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AN IMPLEMENTATION OF STAGE-FALL-DISCHARGE RELATIONSHIP ON DIGITAL COMPUTERS

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The stage-discharge relation is defined by the complex interaction of channel characteristics, including area, shape, slope, and roughness of the cross-section. The combination of these effects has been designated control. The control is permanent if the stage-discharge relation it defines does not change with time, otherwise it is impermanent. The latter is designated a shifting control. The stage-discharge relation for a permanent control is established by simultaneous measurements of the two parameters stage and discharge, the relation is often described mathematically by a parabolic formula.

Controls may change because of the effects of a changing channel, backwater, rapidly changing stage, variable channel storage, and the freezing and breaking of ice. Backwater is produced when the normal water slope is decreased as the result of the normal stage being increased at some point downstream. This can result from the operation of a dam, an increase in discharge in a downstream tributary, or a rise in the stream into which the gaged stream empties. The effect of backwater is determined by introducing the slope or the fall through a reach of the channel downstream from the hydrometric station as a third parameter. This is graphically incorporated into a three-dimensional stage-fall-discharge relationship.

In Norway this relation is established by using Manning's formula and simultaneous measurements of discharge, stage, and fall (Hansson n. d., Westerberg 1920). In other methods a computational conversion of the measurements to a normal fall is made (Linsley, Kohler & Paulhus 1958). These methods give a graphical representation of the stage-fall-discharge relationship. They demand much work, are cumbersome in use, and badly adapted for conversion of stage measurements to discharges on digital computers.

The following describes a method well fitted for such conversions on computers. An eventual correction of the formulae used is also possible in a simple manner.

Theoretical description of the method

For a river cross-section the mean velocity v is given by

(1) $$v \equiv k \cdot R^m \cdot I^p.$$

R is the hydraulic radius, I the fall of the energy line, k a friction parameter, dependent on the roughness and degree of developed turbulence, m and p are constants, expressing physical properties of the section.
The mean velocity can also be written as

(2) $$v = \frac{q}{F},$$

where q is the discharge, and F the cross-section area. Combining formulae (1) and (2):

(3) $$q = F \cdot k \cdot R^m \cdot I^p.$$

k, R, and F are dependent on the stage in the section, h_T or

(4) $$q = f(h_T) \cdot I^p.$$

The assumption that the fall of the energy line equals the fall of the water surface, gives

(5) $$I = \frac{dh}{dl}.$$

dh is the stage-variation over an infinitely small channel length dl. A combination of (4) and (5) gives

(6a) $$q = f(h_T) \cdot \left(\frac{dh}{dl}\right)^p.$$

In practice the fall has to be measured over so long a reach of the channel L that it can be registered on stage-indicators.
Calling this fall Δh, (6a) gives

(6) $$q = f(h_T) \cdot \left(\frac{\Delta h}{L}\right)^p,$$

or

(7) $\Delta h = (f(h_T))^{\frac{1}{p}} \cdot L \cdot q^{\frac{1}{p}}.$

For discharges at constant mean stage, *hm*, over the reach in question, both L and $f(h_T)^{\frac{1}{p}}$ are constants. (7) can then be written as

(8) $h = \text{const.} \cdot q^{\frac{1}{p}} = C \cdot q^n.$

C can be called a friction parameter, and is dependent only on the stage. $n = \frac{1}{p}$. For a given n, a variable mean stage effects a variation in C alone. Thus a relation exists

(9) $C = f_1\,(hm)$

between the mean stage and the friction parameter.

If a combination of the stage, fall, and discharge measurements can give an analytical expression for this relation, it will be relatively simple to compute the discharge corresponding to two gauge-readings or one gauge-reading and the fall through the reach in question.

The procedure will then be: From the measurements of the three parameters q, dh, and hm, the friction parameter C is computed for various values of n by (8). A regression analysis with C and hm as variables gives the best possible value of n. The values of C for this n are then combined with the hm's to decide the analytical function (9). A parabolic form of this function is generally satisfactory.

Practical investigations in the usefulness of the method

To investigate the efficiency of this method, stage, slope, and discharge measurements from three hydrometrical stations were used:

The Strengen-Hogga in Eidselv, the Ertesekken in Vorma, and the Mosseelv gauges in Mosseelv.

