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Improved Zonal Method Predictions in a Rectangular Furnace by Smoothing the Exchange Areas

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Abstract

A numerical study was conducted by the means of the Hottel's zone method based on the concept of the exchange areas associated with the weighted sum of grey gases model (WSGG model). 2D computational code has been developed in order to fulfill these areas by direct numerical integration then the conservation constraints (summation rules) are enforced with the Larsen and Howell's least squares and the generalized Lawson's smoothing methods which has not been previously published to the best knowledge of the authors. The Farag's WSGG model parameters are used in a first test case to simulate a non-grey semi-transparent media containing only a carbon dioxide (CO_2) as radiating specie. The radiative pattern is then applied to a homogeneous and non-isothermal CO_2-H_2O gas mixture using both the Truelove's mixed grey gas model parameters and the Smith et al's parameters. The two test cases are investigated under the same geometrical conditions. In this paper, attention is focused on the effect of two direct exchange areas smoothing procedures on the accuracy of the global radiative modeling. The predicted wall net radiative heat flux distributions are presented and compared against benchmark solutions in literature. The grid dependence study shows that the results did not fully achieve grid independence. However, it has been checked that further grid refinement does not affect the qualitative conclusions of this study but greatly increases the computing time. It is concluded from the agreement shown, that the zone method of analysis is a realistic mathematical model which can be used with some confidence for the calculation of the radiative heat transfer in furnaces.

Key words: Zone method, exchange areas, smoothing methods, rectangular enclosure.

Introduction

The radiative heat transfer was always a challenge for the engineers and researchers who attempted for many years to overcome this problem by the development of many radiative properties models and numerical methods. From the most popular alternatives, the zoning method has been widely used as an accurate solution method for the radiative transfer equation (RTE) in many practical engineering calculations. The most radiating gaseous combustion products in the industrial natural-gas or fuel combustion furnaces are mainly the carbon dioxide (CO_2) and the water vapor (H_2O). The statistical narrow band model (SNB model) and the wide band model (WBM) are among the most accurate spectral gas radiative properties models allowing the real nature (non-greyness) of the furnaces gas to be taken into account. These models are time consuming and then not practical for engineering calculations. Global models are another class of non-grey gas models which are less accurate than the first one but too computationally economical. The Modak's gas emissivity model (1979) for homogeneous and isothermal CO_2-H_2O and soot mixture, the Leckner's combustion model for fire combustion products (1972) and the Hottel's weighted sum of grey gases model (WSGG model) (1967) for gas-soot mixtures are the most widely used global models in practical problems. During the past two decades, the WSGG model still receives more and more survey from the researchers and engineers and it has been regarded as a highly attractive and practical non-grey gas model owing to its simplicity to be incorporated in any RTE solution solvers, its reasonable accuracy and its low computation time for the engineering applications. Recently, this model has been shown to be a good bridge between accuracy and computation load compared to the SNB and the correlated-k distribution method (C-K) (Goutiere et al, 2000; Coelho, 2002). The most widely used parameters based on the concept of the WSGG model are those evaluated from Truelove (1976), Farag (1982), Smith et al (1982) and Soufiani and Djavdan (1994).

The main goal of this study is making a comparison between the current results given by a 2D computational code allowing the estimation and the normalization of the direct exchange areas (DEA) in an isothermal rectangular black-walled enclosure with the benchmark solutions provided by Goutiere et al (2000). The direct numerical integration is used to estimate the DEA then two iterative smoothing processes namely the least squares method associated to the Lagrange multipliers due to Larsen and Howell (1986) and the generalized Lawson's smoothing method (1995) which is an alternative to the Leersum's algorithm (1989) are compared for two test case problems. In this paper, the effect of the WSGG model parameters suggested by Truelove (1976) and Smith et al (1982) with three and four grey gases and those evaluated from Farag (1982) with seven grey gases on the full radiative modeling accuracy is discussed.

Mathematical radiation model

Weighted sum of grey gases model The weighted sum of grey gases model (WSGG model) has initially been developed by Hottel and Sarofim (1967) within the framework of the zonal method. Modest (1991) has shown that the former may be extended to be used with any radiative transfer equation (RTE) solution method. The WSGG model consists of replacing the real gas by a set of several virtual gas components, each one with its own constant absorption coefficient. Through the choice of an appropriate set of weights and absorption coefficients, the radiative behaviour of the hypothetical gas is made somewhat like the real gas. Recently, new mathematical models have been developed to handle radiative heat transfer in combustion systems based on the concept of WSGG model (Khan et al, 1997a and 1997b; Bressloff, 1999; Trivic, 2004; Cumber, 2005). The total gas emissivity is obtained by a summation of all the grey gas emissivities over a small number of grey gases, one of them is necessary a clear gas in order to account for the windows in the spectrum between the spectral regions that have absorption. Two or three grey gases plus one clear gas are quite sufficient (Johnson and Beer, 1973; Soufiani and Djavdan, 1994; Goutiere et al, 2000; Coelho, 2002). Thus, the total gas emissivity is given by

