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# Magnetohydrodynamic Transient Convection Flow past a Vertical Surface Embedded in a Porous Medium with Oscillating Temperature

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**Abstract**

This paper presents an analytical study of the transient hydromagnetic and thermal behaviour of free convection flow past a vertical plate embedded in a porous medium. The governing equations are solved in closed form by the Laplace-transform technique. The results are obtained for temperature, velocity, penetration distance, Nusselt number, and skin-friction. The effects of various parameters on the flow variables are discussed and presented in graphs.

**Key words:** Natural convection, Magnetohydrodynamic flow, Porous medium, Oscillating temperature.

## Introduction

Buoyancy forces that arise from density differences in a fluid cause free convection. These density differences are a consequence of temperature gradients within a fluid. Free convection flow is a significant factor in several practical applications that include, for example, cooling of electronic components, in designs related to thermal insulation, material processing, and geothermal systems. Transient natural convection is of fundamental interest in many industrial and environmental situations such as air conditioning systems, human comfort in buildings, atmospheric flows, motors, thermal regulation process, cooling of electronic devices, and security of energy systems. Oscillatory flow has applications in the technological field, and industrial and aerospace engineering. Hydromagnetic flow is encountered in heat exchangers, pumps, flow meters, in designing communications and radar systems, and in nuclear engineering in connection with the cooling of reactor and MHD accelerators. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fibre and granular insulation, electronic system cooling, cool combustors, and porous material regenerative heat exchangers. Books by Nield and Bejan (1999), Bejan and Kraus (2003), and Ingham et al. (2004) excellently describe the extent of the research information in this area.

Extensive research has been published on free convection flow past a vertical plate. Free convection at a vertical plate with transpiration was investigated by Kolar and Sastri (1998). Ramanaiah and Malarvizhi (1992) considered natural convection adjacent to a surface with 3 thermal boundary conditions. A numerical study for natural convective cooling of a vertical plate was presented by Camargo et al. (1996) with different boundary conditions. The more difficult problem of transient free convection flow past a semi-infinite isothermal vertical plate was first studied by Siegel (1958) using an integral method. The experimental confirmation of these results was presented by Goldstein and Eckert (1960). Another review of transient natural convection was presented by Raithby and Hollands (1985), wherein a large number of papers on this topic were referred to. In reference to transient convection, Gebhart et al. (1988) introduced the idea of leading edge effect in their book. They explained that the transition from conduction to convection begins only when some effects from the leading edge have propagated up the plate as a wave, to a particular point in question. Before this time, the fluid in this region effectively does not know that the plate has a leading edge. Later on, numerous investigators considered transient convective flow past a vertical surface by applying different boundary conditions and techniques. Transient convective heat transfer was pioneered by Padet (2005). The flow past a vertical plate with sudden change in surface temperature was examined by Harris et al. (1998). Das et al. (1999) analysed transient free convection flow with periodic temperature variation of the plate by Laplace-transform technique. In all the studies cited above, the effects of magnetic field and porous medium on the flow are ignored.

Many studies have been carried out to investigate the magnetohydrodynamic transient free convective flow. Gupta (1960) first discussed the transient natural convection flow from a plate in the presence of magnetic current. Chowdhury and Islam (2000) investigated magnetohydrodynamic free convection flow past a vertical surface by Laplace-transform technique. Aldoss and Al-Nimr (2005) analysed transient hydromagnetic free convection flow over a surface. All the above studies are concerned with the absence of porous medium in the flow.

In recent years, only a few studies have been performed on transient convective flows in porous media. A detailed review of the subject, including an exhaustive list of references, can be found in the papers by Bradean et al. (1998) and Pop et al. (1998). Transient natural convection flow past a plate was pioneered by Cheng and Pop (1984).

In industrial applications, quite often the plate temperature starts oscillating about a non-zero mean temperature. Many works reported in the literature deal with oscillating plate temperature. Free convection flow past a vertical plate with surface temperature oscillations was considered by Li et al. (2001). The response of the boundary layer flow past a semi-infinite flat plate to harmonic oscillations in the plate temperature was examined by Takhar and Soundalgekar (1989). Hossain et al. (1998) elucidated the flow past a plate with surface temperature oscillations in the presence of a magnetic field.

Hence, we have extended the problem described by Das et al. (1999) by applying a magnetic field in transverse direction of the plate and assuming that the plate is embedded in a porous medium. Under these conditions, how is transient free convection flow past an infinite vertical plate affected? This has not been studied in the literature so far. Therefore, this paper deals with the study of magnetohydrodynamic transient free convection flow when the mean plate temperature is superposed by periodic plate temperature with frequency $\omega$' and the plate is embedded in a porous medium.

