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The role of nonlinear Landau damping and the bounced motion of protons in the formation of dissipative structures in the solar wind plasma

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Abstract. The present work examines the effects arising from the nonlinear Landau damping and the bounced motion of protons (trapped in the mirror geometry of the geomagnetic field) in the formation of nonlinear Alfvénic structures. These structures are observed at distances 1-5AU in the solar wind plasma (with $\beta \sim 1$). The dynamics of formation of these structures can be understood using kinetic nonlinear Schrodinger (KNLS) model. The structures emerge due to balance of nonlinear steepening (of large amplitude Alfvén waves) by the linear Landau damping of ion-acoustic modes in a finite β solar wind plasma. The ion-acoustic mode is driven nonlinearly by the large amplitude Alfvén waves. At the large amplitudes of Alfvén wave, the effects due to nonlinear Landau damping become important. These nonlinear effects are incorporated into the KNLS model by modifying the heat flux dissipation coefficient parallel to the ambient magnetic field. The effects arising from the bounced motion (of mirroring protons) are studied using a one-dimensional Vlasov equation. The bounced motion of the protons can lead to growth of the ion-acoustic mode, propagating in the mirror geometry of the geomagnetic field. The significance of these studies in the formation of dissipative quasistationary structures observed in solar wind plasma is discussed.

1 Introduction

The spacecraft observations on the Alfvénic fluctuations (level $\frac{\Delta B}{B} \sim 1$) observed in the solar wind plasma reveal that these fluctuations are magnetohydrodynamic (MHD) waves with frequency in the range 3×10^{-4} Hz to 3×10^{-2} Hz (Belcher and Davis, 1971). Here, Alfvén waves refer to shear Alfvén waves and fast magnetosonic modes. The observed signals are believed to be a result of the nonlinear evolution of MHD waves that are excited in the solar wind plasma at distances 1-5AU (1AU=1.5 × 10^8km). Nonlinear evolution can lead to formation of dissipative wave forms, such as S-type and arc- polarized Alfvénic structures with a small compressional component (Tsurutani et al., 1994).

The Alfvénic fluctuations deposit a large amount of momentum and energy into the Earth's magnetosphere and excite geomagnetic activity observed during magnetospheric substorms (Tsurutani et al., 1987). Better understanding of the nature of the magnetic fluctuations is crucial to understand the nature of modes of energy propagation within solar wind plasma. The forms of energy modes determine the interaction of the solar wind with the geomagnetic field. The mechanism of formation of the nonlinear and *quasistationary* Alfvénic structures (that are sporadically interdispersed over the Alfvénic fluctuations) can be studied using kinetic nonlinear Schrodinger (KNLS) model (Medvedev and Diamond, 1996; Medvedev et al., 1997a;b; Vasquez and Hollweg, 1996). It is believed that these structures emerge when the nonlinear steepening of the large amplitude Alfvén wave is balanced by linear Landau damping of the ion-acoustic waves.

The above nonlinear steepening of the Alfvén waves takes place when the ponderomotive force (of the Alfvén waves) squeezes plasma out of the regions of large amplitude Alfvén modes. This results in a decrease in the plasma density and an increase in the local Alfvénic velocity. The amplitude-dependent velocity can lead to nonlinear steepening of the wave (Hasegawa, 1975; Medvedev and Diamond, 1996; Medvedev et al., 1997a;b). The KNLS equation describes the nonlinear dynamics of an envelope of Alfvén waves excited in a finite β plasma (Mjϕlhus and Wyller, 1988; Rogister, 1971; Spangler, 1989; 1990). The S-type and arc-polarized phase profiles (of the dissipative structures) are obtained when the KNLS equation is solved with different initial conditions, e.g. sense of polarizations and the angles of propagation of Alfvén waves (Galinsky et al., 1997; Medvedev

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et al., 1997b; Tsurutani et al., 1994; Vasquez and Hollweg, 1996).

Alfvén waves are transverse in nature. They do not Landau damp by resonant wave-particle interactions. However, their energy is dissipated in a finite $\beta(\sim 1)$ solar wind plasma, when the ion-acoustic mode (driven nonlinearly by the ponderomotive forces that are exerted by the Alfvénic turbulence) damps by resonant linear Landau damping (Hollweg, 1971; Mjϕlhus and Wyller, 1988; Spangler, 1989; 1990). The KNLS is an integral equation and describes wave dynamics, which is not solely determined by the local spatial derivatives of the perturbed field. It has a dissipative nonlinearity which is temporally non-local in nature. The nonlocality arises due to the finite ion transit time through the wave envelope. Thus the nature of dissipation is linear, but the ion-acoustic mode (which sinks energy from the Alfvén waves) is driven nonlinearly by ponderomotive forces of the Alfvén waves. It is for this reason that the damping is sometimes referred to as nonlinear damping.