$$\varepsilon_g(T) = \sum_n a_{g,n}(T)[1 - e^{K_{g,n}L}] \qquad (1)$$

The weighting factors $a_{g,n}$(T) are low order polynomials in temperature. In this paper, the Truelove's linear expressions and the two and three order polynomial weighting factors due to Farag and Smith et al are used (Khan et al, 1997a and 1997b; Bressloff, 1999). For a given grey gas absorption coefficient $K_{g,n}$, the direct exchange areas are estimated based on the Hottel's zone method and the Olsommer et al's alternative (1997).

Zoning method of Hottel Perhaps the most widely used method for calculating radiative transfer in nonisothermal enclosures is the zone method as developed by Hottel and Cohen (1958) and Hottel and Sarofim (1967). Application of the zone method requires the whole gas volume to be divided into a given number of smaller volumes with uniform properties such as temperature, composition, emissivity and transmissivity. Central to the zone method are exchange areas. Direct exchange areas (DEA) give a measure of the amount of radiation emitted by one zone which is directly intercepted by another one. Total exchange areas (TEA) are a measure of the amount of the radiation emitted by one zone which is eventually absorbed by another zone (Khan et al, 1997b; Murty and Murty, 1991). DEA and TEA depend on the geometric orientation of the zones, the gas attenuation coefficient K_t and the surface emissivities. For a given attenuation coefficient, the DEA and TEA are independent of the gas zone temperatures. Hence, they need to be calculated only once before solving the zone energy balance equations for temperatures and heat fluxes. These equations are formulated for the radiation interchanges between all surface to surface, surface to volume, volume to surface and volume to volume zones. Then, the DEA are expressed by the following multiple integrals (Larsen and Howell, 1986; Olsommer et al, 1997; Méchi et al, 2004; Yuen, 2006)

$$\begin{cases} \overline{s_i s_j} = \int\limits_{A_i}\int\limits_{A_j} \frac{\mu_i \mu_j e^{-K_t r_{ij}}}{\pi r_{ij}^2} dA_i dA_j = \overline{s_j s_i} \\ \overline{g_i s_j} = \int\limits_{V_i}\int\limits_{A_j} \frac{K_t \mu_j e^{-K_t r_{ij}}}{\pi r_{ij}^2} dV_i dA_j = \overline{s_j g_i} \\ \overline{g_i g_j} = \int\limits_{V_i}\int\limits_{V_j} \frac{K_t^2 e^{-K_t r_{ij}}}{\pi r_{ij}^2} dV_i dV_j = \overline{g_j g_i} \end{cases} \quad (2)$$

From the definition, the DEA are temperature independent and must satisfy the reciprocity relationships (Eq. 2). Besides, they should obey to the conservation constraints or summation rules which are written for a surface zone j (Hottel and Sarofim, 1967)

$$\sum_i \overline{s_j s_i} + \sum_i \overline{s_j g_i} = A_j \qquad j = 1...\Gamma_s \quad (3)$$

and for a volume zone k

$$\sum_i \overline{g_k s_i} + \sum_i \overline{g_k g_i} = 4K_t V_k \quad k = 1...\Gamma_g \quad (4)$$

In this work, the surface and volume zones are further subdivided into smaller volume and surface elements (Figure 1). Then, the DEA between two finite surface or volume zones are evaluated using the direct numerical integration by making a simple summation of the direct exchange areas between these smaller elements (Rhine and Tucker, 1991; Olsommer et al, 1997; Batu and Selçuk, 2002; Méchi et al, 2004). Therefore, each of the previous integrals (Eq. 2) is transformed into discrete sum of the integrand, assumed to be constant for a uniform spatial grid mesh sufficiently fine

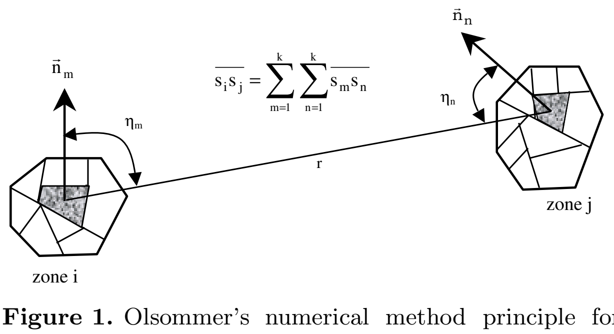

Figure 1. Olsommer's numerical method principle for evaluation of direct exchange areas: case of two surface zones (1997).

$$\overline{s_m s_n} = \frac{\mu_m \mu_n \exp\left[-K_t r_{mn}\right]}{\pi r_{mn}^2} \quad (5)$$

where m and n denote the subdivisions of the zones i and j, respectively.