## Mathematical analysis

We consider a 2-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite vertical plate that is embedded in a porous medium. The x'-axis is taken on the infinite plate and parallel to the free stream velocity and y'-axis normal to it. Initially, the plate and the fluid are at the same temperature $T^{'}_{\infty}$. At time t' > 0, the plate temperature is raised to $T^{'}_{w}$ and a periodic temperature is assumed to be superimposed on this mean constant temperature of the plate. A magnetic field of uniform strength is applied in the transverse direction of the plate and the induced magnetic field is neglected. Then, neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation), the problem can be governed by the following set of equations:

$$\frac{\partial T^{'}}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y'^2} \tag{1}$$

$$\frac{\partial u^{'}}{\partial t'} = \nu\frac{\partial^2 u^{'}}{\partial y'^2} + g\beta\,(T^{'} - T^{'}_{\infty}) - \frac{\sigma B_0^2 u^{'}}{\rho} - \frac{\nu u^{'}}{K'} \tag{2}$$

with the following initial and boundary conditions:

$$\begin{aligned} & u^{'} = 0,\ T^{'} = T^{'}_{\infty} && for all y^{'}, t^{'} \le 0 \\ & u^{'} = 0\ ,\ T^{'} = T^{'}_{w} + \in (T^{'}_{w} - T^{'}_{\infty})\, cos\omega^{'} t^{'} && \\ & && at y^{'} = 0,\ t^{'} > 0 \\ & u^{'} \to 0,\ T^{'} \to T^{'}_{\infty} && as y^{'} \to \infty\ ,\ t^{'} > 0 \end{aligned} \tag{3}$$

The temperature distribution is independent of the flow and heat transfer is by conduction alone. This is true for fluids in the initial stage due to absence of convective heat transfer or at small Grashof number flow (Gr $\le$ 1).

We introduce the non-dimensional variables

$$Pr = \frac{\mu C_p}{\kappa}\ ,\ t = \frac{t^{'}}{t_R},\ \omega\ =\ \omega^{'} t_R\ ,$$

$$y = \frac{y^{'}}{L_R}\ ,\ u = \frac{u^{'}}{U_R}\ ,\ \theta\ =\ \frac{T^{'} - T^{'}_{\infty}}{T^{'}_{w} - T^{'}_{\infty}}\ ,$$

$$K = \ \frac{U_R^2\, K^{'}}{\nu^2}\ ,\ M = \frac{\sigma\ B_0^2\, \nu}{\rho U_R^2}\ ,$$

$$\Delta T = T^{'}_{w} - T^{'}_{\infty},\ U_R = \ (\nu g \beta \Delta T)^{1/3},$$

$$L_R\ = \left( \frac{g\beta\Delta T}{\nu^2} \right)^{-1/3}\ ,\ t_R =\ (g\beta\Delta T)^{-2/3}\, \nu^{1/3}, \tag{4}$$

All the physical variables are defined in the nomenclature.

With the help of Eq. (4), the governing equations with the boundary conditions reduce to

$$Pr\frac{\partial \theta}{\partial t}\ =\ \frac{\partial^2 \theta}{\partial y^2} \tag{5}$$

$$\frac{\partial u}{\partial t}\ =\ \frac{\partial^2 u}{\partial y^2}\ +\ \theta\ -\ \left( M + \frac{1}{K} \right) u \tag{6}$$

with the following initial and boundary conditions:

$$u = 0, \theta\ =\ 0 \qquad for all y, t\ \leq\ 0 \tag{7}$$

$$u = 0, \theta\ =\ 1 +\ \in cos\omega t \qquad at y = 0, t > 0$$

$$u \rightarrow\ 0\ ,\ \theta\ \rightarrow\ 0 \qquad as y \rightarrow\ \infty\ ,\ t >\ 0 \tag{8}$$

On taking Laplace transform of Eq. (5), Eq. (6), and Eq. (8), we get

$$\frac{d^2\bar{\theta}}{dy^2}\ -\ pPr\bar{\theta}\ =\ 0 \tag{9}$$

$$\frac{d^2\bar{u}}{dy^2}\ -\ (p + M')\bar{u} = -\bar{\theta}\ (y, p) \tag{10}$$

$$\left.\begin{array}{ll} \bar{u}\ =\ 0\ ,\ \bar{\theta}\ =\ \frac{1}{p}\ +\ \frac{\in p}{p^2+\omega^2} & at y = 0, t > 0 \\ \bar{u} \rightarrow 0, \bar{\theta}\ \rightarrow\ 0 & as\ y \rightarrow\ \infty\ ,\ t > 0 \end{array}\right\} \tag{11}$$

