form. As a result, we should consider both $D^* D_1$ and $D^* D_2$ states, then solve the two channels coupled Schrödinger equation to find whether there is solution whose energy eigenvalue lies below the $D^* D_1$ threshold.

The model parameters employed are $m_{\mathrm{u}} = m_{\mathrm{d}} = 0.334$ GeV, $m_{\mathrm{c}} = 1.776$ GeV, $m_{\mathrm{b}} = 5.102$ GeV, $b = 0.18$ GeV^2 and $\mu = 0.28$ GeV, which is a set of fairly conventional quark model parameters. Moreover, we use a running coupling constant $\alpha_{\mathrm{s}}(Q^2)$, which is given by

$$\alpha_{\mathrm{s}}(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(A + Q^2/B^2)} \tag{10}$$

with $A = 10$ and $B = 0.31$ GeV. Theoretical estimates for the harmonic oscillator parameter β scatter in a relative large region 0.3—0.7 GeV. Many recent quark model studies of meson and baryon decays use a value of $\beta = 0.4$ GeV[9], therefore we assume $\beta_{\mathrm{A}} = \beta_{\mathrm{B}} = 0.4$ GeV in this work.

The two $J = 1$ heavy-light mesons D_1 and D_1' are the coherent superpositions of quark model $^3P_1$ and $^1P_1$ states

$$\begin{aligned} |D_1\rangle &= \cos\theta\,|^1P_1\rangle + \sin\theta\,|^3P_1\rangle , \\ |D_1'\rangle &= -\sin\theta\,|^1P_1\rangle + \cos\theta\,|^3P_1\rangle . \end{aligned} \tag{11}$$

Little is known about the mixing angle θ at present. In the heavy quark limit, the mixing angle is predicted to be $-54.7°$ or $35.3°$, if the expectation of heavy quark spin-orbit interaction is positive or negative[40]. Since the former implies that the 2^+ state is greater in mass than the 0^+ state, and this agrees with experiment, we employ $\theta = -54.7°$ in the following.

There are various methods of integrating the multichannel Schrödinger equation numerically. In this work we shall employ two packages MATSCS[11] and FESSDE2.2[12] to perform the numerical calculations so that the results obtained by one program can be checked by another. The first package is a Matlab software, and the second is written in Fortran 77. Both packages can fastly and accurately solve the eigenvalue problem for systems of coupled Schrödinger equations, and the results obtained by two codes are the same within

errors.

The numerical results are listed in Table 2. We find there is a $J^P=0^-$ state with mass 4 411.9 MeV, which is below the D^*D_1 threshold. In addition, a 1^- state with mass 4 425.8 MeV is found, its dominant component is D^*D_1, and its rms is 1.32 fm. Both the 0^- state and 1^- state are widely extended in space, it is a good feature of molecular state. The wavefunctions for the two states are shown in Fig. 2. For $J^P=2^-$, the mass of the lowest state is about 4 437.8 MeV. It is just above the D^*D_1 threshold, and its dominant component is D^*D_1, consequently it looks like a virtual state.

The measured mass and width of $Z^+(4\,430)$ are fitted to be $4\,433\pm4(\text{stat})\pm1(\text{syst})$ MeV and $44^{+17}_{-13}(\text{stat})^{+30}_{-11}(\text{syst})$ MeV respectively. It is encouraging that the present analysis predicts molecules around 4 430 MeV. More precise measurements of its mass and width, partial wave analysis are important to understand the nature of $Z^+(4430)$. If it is below the D^*D_1 threshold, $Z^+(4430)$ as a $J^P=1^-$ molecule is favored, and $J^P=0^-$ can not be ruled out. Otherwise it may be a $J^P=2^-$ virtual state. It is very interesting and important to investigate whether $Z^+(4430)$ could be a virtual state and the characteristic properties if it is[13].