Strengen-Hogga: The distance between the two gauges is about 4.8 kms. 51 measurements of stage, slope, and discharge were used in the computations. For values of n in (8) between 1.50 and 3.00, correlation analysis were made between hm and C. Fig. 1a shows the values of the correlation coefficient for different values of n. The highest value was attained for $n = 2.18$-2.20. Fig. 1b shows the relation between hm and C for $n = 2.20$, and the computed values of

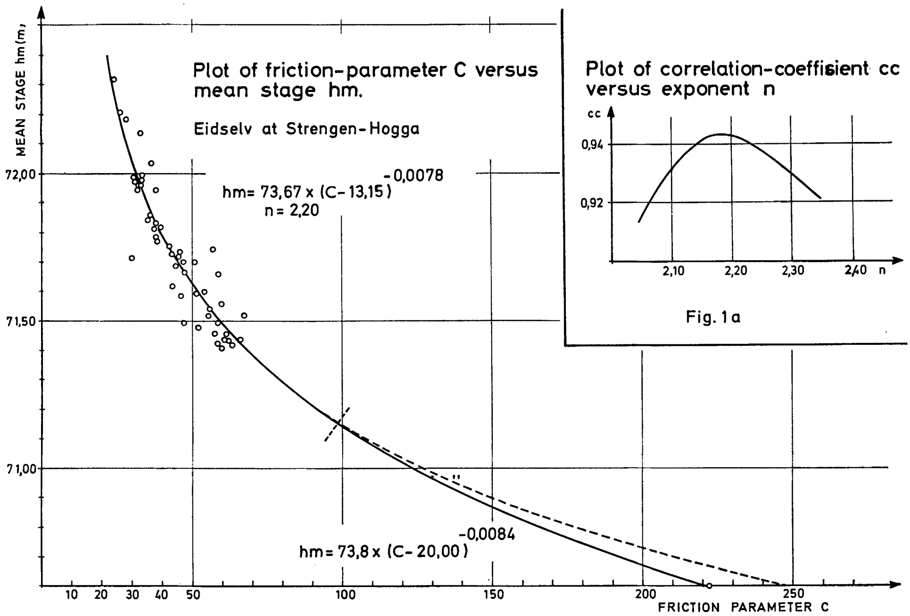

Fig. 1 a

Fig. 1 b

C. Computed and measured discharges, and their deviations in per cent, are given in Table 1a. Table 1b and 1c give the sum of measurements with deviations between the values –7, –5, ..., 5, and 7 per cent. One of the measurements is made at mean-stage 70.60 m. By means of this measurement, and comparisons with near-by gauges, the friction-curve in Fig. 1b was segmented as shown. Lack of measurements bounds the reliability of the lower part of the curve.

Ertesekken: The distance between the two gauges in Ertesekken is 0.8 kms. A value of $n \equiv 2.01$ gave the best correlation between *hm* and *C*. 105 measurements were used, and the friction-curve $C \equiv f_1(hm)$ was segmented in 4 parts. Table 2 gives the sum of the total number of measurements within the same deviation values as in Table 1b and 1c. The distribution of the measurements, as regards both mean-stage and discharge, should give satisfactory results over the whole observed mean-stage interval.

Mosseelv gauges: The distance between the gauges is 2.5 kms. 33 discharge-measurements were used, but 5 of these were excluded because of too high deviation values. This exclusion resulted in an increase of the highest correlation coefficient from 0.928 to 0.992. The corresponding *n* values were 1.78 and 1.79

Table 1a.