Solving Eq. (9) and Eq. (10) with the help of Eq. (11), we get

$$\bar{\theta}\ (y, p) = \frac{\exp(\ - y\sqrt{pPr})}{p}\ +\ \frac{\in p \exp(\ - y\sqrt{pPr})}{p^2 + \omega^2} \tag{12}$$

$$\bar{u}(y, p) = \frac{\exp(\ - y\sqrt{p + M'})}{p(Pr - 1)(P - c)}\ -\ \frac{\exp(\ - y\sqrt{pPr})}{p(Pr - 1)(P - c)}$$

$$+ \frac{\in p\exp(\ - y\sqrt{p + M'})}{(Pr - 1)(p^2 + \omega^2)(P - c)}$$

$$-\ \frac{\in p\exp(\ - y\sqrt{pPr})}{(Pr - 1)(p^2 + \omega^2)(P - c)} \tag{13}$$

where p is the Laplace transformation parameter.

Inverting Eq. (12) and Eq. (13), we get

$$\theta\ =\ erfc(\eta\sqrt{Pr}\,)\ +\ \frac{\in}{2}\{g(\eta\sqrt{Pr},\ i\omega)$$

$$+\, g(\eta\sqrt{Pr},\ - i\omega)\} \tag{14}$$

For Pr $\neq$ 1

$$u = \ -\ \frac{1}{M'}\{\exp(\ - M^{'}t)g(\eta, M^{'})\} + \frac{1}{M'}\, erfc(\eta\sqrt{Pr})$$

$$-\ \frac{\in\ \exp(\ - M^{'}t)}{2(Pr - 1)\,(c^2 + \omega^2)}\{(c - i\omega)\, g(\eta\ ,\ M^{'}\ - i\omega)$$

$$+\, (c + i\omega)\, g(\eta,\ M^{'}\ + i\omega)\}$$

$$+\ \frac{\in}{2(Pr - 1)\,(c^2 + \omega^2)}\{(c - i\omega)\, g(\eta\sqrt{Pr},\ -i\omega)$$

$$+ (c + i\omega)\, g(\eta\sqrt{Pr}, i\omega)\}$$

$$+ \left\{ \frac{1}{M'}\ +\ \frac{c\in}{(Pr - 1)\,(c^2 + \omega^2)} \right\}\{\exp(\ - M^{'}t)g(\eta, e)$$

$$-\, g(\eta\sqrt{Pr}, c)\} \tag{15}$$

where

$$\eta = \frac{y}{2\sqrt{t}}, M^{'} = M + \frac{1}{K}, c = \frac{M^{'}}{Pr-1}, e = \frac{M^{'}Pr}{Pr-1}$$

and for Pr = 1,

$$u = \ - \frac{1}{M'} \{\exp(-M^{'}t)g(\eta, M^{'})\} + \frac{1}{M'} erfc(\eta) \tag{16}$$

Initially, the heat is transferred through the plate by conduction. However, a little later, convection currents start flowing near the plate. Hence, it is essential to know the position of a point on the plate where the conduction mechanism changes to a convection mechanism. In the literature, this is studied as a leading edge effect. The distance of this point of transition from conduction to convection is given by

$$X_p = \int_0^t u(y,t)dt$$

or, in terms of the Laplace transform and its inverse,

$$X_p = L^{-1}\left[\frac{1}{p}L\{u(y,t)\}\right]$$

$$X_p = L^{-1}\left[\frac{\{\bar{u}(y,p)\}}{p}\right]$$

$$X_p = L^{-1}\left\{\frac{\exp(-y\sqrt{p+M'})}{p^2(Pr-1)(P-c)}\right\}$$

$$+L^{-1}\left\{\frac{\exp(-y\sqrt{pPr})}{p^2(Pr-1)(P-c)}\right\}$$

$$+L^{-1}\left\{\frac{\in\exp(-y\sqrt{p+M'})}{(Pr-1)(p^2+\omega^2)(P-c)}\right\}$$

$$-L^{-1}\left\{\frac{\in\exp(-y\sqrt{pPr})}{(Pr-1)(p^2+\omega^2)(P-c)}\right\} \tag{17}$$