Similar summations can be drawn for volume-to-surface and volume-to-volume direct exchange areas. With such numerical technique, DEA are evaluated with inherent numerical errors and because the equations (3) and (4) hold in the limit of zero numerical errors in the evaluation of such areas, smoothing methods are very indispensable to overcome this drawback. Also, with a finer spatial grid size, the direct numerical integration yields sufficiently accurate results (Olsommer et al, 1997; Méchi et al, 2004).

The TEA are deduced from the adjusted DEA using some explicit matrix relations (Noble, 1975; Kim and Smith, 1985; Méchi et al, 2004). Finally, the heat fluxes are calculated on each surface or volume zone by introducing the so-called directed flux areas (DFA) and the net radiative heat flux on the surface zone A_i is given by (Johnson and Beer, 1973; Khan et al, 1997a and 1997b; Olsommer et al, 1997)

$$Q_i = \sum_{j=1}^{\Gamma_s} \overrightarrow{S_j S_i} E^o_{s,j} + \sum_{j=1}^{\Gamma_g} \overrightarrow{G_j S_i} E^o_{g,j} - \mathrm{A}_i \varepsilon_i E^o_{s,i} \quad (6)$$

$\overrightarrow{S_i S_j}$ and $\overrightarrow{G_i S_j}$ are the directed flux areas (DFA) for surface-to-surface and gas-to-surface radiative exchange respectively.

Smoothing methods

Least squares smoothing method Larsen and Howell (1986) have suggested a least squares smoothing procedure using Lagrange multipliers for adjusting the direct exchange areas in order to satisfy the conservation constraints. The initial estimates of direct exchange areas may be represented in the form of a symmetric M_x by M_x square matrix

$$[\mathrm{X}] = \begin{bmatrix} [\,\overline{ss}\,]\,[\,\overline{sg}\,] \\ \\ [\,\overline{sg}\,]^T\,[\,\overline{gg}\,] \end{bmatrix} \quad (7)$$

The minimisation of an objective function defined by Larsen and Howell (1986) leads to the revised estimates x'_{ij}

$$x'_{ij} = x_{ij} + w_{ij}\left(\lambda_i + \lambda_j\right) \quad (8)$$

We note that the weighting factors w_{ij} must be symmetric to preserve the symmetry of the adjusted direct exchange areas. In this paper, $w_{ij} = x_{ij}^2$ is adopted. The Lagrange multipliers can be obtained by solving a system of simultaneous linear equations (Larsen and Howell, 1986). The individual and relative merits and limitations of this method have been sufficiently discussed by Murty and Murty (1991).

Generalized Lawson's improved smoothing method This method was proposed by Lawson (1995) as an alternative method to the Leersum (1989) one. It stands up by the possibility to modify each DEA according to its size and guarantees that no modified value is negative avoiding, consequently, the shortcomings of the conventional smoothing algorithm. Lawson (1995) developed his improved method for only a set of surface-to-surface DEA (transparent medium). The smoothed DEA for surface-to-surface radiative exchange are given by Lawson (1995)

$$\overline{s_i s_j}' = \overline{s_i s_j} \frac{A_i}{\sum\limits_k \overline{s_i s_k}} \tag{9}$$

Nevertheless, this method can be extended to a semi-transparent absorbing medium when the DEA for surface-to-gas and gas-to-gas are not zero. Thus, additional constraints for each gas volume V_i must be written. Analogous expressions of the smoothed DEA for surface-to-surface, surface-to-gas and gas-to-gas radiative exchange are respectively

$$\begin{cases} \overline{s_i s_j}' = \overline{s_i s_j} \frac{A_i}{\sum\limits_k \overline{s_i s_k} + \sum\limits_k \overline{s_i g_k}} \\ \overline{s_i g_j}' = \overline{s_i g_j} \frac{A_i}{\sum\limits_k \overline{s_i s_k} + \sum\limits_k \overline{s_i g_k}} \\ \overline{g_i g_j}' = \overline{g_i g_j} \frac{V_i}{\sum\limits_k \overline{g_i s_k} + \sum\limits_k \overline{g_i g_k}} \end{cases} \tag{10}$$

The starting point in the smoothing process is to check the first row of the approximate DEA matrix using equation (10), initially calculated by the direct numerical integration method previously described. This alters the symmetry of the original matrix which is restored with the reciprocity property. Since the other rows are changed, the DEA in the second row are smoothed by the same equation, taking into account of the new DEA modified by reciprocity enforcement. By the same way, both the conservation and reciprocity conditions are applied for all the original matrix rows. After the first iteration of the Lawson's algorithm is achieved, the smoothing process is restarted for the new set of adjusted DEA until the iterative process is deemed to have converged. In this study, the maximum discrepancy between the set of the last modified DEA and the previous ones for all the zone pairs is chosen to be less than 10^{-10}. The exchange areas smoothing effect on the zone method predictions is studied hereafter based on the least squares (Larsen and Howell, 1986) and the generalized Lawson's smoothing methods (1995).