Table 2 The mass, the root of mean square radius(rms) and the ratio of D^*D_1 probability to D^*D_2 probability for the eigenstates of the D^*D_1 and D^*D_2 system

	mass/MeV	rms/fm	$P(D^*D_2):P(D^*D_1)$
$J=0$	4 411.9	0.94	0
$J=1$	4 425.8	1.32	0.042
$J=2$	4 437.8	2.15	0.003

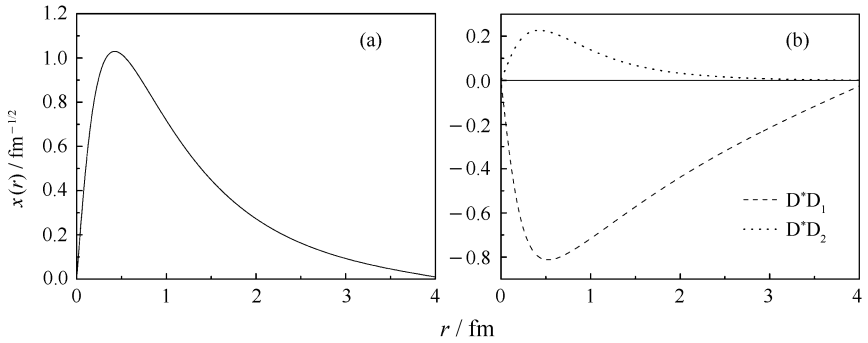

Fig. 2 The radial wave functions $\chi(r)=rR(r)$ for the molecular states of the D^*D_1 and D^*D_2 system, (a) and (b) respectively corresponds to $J^P=0^-$ and $J^P=1^-$ molecule.

4 Bottom Analog Z^+_{bb} and Z^{++}_{bc}

The bottom analog Z^+_{bb} denotes the state obtained by replacing both the charm quark and antiquark in $Z^+(4430)$ with bottom quark and antiquark. The state Z^{++}_{bc} is obtained by replacing the charm antiquark in $Z^+(4430)$ with bottom antiquark, which carrying two unit electric charge. The state replacing charm quark with bottom quark is conjugated to Z^{++}_{bc}, and the static properties are the same as those of Z^{++}_{bc}. Consequently we only need to discuss one of them, where we focus on Z^{++}_{bc}. Performing the coupled-channel analysis analogous to section 3, the numerical results are shown in Tables 3 and 4 respectively. It is obvious that the predictions for the masses of these states are consistent with the results from potential model and QCD sum rule.

Table 3 The mass, the root of mean square(rms) radius and the ratio of $B^* B_2$ probability to $B^* B_1$ probability for Z_{bb}^+. The mass predictions in Ref.[14] and the last paper of Ref.[3] are shown as well

	mass/MeV	mass in Ref.[14]/MeV	mass in Ref.[3]/MeV	rms/fm	$P(B^* B_2) : P(B^* B_1)$
$J=0$	10 886.4	10 730 ±100	10 740±120	0.31	0
$J=1$	10 906.5		——	0.32	0.254
$J=2$	10 982.5		——	0.37	0.027

Table 4 The mass, the root of mean square(rms) radius and the ratio between different components for Z_{bc}^{++}. The mass prediction in Ref.[14] is also listed

	mass/MeV	mass in Ref.[14]/MeV	rms/fm	$P(D^* B_1) : P(D^* B_2) : P(D_1 B^*) P(D_2 B^*)$
$J=0$	7 674.9	7 630±100	0.59	1 : 0 : 0.152 : 0
$J=1$	7 651.4		0.49	1 : 0.417 : 0.387 : 0.667
$J=2$	7 656.9		0.49	1 : 194.503 : 180.221 : 93.277

5 Conclusions

We have dynamically studied $Z^+(4430)$ and analogous heavy flavor states in a quark model. We have included the spin-spin interaction in addition to the screened color-Coulomb and screened linear confinement interactions in our model. The interactions between two mesons induce states mixing effects, we have solved the coupled channel Schrödinger equation to find the eigenvalues and corresponding eigenfunctions of the system, where the second order perturbation theory can not be used anymore.

For the $D^* D_1$ and $D^* D_2$ system, our model predicts molecule state around 4 430 MeV. More precise measurements of $Z^+(4430)$ mass and width, partial wave analysis are helpful to understand its nature. The bottom analog Z_{bb}^+ and Z_{bc}^{++} are investigated as well. The masses predicted in our model are in agreement with predictions from potential model[14] and QCD sum rule[3]. We propose to search for these states at Tevatron and LHC.

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