On solving Eq. (17), we have

For Pr $\neq$ 1

$$X_p = \frac{\eta}{2M'}\sqrt{\frac{t}{M'}}\Big\{ -\exp(2\eta\sqrt{M't})erfc(\eta+\sqrt{M't})$$

$$+\exp(-2\eta\sqrt{M't})erfc(\eta-\sqrt{M't})\Big\}$$

$$+\frac{t}{M'}\Bigg\{(1+2\eta^2Pr)erfc(\eta\sqrt{Pr})$$

$$-2\eta\sqrt{\frac{Pr}{\pi}}\exp(-\eta^2Pr)\Bigg\} + \frac{1}{M'c}erfc(\eta\sqrt{Pr})$$

$$-\left(\frac{t}{M'}+\frac{1}{M'c}\right)\exp(-M^{'}t)g(\eta, M^{'})$$

$$-\frac{i\in\exp(-M^{'}t)}{2\omega(Pr-1)(c^2+\omega^2)}\{(c-i\omega)g(\eta, M^{'}-i\omega)$$

$$-(c+i\omega)g(\eta, M^{'}+i\omega)\}$$

$$+\frac{i\in}{2\omega(Pr-1)(c^2+\omega^2)}\{(c-i\omega)g(\eta\sqrt{Pr}, -i\omega)$$

$$-(c+i\omega)g(\eta\sqrt{Pr}, i\omega)\}$$

$$+\left\{\frac{1}{M'c}+\frac{\in}{(Pr-1)(c^2+\omega^2)}\right\}\{\exp(-M^{'}t)g(\eta, e)$$

$$-g(\eta\sqrt{Pr}, c)\} \tag{18}$$

and for Pr = 1,

$$X_p = \frac{\eta}{2M'}\sqrt{\frac{t}{M'}}\Big\{\exp(-2\eta\sqrt{M't})erfc(\eta-\sqrt{M't})$$

$$-\exp(2\eta\sqrt{M't})erfc(\eta+\sqrt{M't})\Big\}$$

$$+\frac{t}{M'}\left\{(1+2\eta^2)erfc(\eta)-2\eta\sqrt{\frac{1}{\pi}}\exp(-\eta^2)\right\}$$

$$-\frac{t}{M'}\exp(-M^{'}t)g(\eta, M^{'}) \tag{19}$$

where

$$g(a,b) = \frac{\exp(bt)}{2}\{\exp(2a\sqrt{bt})erfc(a+\sqrt{bt})$$

$$+\exp(-2a\sqrt{bt})erfc(a-\sqrt{bt})\}$$

where a = $\eta$ or $\eta\sqrt{Pr}$ and b = M' or i $\omega$ or − i $\omega$ or M' + i $\omega$ or M' − i $\omega$ or e or c.

We have extended the problem of Das et al. (1999) and we have calculated skin- friction and Nusselt number for the flow, which they did not calculate. Now, on setting M = 0 and K→ ∞ and by taking the limit M'→0 our results for the velocity (Eq. (15)) are comparable with that of Das et al. (1999), but the penetration distance (Eq. (18)) is not comparable. It appears that there are some printing mistakes in the expression of the penetration distance of Das et al. (1999). In the expression of the penetration distance of Das et al. (1999) the term $\frac{1}{\mathrm{Pr}-1}\left\{\frac{\varepsilon}{\omega^2}erfc(\eta)\right\}$should be added and the term $-\frac{1}{\mathrm{Pr}-1}\left\{\frac{\varepsilon}{2\omega^2}erfc(\eta\sqrt{\mathrm{Pr}})\right\}$should be $-\frac{1}{\mathrm{Pr}-1}\left\{\frac{\varepsilon}{\omega^2}erfc(\eta\sqrt{\mathrm{Pr}})\right\}$.

Further, our graphs for the velocity and the penetration distance are not comparable with that of Das et al. (1999). In calculating the numerical results for the velocity and the penetration distance they assigned the values to $\omega$t, t, and $\omega$ separately, and the value given to $\omega$ does not match the values of $\omega$t and t, taken altogether, which is not the appropriate way to fix these material parameters. In our analysis, we assigned the values to $\omega$t and t; after that, from these values, the $\omega$ was set.

In expressions, erfc ($x_1$ + i$y_1$) is the complementary error function of complex argument, which can be calculated in terms of tabulated functions (Abramowitz and Stegun, 1970). The table given in Abramowitz and Stegun (1970) does not give erfc ($x_1$ + i$y_1$) directly but an auxiliary function $W_1$ ($x_1$+ i$y_1$), which is defined as

$$erfc(x_1+iy_1)=W_1(-y_1+ix_1)\exp\{-(x_1+iy_1)^2\}$$

Some properties of $W_1$ ($x_1$ + i$y_1$) are

$$W_1(-x_1+iy_1)=W_2(x_1+iy_1)$$

$$W_1(x_1-iy_1)=2\exp\{-(x_1-iy_1)^2\}-W_2(x_1+iy_1)$$

where $W_2(x_1+iy_1)$ is complex conjugate of $W_1(x_1+iy_1)$.