Results and Discussion

The basic problem studied in this paper was already investigated by Goutiere et al (2000) and we limit the attention here in two test case problems which are both treated under the same geometry and thermal and radiative boundary conditions imposed in the surrounding enclosure (cold and black walls) (Figure 2).

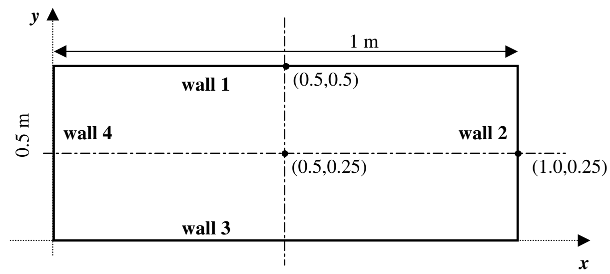

Figure 2. Geometric characteristics of the homogeneous medium.

However, the gaseous combustion products in the participating medium are modified. The first test problem consists in studying the radiative transfer in an isothermal rectangular enclosure of 1×0.5 m^2, where the carbon dioxide with uniform concentration is maintained at 1000 K. Then, a CO_2-H_2O gas mixture is undertaken for a prescribed non-uniform temperature field. For both test cases, net radiative heat flux distributions were compared with the results of reference (Goutiere et al, 2000) based on the statistical narrow band model (SNB) (Soufiani and Taine, 1997), the weighted sum of grey gases model due to Smith et al (1982) and Soufiani and Djavdan (1994), and the spectral line based weighted sum of grey gases model (SLW) (Denison, 1994).

First test case

The carbon dioxide concentration is supposed to be uniform in all the rectangular enclosure with partial pressure of 0.1 atm. The weighted sum of gray gases model (WSGG model) parameters evaluated from Farag (1982) are incorporated into the developed 2D computational code. On Figure 3 are represented the net radiative heat fluxes in wall 1 and 2 for different spatial grid meshes using the two smoothing methods. With the least squares method, the net radiative wall heat fluxes in both wall sides are underestimated and a maximum discrepancy of about 20% between results obtained using the two different smoothing methods is outlined. The Lawson's smoothing method gives fairly good accuracy even with the small grid size (21×11) compared to the 61×31 spatial grid used by Goutiere et al (2000). Besides, 31×15 seems to be an optimal grid mesh providing accurate results.

Figure 4 depicts that the current results obtained by the Lawson's smoothing method are in good agreement with those of reference for all the spatial grids used here. Based on the results presented in the Table 1, the generalized Lawson's smoothing method predicts wall heat fluxes better than the least squares method and the discrepancies between the calculated results and those of reference (Goutiere et al, 2000) are about 7% in wall 1 and 3% in wall 2 but they exceed 11% with the least squares method.

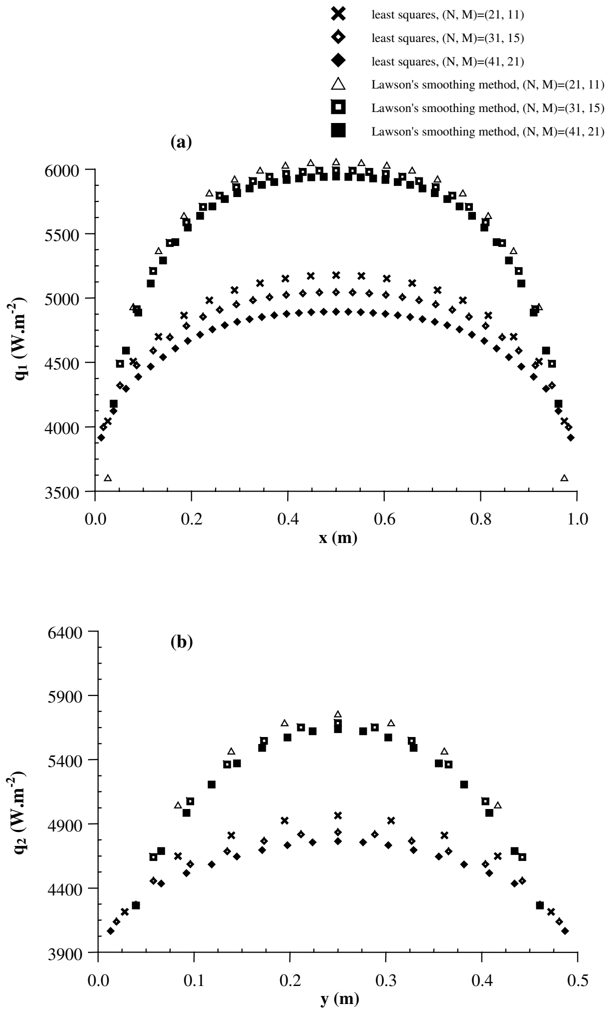

Figure 3. Wall radiation heat fluxes for different spatial grids using the least squares (Larsen and Howell, 1986) and the generalized Lawson (1995) smoothing methods: (a) wall 1; (b) wall 2.