A fully kinetic treatment to study propagation of nonlinear low frequency waves in a high β plasma was undertaken by Rogister (1971). For parallel propagation his results coincide with equations obtained by Mjϕlhus and Wyller (1988) and Spangler (1989;1990). Mjϕlhus and Wyller used the guiding center approach to calculate anisotropic pressure and included it in the two fluid model. Spangler used Vlasov equation to compute the second order perturbed distribution function which is inserted in the fluid model. Both models involve a nonlocal integral and the coefficients of the integral term coincide with those of Rogister. In the KNLS equation the fluid model describes the nonlinear steepening, while the kinetic model describes the linear Landau damping.

As the amplitude of the wave is increased, the particles can get trapped and execute periodic motion in the potential trough of the wave. The energy exchange between the wave and the oscillating particles results in nonlinear oscillation of the damping constant. Thus the trapped particles can sustain the wave amplitude which would otherwise decay exponentially due to linear Landau damping effects (Isichenko, 1997; Manfredi, 1997; Wharton et al., 1968). The oscillating particles can interact by phase mixing effects and modify the particle distribution function. As a result, the approximation of linear Landau damping breaks down and effects due to nonlinear damping should be considered.

In the past, the role of nonlinear Landau damping has been studied by incorporating the trapped particle fraction directly into the KNLS model (Medvedev et al., 1998). This approach is consistent with the approximations underlying the KNLS equation at asymptotically large times. However, an alternative approach (with better physical justification) is to study the effects of nonlinear Landau damping by including the nonlinear damping rate directly into the KNLS model and not by including the trapped particle fraction. The present work is a step towards developing this approach.

The particles trapped in the potential trough of the wave will reduce the number of carriers available for heat conduction and thus modify the heat flux coefficient parallel to the ambient magnetic field (Lee and Diamond, 1986). This supports the view that the nonlinear damping effects should be incorporated into the KNLS model through the parallel heat flux coefficient and not by including the trapped particle fraction (Lee and Diamond, 1986; Hammett and Perkins, 1990). The heat flux coefficient appears in the KNLS formalism through the coefficients m_1 and m_2. These coefficients describe the nonlinear steepening and the damping effects, respectively. The modified coefficients (including nonlinear damping effects) will redefine the balance of nonlinear steepening and damping effects which is crucial to the formation of *quasistationary* structures.

The limitations of our approach are: (1) it does not derive KNLS equation to include full nonlinear wave-particle interactions; (2) it is based on the expressions for nonlinear damping rate, which are good only to a leading order in the parameter $\frac{\tau}{\tau_l}$. Here, τ is period of the oscillating particle in the potential trough of the wave and τ_l is the inverse of linear Landau damping rate (γ_l^{-1}). Our approach modifies the KNLS equation by incorporating the nonlinear damping rate into the parallel heat flux coefficient occuring in the coefficients m_1 and m_2 (of the KNLS equation). This approach is simple and describes the evolution of the nonlinear structures at all times, in contrast to earlier studies which describe the evolution at asymptotically large times only (Medvedev et al., 1998). Therefore, these studies hold the potential to provide new insights into the mechanism of formation of nonlinear structures observed in solar wind plasma.

The current studies on the interaction of ion-acoustic mode with protons assume that the ambient magnetic field (parallel to the direction of propagation of the mode) is a straight line. It has been shown that the ion-acoustic mode can grow (instead of Landau damp) in the presence of a beam of mirroring protons in the geomagnetic field. These effects play an important role when the bounce frequency (ω_{bm}) of a mirroring particle is comparable to the frequency of the ion-acoustic mode at distances of $\sim$ 1 AU (Hasegawa, 1975; Nishihara et al., 1969). These studies have ramification for improving current understanding of the role of wave-particle interactions in the formation of *quasistationary* structures observed in solar wind plasma.

The present work will provide analytical formalism to modify the KNLS equation to study the effects arising from the nonlinear Landau damping (at 1-5 AU) as well as those from the mirror geometry (of the geomagnetic field) in the formation of dissipative structures at 1 AU. Detailed numerical studies examining the role of these effects on the formation of Alfvénic structures will be reported separately. In section 2 of this paper, the KNLS equation will be discussed briefly. The KNLS model will be modified to incorporate effects arising from nonlinear Landau damping. In section 3, the role of mirror geometry on the linear Landau damping of the ion-acoustic mode will be investigated. The summary remarks and conclusions will be given in section 4 of the paper.

2 Theoretical framework

The mechanism of formation of the dissipative wave forms observed in the solar wind plasma can be understood using the KNLS equation:

$$\frac{\partial b}{\partial t_e}+\frac{V_A}{2}\frac{\partial}{\partial z}(U_{NL}b)+i\frac{V_A^2}{2\Omega_i}\frac{\partial^2 b}{\partial z^2}=0. \quad (1)$$

The equation describes the nonlinear evolution of an envelope of Alfvén waves excited in the solar wind plasma. Here, $b=\frac{b_x+ib_y}{B_0}$ is the wave magnetic field, V_A and Ω_i are the Alfvén speed and proton ion-cyclotron frequency, respectively. The quantity $t_e=(\frac{b^2}{B_0^2})t$ represents slow envelope evolution time. U_{NL} is a nonlinear velocity perturbation operator and is given as:

$$U_{NL}=(m_1+m_2H)|b|^2. \quad (2)$$

$$H[F(z)]=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{P}{(z'-z)}F(z'). \quad (3)$$