SKIN-FRICTION: In non-dimensional form, the skin-friction is given by

$$\tau=-\frac{\partial u}{\partial y}\bigg|_{y=0}$$

For Pr ≠ 1

$$\tau=\frac{1}{M'}\sqrt{\frac{Pr}{\pi t}}-\frac{1}{2M'}f(M')$$

$$+\frac{\in\exp(-i\omega t)(c-i\omega)}{4(Pr-1)(c^2+\omega^2)}\{\sqrt{Pr}f(-i\omega)-f(M'-i\omega)\}$$

$$+\frac{\in\exp(i\omega t)(c+i\omega)}{4(Pr-1)(c^2+\omega^2)}\{\sqrt{Pr}f(i\omega)$$

$$-f(M'+i\omega)\}+\left\{\frac{1}{M'}+\frac{c\in}{(Pr-1)(c^2+\omega^2)}\right\}$$

$$\left\{\frac{\exp(-M't)}{\sqrt{\pi t}}-\sqrt{\frac{Pr}{\pi t}}+\sqrt{\frac{M'Pr}{Pr-1}}exp\right.$$

$$\left.\left(\frac{M't}{Pr-1}\right)\left(erf\sqrt{\frac{M'Prt}{Pr-1}}-erf\sqrt{\frac{M't}{Pr-1}}\right)\right\} \tag{20}$$

and for Pr = 1,

$$\tau=-\frac{1}{2M'}f(M')+\left\{\frac{1}{M'}\right\}\left\{\frac{exp(-M't)}{\sqrt{\pi t}}\right\} \tag{21}$$

NUSSELT NUMBER: From the temperature field, the rate of heat transfer in non-dimensional form is expressed as

$$Nu=-\frac{\partial\theta}{\partial y}\bigg|_{y=0}$$

$$Nu=\sqrt{\frac{Pr}{\pi t}}+\frac{\in\sqrt{Pr}}{4}\{exp(-i\omega t)f(-i\omega)$$

$$+exp(i\omega t)f(i\omega)\} \tag{22}$$

where

$$f(d)=2\sqrt{d}erf\sqrt{dt}+\frac{2}{\sqrt{\pi t}}exp(-dt)$$

$$d=M'or-i\omega ori\omega orM'-i\omega orM'+i\omega$$

## Discussion

In order to gain physical insight into the problem, the value of $\in$ is chosen as 1.0 and the values of Prandtl number are chosen as 0.71, 1, and 7, which represent air, electrolytic solution, and water, respectively, at 20 °C temperature and 1 atmospheric pressure. Figure 1 displays the transient temperature profiles against $\eta$ (distance from the plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically. The temperature falls owing to an increase in the value of $\omega$t for both air (Pr = 0.71) and water (Pr = 7). The magnitude of temperature for air (Pr = 0.71) is greater than that for water (Pr = 7); this is due to the fact that the thermal conductivity of fluid decreases with increasing Pr, resulting in a decrease in thermal boundary layer thickness.