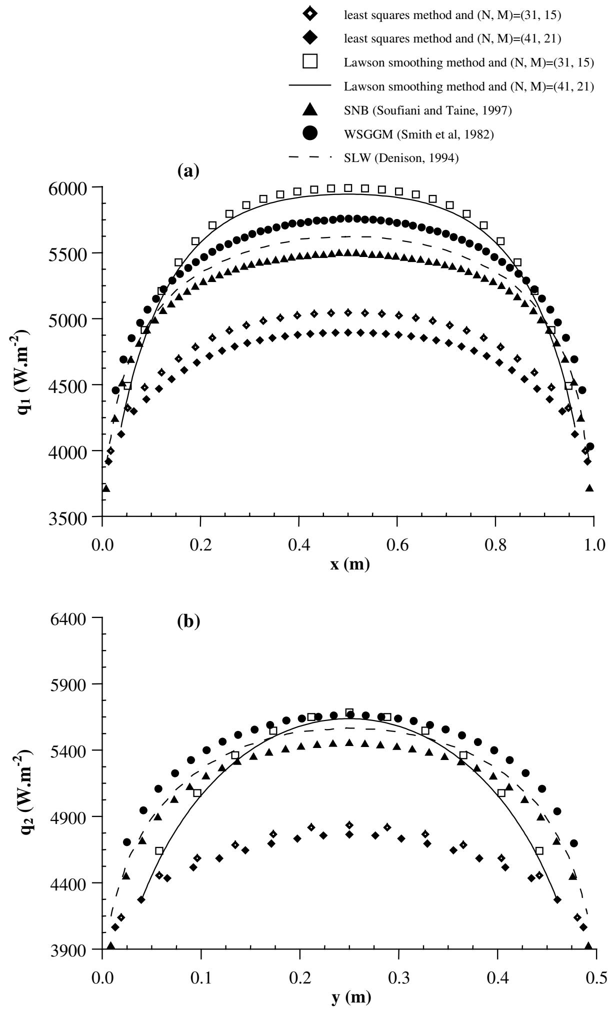

Figure 4. Comparison between the predicted net radiative wall heat fluxes with those of reference for the isothermal and homogeneous case of CO_2 (case 1 in Goutiere et al, 2000): (a) wall 1; (b) wall 2.

Table 1. Numerical predictions based on the two different smoothing methods, case of isothermal and homogeneous CO_2 with the Farag's WSGG model parameters (7 gg) (1982).

Real gas models		q_1 (W.m^{-2})		q_2 (W.m^{-2})	
		At (0.5, 0.5)	Discrepancies (%)	At (1, 0.25)	Discrepancies (%)
SNB (Soufiani and Taine, 1997)		5537		5479	
WSGGM (Smith et al, 1982)		5760	4.0	5664	3.4
SLW (Denison, 1994)		5628	1.6	5565	1.6
Current findings with least squares smoothing method	**21×11**	5178	-6.5	4964	-9.4
	31×15	5044	-8.9	4834	-11.8
	41×21	4894	-11.6	4764	-13.0
Current findings with Lawson's smoothing method	**21×11**	6057	9.4	5755	5.0
	31×15	5990	8.2	5684	3.7
	41×21	5943	7.3	5637	2.9

Second test case

In this test case problem, previously described by Goutiere et al (2000), a homogeneous CO_2-H_2O gas mixture is studied for a prescribed non-uniform temperature distribution. Truelove's mixed grey gas model (WSGG model) is adopted with three and four grey gases (3 gg and 4 gg).

The Figures 5 and 6 illustrate the variations of the net radiative heat fluxes obtained on the wall 1 and 2 of the rectangular enclosure with respect to the number of grey gases and the spatial grids for each smoothing method. Figure 5a shows that changing from 3 to 4 grey gases has a significant influence on the results particularly in the hot region and then shifted at high x coordinate where the temperature decreases by more than 30%. In this cold region, optically thin medium assumption is verified and radiative heat transfer is not strongly affected by increasing the number of gray gases to four. The same behavior is obtained using the Lawson's smoothing method but without outstanding modifications (Figure 5b). We note that the numerical results are very dependent on the grid size particularly for the coarsest grids and only those calculated with the finest ones are reported on Figures 5 and 6. In fact, with 31×15 and 41×21 grids, we attain perhaps the same findings and no significant change is expected when increasing enough the grid size. Then, the predicted net radiative heat flux distributions carried out in the wall 1 and 2 using these finer spatial grids are represented on Figure 7, together with those of reference (Goutiere et al, 2000). The aim of this comparison is to check out the accuracy of the developed 2D computational code and the degree of the confidence that we can have into the mathematical radiation model presented in this work. The least squares method with 4 gg WSGG model evaluated from Truelove (1976) yields fairly accurate results with reasonable computation load compared to those of reference (Goutiere et al, 2000). Moreover, the Lawson's smoothing method produces numerical results in close agreement with the reference solution calculated by the parameters of Smith et al (1982). With these parameters, more accurate predictions are obtained using the least squares method with comparison to the reference solution given by the SNB model (Soufiani and Taine, 1997) which is regarded as the most accurate spectral radiative gas model after the line-by-line method (Goutiere et al, 2000).