Measured and computed discharges at the Strengen-Hogga gauges in Eidselv

Mean stage (m)	Slope (m)	Measured discharge q (m3/s)	Computed discharge q_c (m3/s)	Deviation $\frac{q_c-q}{q} \times 100$ (%)	Mean stage (m)	Slope (m)	Measured discharge q (m3/s)	Computed discharge q_c (m3/s)	Deviation $\frac{q_c-q}{q} \times 100$ (%)
71.97	1.84	414.2	408.6	−1.3	71.71	0.39	210.8	176.3	−16.4
71.96	1.84	411.9	406.6	−1.3	71.99	0.39	208.0	204.3	−1.8
71.96	1.84	406.4	406.6	0	71.71	0.48	192.0	204.3	0.6
71.94	1.83	406.3	402.6	−0.9	71.70	0.54	191.3	202.6	5.9
71.85	1.83	385.8	384.5	−0.3	71.66	0.37	167.0	167.2	0.1
71.83	1.83	378.5	380.4	0.5	71.81	0.27	160.0	157.7	−1.4
71.78	1.83	377.2	370.2	−1.8	72.03	0.21	145.2	157.2	8.2
71.77	1.82	375.3	366.3	−2.4	71.72	0.25	144.6	144.8	0.2
71.75	1.82	361.8	362.2	0.1	71.78	0.20	134.0	135.0	0.7
71.98	1.42	359.2	365.0	1.6	71.84	0.17	133.0	129.9	−2.4
71.97	1.43	358.3	365.3	1.9	70.60	0.72	112.0	106.9	−4.6
71.62	1.73	347.8	329.1	−5.4	71.55	0.17	104.0	109.9	5.6
71.58	1.73	338.3	321.2	−5.0	71.45	0.15	101.0	97.4	−3.6
72.13	1.13	323.7	353.1	9.0	71.62	0.11	100.0	97.1	−5.9
72.20	0.84	312.8	317.1	1.4	71.49	0.11	96.2	86.8	−9.8
72.32	0.74	305.0	313.4	2.8	71.59	0.12	93.6	95.8	2.4
72.18	0.76	289.0	300.5	4.0	71.40	0.09	78.4	74.8	−4.6
71.51	1.40	284.0	278.5	−1.9	71.47	0.09	75.0	69.8	−7.0
71.94	0.92	275.8	293.8	6.5	71.74	0.07	71.5	82.1	14.9
71.41	1.38	274.9	259.4	−5.6	71.43	0.08	71.5	72.0	0.7
71.84	0.75	260.3	255.0	−2.0	71.51	0.08	70.9	75.8	6.9
71.69	0.93	254.2	258.2	1.8	71.43	0.07	69.9	68.0	−2.7
71.59	0.95	245.6	246.1	0.2	71.49	0.05	61.0	60.7	−0.6
71.42	1.06	237.8	231.6	−2.6	71.66	0.05	60.7	67.3	10.9
71.53	0.91	233.9	232.6	−0.6	71.55	0.04	54.8	56.7	3.0
71.73	0.71	225.4	234.1	3.9					

respectively. Table 3 gives the total number of measurements within the various deviation classes. The measurements are taken over a sufficient mean-stage and discharge interval, no segmenting of the friction curve was requisite.

CONCLUSIONS

Discharge conversions on digital computers for gauges where a slope-stage-discharge relationship is recommended seem to give good results by employment of a relationship between the mean-stage and the friction parameter. For the three hydrometrical stations investigated, the method gave satisfactory ac-

Table 1b.
Total number of measurements (t) with deviations within different deviation-intervals (q)

q	t		q	t
0-1 %	9		-1 - 0 %	4
1-3 %	6		-3 - -1 %	11
3-5 %	3		-5 - -3 %	3
5-7 %	4		-7 - -5 %	5
>7 %	4		<-7 %	2

Table 1c.
q in absolute values

q	t
0-1 %	13
1-3 %	17
3-5 %	6
5-7 %	9
>7 %	6

Table 2.
The Ertesekken gauges in Vorma
Total number of measurements (t) with deviations within different deviation-intervals (q)

q	t		q	t
0-1 %	12		-1 - 0 %	9
1-3 %	22		-3 - -1 %	21
3-5 %	6		-5 - -3 %	9
5-7 %	7		-7 - -5 %	4
>7 %	7		<-7 %	8

q in absolute values

q	t
0-1 %	21
1-3 %	43
3-5 %	15
5-7 %	11
>7 %	15

Table 3.
The Mosseelv gauges in Mosseelv
Total number of measurements (t) with deviations within different deviation-intervals (q)

q	t		q	t		q	t
0-1 %	1		−1 - 0 %	3		0-1 %	4
1-3 %	6		−3 - −1 %	6		1-3 %	12
3-5 %	2		−5 - −3 %	3		3-5 %	5
5-7 %	2		−7 - −5 %	1		5-7 %	3
>7 %	2		<−7 %	2		>7 %	4

cordance between computed and measured discharge. The method is simple to use, conversions can be made with only two equations, (8) and (9). A correction of the friction curve on the basis of later measurement is simple. As for all slope-stage-discharge methods, the discharge measurements have to be made over a sufficiently long mean-stage and discharge interval.

REFERENCES

Hanson, H (n. d.) Instruktion för beräkning av vattenföringen enligt tvåskalemetoden. SMHI, Stockholm.

Westerberg, N. (1920) Vattenmängdsmätning enligt tvåskalemetod. *Tek. Tidsskr., Väg- och Vattenbyggnadskonst.* H. 6.

Linsley, R. K., Kohler, Max A. & Paulhus, Joseph L. M. (1958) *Hydrology for engineers.* McGraw-Hill, New York.

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