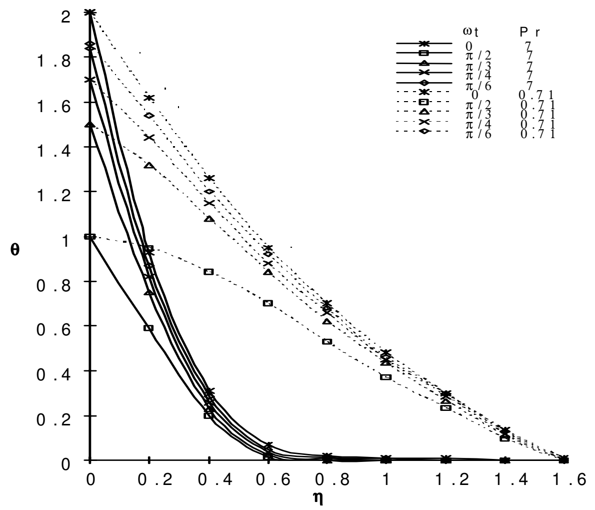


**Figure 1.** Transient temperature profiles.

Figure 2 reveals the effects of t and Pr on the transient velocity profiles. It is evident from the figure that the velocity increases with an increase in time for both air and water. Furthermore, the velocity increases and attains its maximum value in the vicinity of the plate and then fades away. The magnitude of velocity for Pr = 0.71 is much higher than that of Pr = 1 and Pr = 7. Physically, this is possible because fluids with high Prandtl numbers have high viscosity and hence move slowly. Figure 3 illustrates the influences of M, K, and Pr on the velocity profiles. It is obvious from the figure that the velocity near the plate exceeds that at the plate, i.e. velocity overshoot occurs for both air and water. The magnitude of the velocity decreases on increasing M. This is because the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force, which tends to resist the flow thus reducing its velocity. On the other hand, the transient velocity increases on increasing K for both Pr = 7 and Pr = 0.71. This is due to the fact that the presence of a porous medium increases the resistance to flow and when K = $\infty$ (i.e. the porous medium effects vanish) the velocity is greater in the flow field. Figure 4 concerns the velocity profiles against $\eta$ for different values of $\omega$t and Pr. It is clear from the figure that the maximum velocity is attained near the plate and then decreases and tends to zero as $\eta \to \infty$. An increase in the phase angle leads to a fall in the velocity for both Pr = 0.71 and Pr = 7. The magnitude of the velocity for air is much higher than that for water. The reason is the same as explained before.

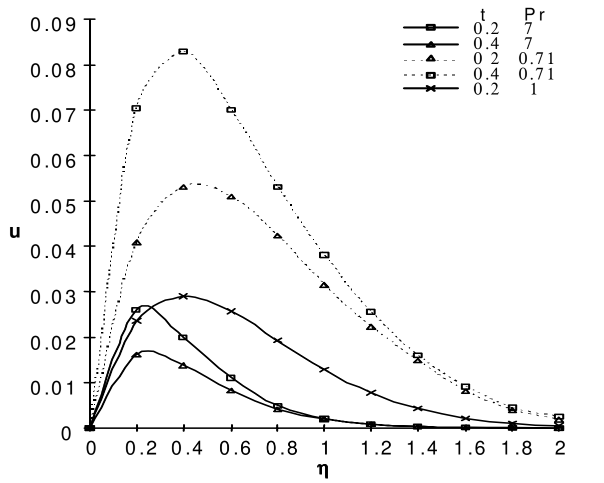


**Figure 2.** Transient velocity profiles when M = 5, K = 0.5, $\omega$t = $\pi/2$.

Figure 5 depicts the penetration distance ($X_p$) against $\eta$ for variations in t and Pr. It is observed that the penetration distance increases with an increase in time for both air and water. This increase in magnitude of penetration distance for air is higher than that of water and electrolytic solution. Figure 6 displays the influences of M, K, and Pr on the penetration distance. The penetration distance decreases owing to an increase in the magnetic parameter for both air and water. The reason is the same as that explained for velocity. The penetration distance is less affected by the magnetic parameter for Pr = 7. In contrast, it increases with increasing permeability parameter. Figure 7 shows the penetration distance against $\eta$ for various values of $\omega$t and Pr. It is found that the penetration distance decreases with an increase in the value of the phase angle for both air and water and it is minimum when the plate is isothermal. Further, it is observed that the penetration distance is more affected in cases of fluids with small Prandtl numbers.