Similarly, in the first test case, it is shown that the Lawson's smoothing method yields reasonable good agreement using seven gray gases (7 gg). Taking into account the present findings carried out in the two test cases studied in this paper, smoothing the DEA either with the least squares or by the generalized Lawson's smoothing method doesn't strongly affect the global behavior of the predicted results but the accuracy depends on the number of grey gases and the WSGG model parameters themselves. The dependence of the accuracy on the number of grey gases has been recently investigated by Trivic (2004).

By anyway, improvements to the radiative heat transfer predictions are performed with smoothing the direct exchange areas (DEA) in multi-dimensional furnaces with standard geometry and containing non-grey gas participating medium. However, it seems that real features of the zone method should be attained with the rigorous least squares method due to its fundamental formulation especially if it is used in conjunction with the Lagrange multipliers.

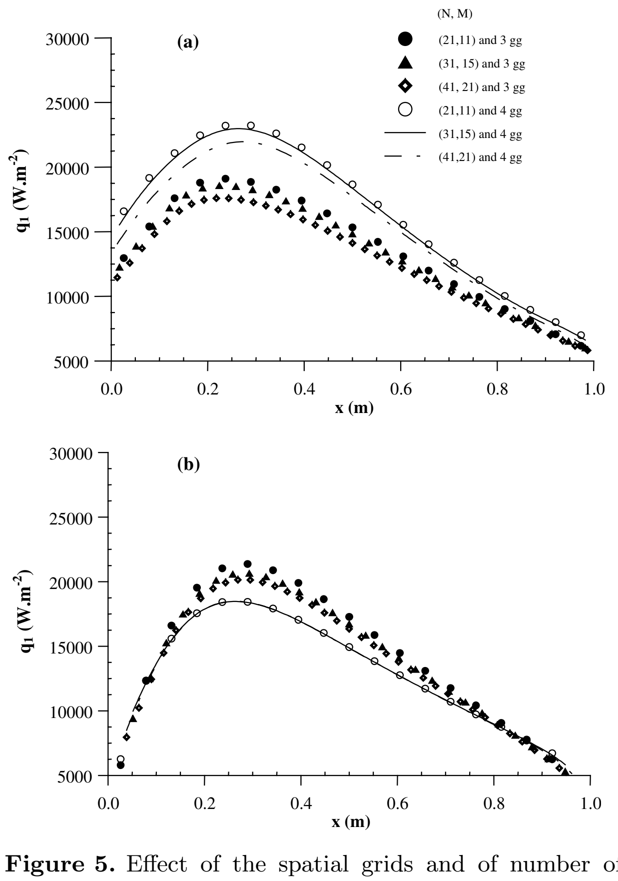

Figure 5. Effect of the spatial grids and of number of grey gases on the accuracy of predicted heat flux q_1 on the wall 1 with two smoothing methods for non-isothermal and homogeneous CO_2-H_2O gas mixture: (a) least squares method (Larsen and Howell, 1986); (b) generalized Lawson smoothing method (1995).