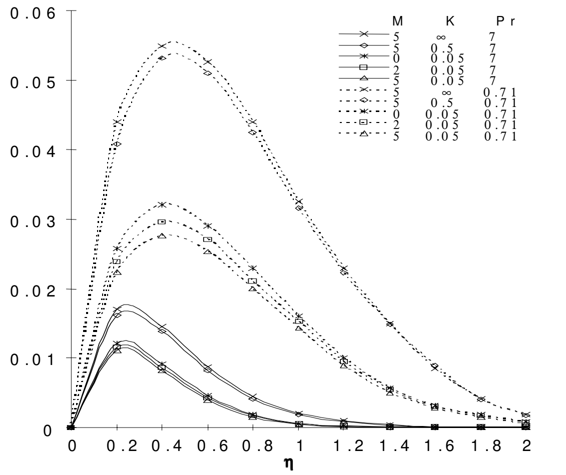


**Figure 3.** Transient velocity profiles when t = 0.2, $\omega$t = $\pi/2$.

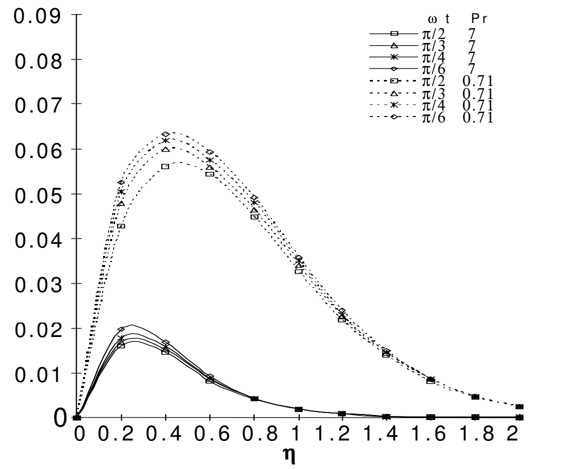


**Figure 4.** Transient velocity profiles when M = 5, K = 0.5, t = 0.2.

Nusselt number is presented in Figure 8. The rate of heat transfer falls with increasing $\omega$t for both Pr = 0.71 and 7. Nusselt number for Pr = 7 is higher than that for Pr = 0.71. The reason is that smaller values of Pr are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than with higher values of Prandtl number, and hence the rate of heat transfer is reduced. Figure 9 reveals the skin-friction against time t for various values of parameters. It is observed that as time passes the skin friction decreases but it increases with M due to enhanced Lorentz force, which imports additional momentum in the boundary layer. The skin-friction decreases with increasing permeability parameter K for both air and water. The magnitude of the skin-friction for water is greater than that for air and electrolytic solution.