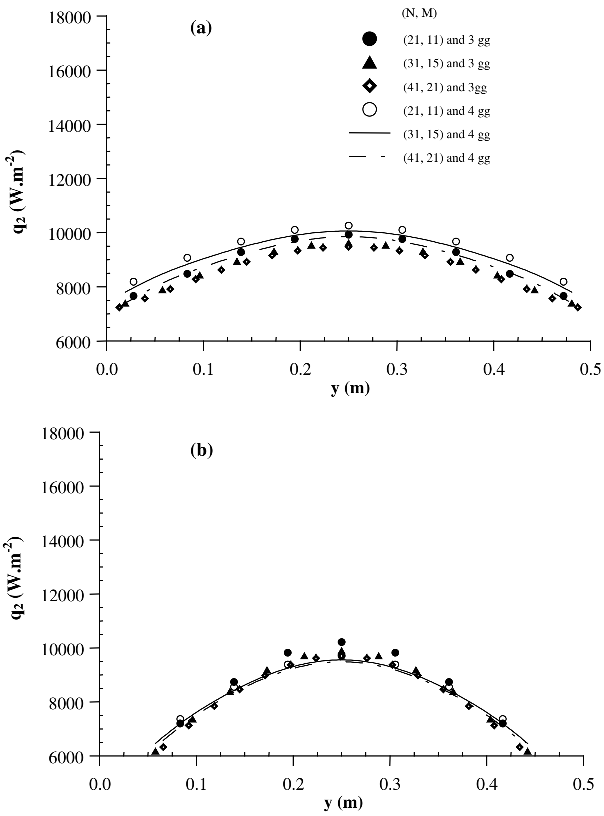

Figure 6. Effect of the spatial grids and of number of grey gases on the accuracy of the predicted heat flux q_2 on the wall 2 with two smoothing methods for non-isothermal and homogeneous CO_2-H_2O gas mixture: (a) least squares method (Larsen and Howell, 1986); (b) generalized Lawson smoothing method (1995).

The net radiative wall heat fluxes calculated in two surface zones for each smoothing method and different spatial grids are reported in Tables 2 and 3 with the corresponding relative errors. Table 3 shows that the discrepancies decrease with the spatial grid size and the relative errors obtained when using the least squares smoothing method are smaller than those resulting from the Lawson's one. For 41×21 grid mesh, the lower relative error in one wall side is about 1% using the least squares method and four grey gases (Table 3).

These improvements in the quality of the results from Table 2 to Table 3 can be explained by the fact that with the WSGG model, 3 gg are adequate but 4 gg are quite sufficient to adjust the DEA to the conservation constraints with the help of the least squares smoothing method which has been already approved by many authors (Johnson and Beer,

Table 2. Numerical predictions based on the two different smoothing methods, case of non-isothermal and homogeneous CO_2-H_2O gas mixture with the Truelove's WSGG model parameters (3 gg) (1976).

Real gas models		q_1 ($W.m^{-2}$)		q_2 ($W.m^{-2}$)	
		At (0.25, 0.5)	Discrepancies (%)	At (1, 0.25)	Discrepancies (%)
SNB (Soufiani and Taine, 1997)		21630		12668	
WSGGM (Smith et al, 1982)		26030	20.3	13868	9.5
WSGGM (Soufiani and Djavdan, 1994)		18330	-15.3	11936	-5.8
Current findings with least squares smoothing method	**21×11**	19043	-12.0	9925	-21.6
	31×15	18512	-14.4	9614	-24.1
	41×21	17565	-18.8	9476	-25.2
Current findings with Lawson's smoothing method	**21×11**	20952	-3.1	10219	-19.3
	31×15	20456	-5.4	9891	-21.9
	41×21	19976	-7.6	9702	-23.4

Table 3. Numerical predictions based on the two different smoothing methods, case of non-isothermal and homogeneous CO_2-H_2O gas mixture with the Truelove's WSGG model parameters (4 gg) (1976).

Real gas models		q_1 ($W.m^{-2}$)		q_2 ($W.m^{-2}$)	
		At (0.25, 0.5)	Discrepancies (%)	At (1, 0.25)	Discrepancies (%)
SNB (Soufiani and Taine, 1997)		21630		12668	
WSGGM (Smith et al, 1982)		26030	20.3	13868	9.5
WSGGM (Soufiani and Djavdan, 1994)		18330	-15.3	11936	-5.8
Current findings with least squares smoothing method	**21×11**	23207	7.3	10262	-19
	31×15	22911	5.9	10060	-20.6
	41×21	21873	1.1	9850	-22.2
Current findings with Lawson's smoothing method	**21×11**	18435	-14.7	9703	-23.4
	31×15	18404	-14.9	9556	-24.5
	41×21	18433	-14.8	9486	-25.1

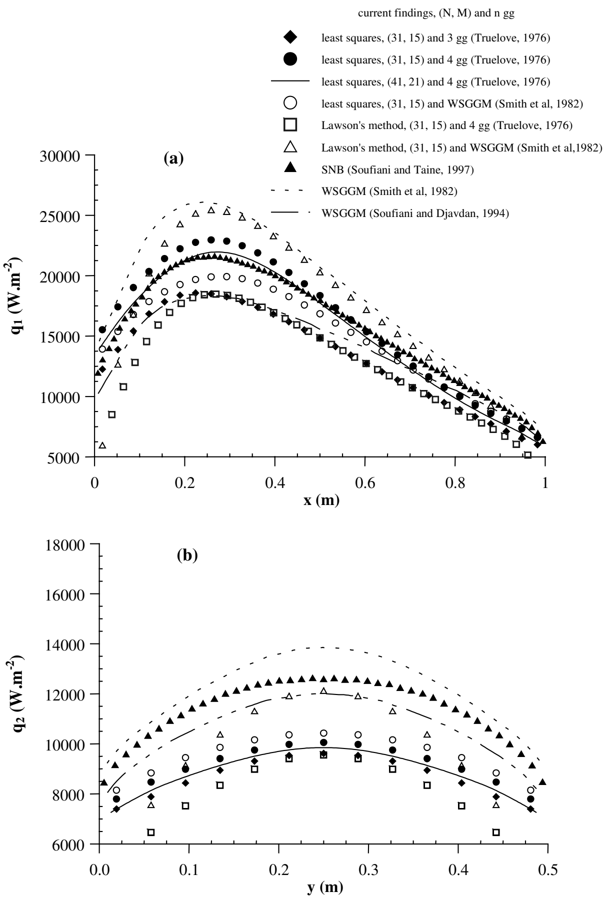

Figure 7. Comparison between the predicted net radiative wall heat fluxes for a CO_2-H_2O gas mixture with those of reference (case 5 in Goutiere et al, 2000): (a) wall 1; (b) wall 2.

1973; Truelove, 1976; Soufiani and Djavdan, 1994; Goutiere et al, 2000; Coelho, 2002).

Conclusions

In this study, a complete mathematical radiation model has been proposed to predict the radiative heat transfer in a rectangular furnace for two test cases: pure carbon dioxide and a CO_2-H_2O gas mixture under uniform and non-uniform temperature fields respectively. Based on the comparison of the current results carried out by a 2D computational code for radiative transfer calculations in furnaces using the well-known zone method of analysis and the most used weighted sum of grey gases model (WSGG model) parameters (Truelove, 1976; Farag, 1982; Smith et al, 1982), smoothing the direct exchange areas (DEA) has a significant effect on the accuracy of the full radiative modelling. Nevertheless, estimation of the DEA by direct numerical integration results in inherent numerical errors and the accuracy of such integration method depends not only on the size of each surface and/or volume zone but also on the inner subdivisions' size. It is shown that with a finer spatial grid, the net radiative wall heat fluxes calculated in two prescribed walls of the rectangular enclosure (walls 1 and 2) based on the Farag's WSGG model parameters and the Lawson's smoothing method are fairly well compared to the benchmark solutions for the first test case (CO_2 only). Fairly good accuracy is also obtained in the second test case of a homogeneous and non-isothermal CO_2-H_2O gas mixture by using four grey gases WSGG model parameters evaluated either from Truelove (1976) or from Smith et al (1982) and the least squares method. It is worth noting that considering more test cases with non-symmetric thermo-radiative boundary conditions and other gas mixtures with more grey gases should allow us to choose for a given problem the appropriate smoothing method.

The new mathematical model and the computational code developed in this paper enable radiative transfer calculations with quite reasonable accuracy which greatly depends on the accuracy of the radiative gas properties. Also, improvements to the zone method predictions are occurred by smoothing the DEA and consequently the total exchange areas. Therefore, it is expected that the present results can be perfected when adopting more accurate non-grey gas models such as the narrow band-based weighted sum of grey gases model (the narrow band based WSGG model) (Kim and Song, 2000) which has been recently applied (Borjini et al, 2007) to non-isothermal and non-homogeneous gas-soot mixtures in multi-dimensional enclosures as an efficient and accurate alternative to the conventional WSGG model due to Hottel.

Nomenclature

A	area of a surface zone (m^2)
a_g	WSGG model weighting factors
$E^o = \sigma T^4$,	blackbody emissive power ($W.m^{-2}$)
K_g	grey gas absorption coefficient (m^{-1})

K_t	attenuation coefficient of the medium (m^{-1})
L	gas path length
M_X	total number of zones in the enclosure
Q	net radiative wall heat flux (W)
r	distance between the centres of two zones i et j (m)
$\overline{ss}, \overline{sg}$ and $\overline{gg}$	surface-to-surface, surface-to-gas and gas-to-gas direct exchange areas (m^2)
$\overrightarrow{S_iS_j}, \overrightarrow{G_iS_j}$	directed flux areas (m^2)
T	temperature (K)
V	volume of a gas zone (m^3)
w_{ij}	weighting coefficients used in the normalization of direct exchange areas
$[X]$	global matrix of estimated direct exchange areas for all the types of radiative exchange
x_{ij}	estimated direct exchange areas (m^2)
x'_{ij}	adjusted direct exchange areas (m^2)
[]	matrix

Greek Symbols

Γ	number of surface or volume zones
ε	total gas or surface emissivity
η	angle between the beam joining the zone centers and normals of the surface zones (rad)
λ	Lagrange multiplier or wave length
μ	$=\cos\eta$
σ	$= 5.6710^{-8}$ $W.m^{-2}.K^{-4}$, Stefan-Boltzmann constant

Subscripts

g	gas
i, j, k	surface or volume zone
n	nth grey gas
s	surface

Superscripts

^{o}	blackbody
T	transpose of matrix

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