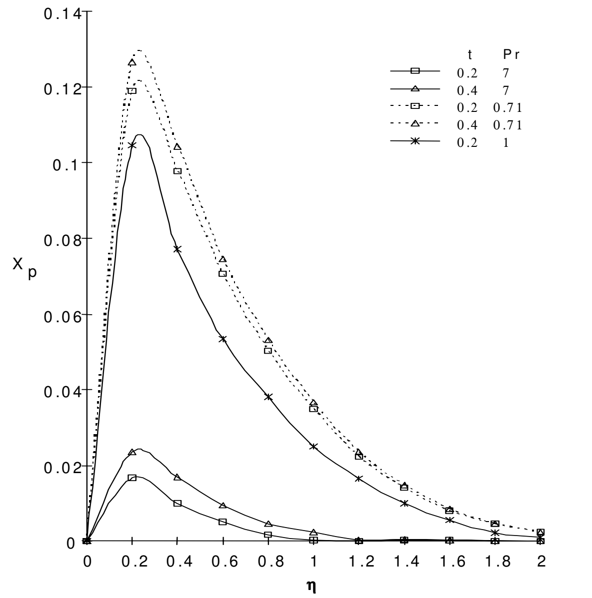


**Figure 5.** Penetration distances when M = 5, K = 0.5, $\omega$t = $\pi/2$.

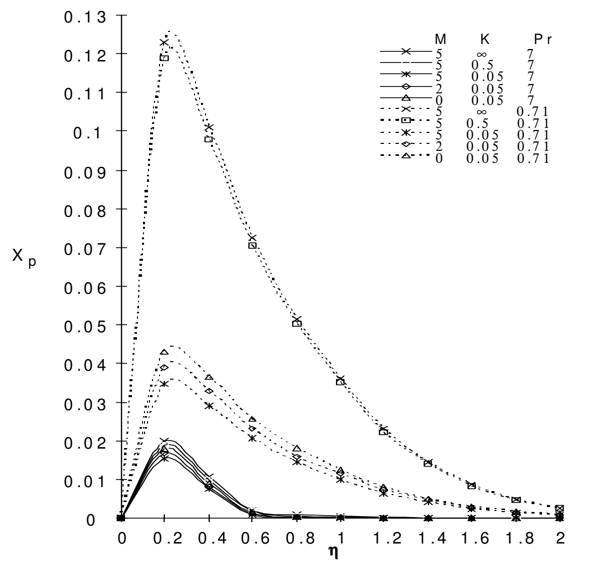


**Figure 6.** Penetration distances when t = 0.2, $\omega$t = $\pi/2$.

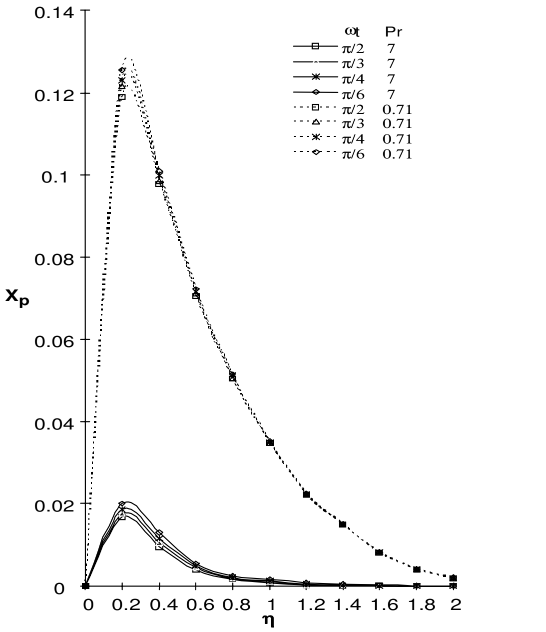


**Figure 7.** Penetration distances when M = 5, K = 0.5.

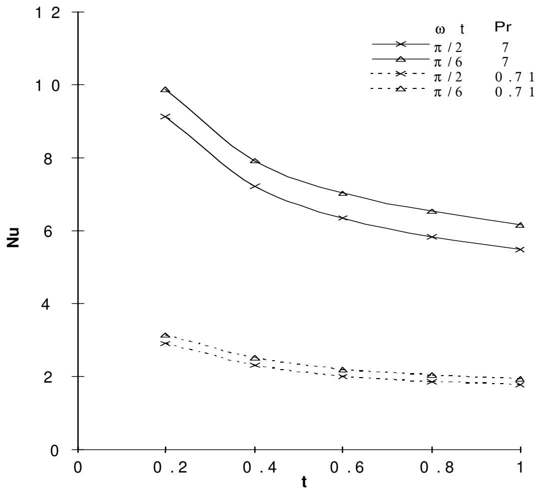


**Figure 8.** Nusselt number.

## Conclusion

We investigated how the presence of a magnetic field and porous medium modifies the flow and heat transfer past a vertical plate, when the surface temperature is oscillating. The resulting governing equations are solved in closed form by the Laplace-transform technique to obtain various closed form solutions. A systematic study on the effects of the various parameters on the flow and heat transfer characteristics is carried out. Some of the important findings obtained from the graphical representation of the results are listed below:

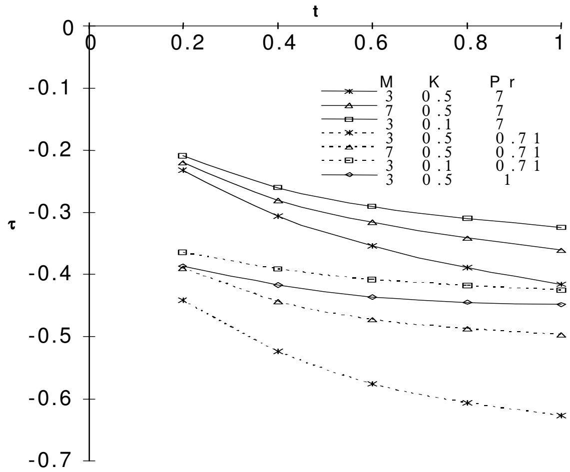


**Figure 9.** Skin-friction when $\omega t = \pi/4$.

(1) The effect of increasing values of Prandtl number is to decrease temperature significantly. The temperature also decreases with an increase in phase angle.

(2) An increase in Prandtl number tends to decrease the velocity and the penetration distance.

(3) The velocity and the penetration distance decrease with an increase in magnetic parameter and phase angle for both air (Pr = 0.71) and water (Pr = 7) while the reverse effect is observed for permeability parameter and time.

(4) Nusselt number is greater for water (Pr = 7) than for air (Pr = 0.71) and it decreases with an increase in the value of the phase angle.

(5) The combined effect of permeability parameter and time is to decrease the skin-friction while the effect of magnetic parameter is to increase it.

The present investigation of flow past a vertical surface can be utilized as the basis of many scientific and engineering applications, including earth science, nuclear engineering, and metallurgy. In nuclear engineering, it finds applications for the design of the blanket of liquid metal around a thermonuclear fusion-fission hybrid reactor. In metallurgy, it can be applied during the solidification process.

### Nomenclature

| | |
|---|---|
| $B_0$ | magnetic field component along y'-axis |
| $C_p$ | specific heat at constant pressure |
| g | gravitational acceleration |
| i | $\sqrt{-1}$ |
| K | permeability parameter |
| $L_R$ | reference length |
| M | magnetic parameter |
| Pr | Prandtl number |
| T' | temperature of the fluid near the plate |
| $T'_w$ | plate temperature |
| $T'_\infty$ | temperature of the fluid far away from the plate |
| t | time in dimensionless coordinate |
| $t_R$ | reference time |
| u | dimensionless velocity component |
| $U_R$ | reference velocity |
| $X_p$ | distance of the transition point from the leading edge |
| x,y | dimensionless coordinates |

### Greek Letters

| | |
|---|---|
| $\beta$ | coefficient of volume expansion |
| $\rho$ | density |
| $\in$ | amplitude |
| $\kappa$ | thermal conductivity of fluid |
| $\sigma$ | electrical conductivity of fluid |
| $\nu$ | kinematic viscosity |
| $\mu$ | viscosity of fluid |
| $\theta$ | dimensionless temperature |
| $\omega$ | frequency of oscillation |
| $\omega$t | phase angle |

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