Here, H is a non-local Hilbert operator which describes the effects arising from the finite transit time of ions in the envelope modulation of Alfvén waves. The coefficients m_1 and m_2 are given as follows:

$$m_1=\frac{(1-\beta^*)+\chi_{//}^2(1-\frac{\beta^*}{\gamma})}{m_0}. \quad (4)$$

$$m_2=-\frac{\chi_{//}\beta^*(1-\frac{1}{\gamma})}{m_0}. \quad (5)$$

$$m_0=4\Big((1-\beta^*)^2+\chi_{//}^2(1-\frac{\beta^*}{\gamma})^2\Big). \quad (6)$$

Here, $\beta^*=\beta(\frac{T_e}{T_i})$, $\beta=\frac{c_s^2}{V_A^2}$, and γ= 3.

The coefficient m_1 characterizes the wave steepening due to nonlinear effects and m_2 specifies the linear Landau damping due to the resonant particles. These coefficients depend strongly on the plasma β value and the electron-ion temperature ratio. The constant $\chi_{//}$ is a heat flux coefficient. The coefficient models dissipation in the fluid along the direction of the ambient magnetic field.

$$\chi_{//}=(\frac{8\beta}{\pi\gamma})^{1/2}(\frac{T_e}{T_i})exp\Big(\frac{T_i-T_e}{2T_i}\Big). \quad (7)$$

The KNLS is a nonlocal, integro-differential equation with no exact analytical solution. Unlike the case of hydrodynamic turbulence (where dissipation sinks energy from small scales) the above U_{NL} operator results in a scale independent (damping acting on all scales) dissipation. As a result, the balance of nonlinear steepening by Landau damping does not lead to formation of shocks, but dissipative structures of S-type and arc-polarized wave forms. The scale-independent damping can result in a turbulent power spectrum with no inertial range.

The nonlinear evolution of Alfvénic fluctuations can lead to the formation of rotational discontinuities (RDs). In a RD, the phase jumps rapidly, while its amplitude is roughly constant. The S-type and arc-polarized RDs result from the nonlinear evolution of quasi-parallel linearly polarized Alfvén waves. The circularly polarized waves do not generate RDs.

2.1 Kinetic effects due to nonlinear particle trapping

The large amplitude Alfvén waves (with linear polarization) can drive density perturbations due to a gradient in the magnetic pressure (that results in ponderomotive forces). For plasma with $\beta\sim 1$, a resonance with ion-acoustic mode exists and these perturbations can become very large (Hollweg, 1971). As a result, the linear Landau damping of the ion-acoustic mode results in the damping of the Alfvén waves. As the amplitude of the Alfvén wave increases, the particles can get trapped in the wave potential (large amplitude) of the ion-acoustic mode. This modifies the ambient particle distribution function. As a result, the approximation of linear Landau damping breaks down. In the present paper, the coefficient $\chi_{//}$ (Eq. 7) will be modified to include effects arising from the nonlinear Landau damping. In the next subsection, we will briefly discuss the particle trapping effects due to a large amplitude ion-acoustic mode which is excited nonlinearly by Alfvén wave.

2.2 Nonlinear Landau damping of the electrostatic wave

We consider a sinusoidal representation for the ion-acoustic mode $\phi(z,t)$ that is excited nonlinearly by the Alfvénic fluctuations in the solar wind.

$$\phi(z,t)=\phi_0 cos(kz-\omega t). \quad (8)$$

Here, ϕ_0 is a constant. Using Galilean transformation:

$$Z=z-\frac{\omega}{k}t, \quad (9)$$

where (z,t) specify co-ordinates in the laboratory frame and (Z,t) denote co-ordinates in the frame co-moving with the wave. In the moving frame the wave potential is given by

$$\phi(Z,t)=\phi_0 cos(kZ). \quad (10)$$

The velocity of the particle in the wave frame is given by

$$v'=v-\frac{\omega}{k}. \quad (11)$$

The total energy of the particle in the wave frame is given by:

$$W = \frac{1}{2}mv'^2 + e\phi_0 cos(kZ). \quad (12)$$

The particle moves along the trajectory defined by W=constant. The particles with negative value of W are trapped while those with positive W are untrapped by the wave potential. The trapped particles bounce back and forth in the potential trough and execute oscillatory motion. For kZ much less than one, the simple harmonic motion of the trapped species (in the moving frame) is described by

$$m\ddot{Z} = -e\phi_0 k^2 Z. \quad (13)$$

The frequency of oscillation ω_b of the trapped particle is given as:

$$\omega_b^2 = \frac{e\phi_0 k^2}{m}. \quad (14)$$

Clearly, the bounce frequency of an oscillating particle trapped near the bottom of the potential trough (of the wave) increases with an increase in the wave amplitude. At small wave amplitudes, ω_b is much less than γ_l. In this case the linear Landau damping treatment holds good. On the other hand, at large wave amplitudes, ω_b is much larger than γ_l, the particle will have completed many cycles of their bounce motion before the wave begins to decay. The phase mixing interactions between the trapped particles can modify the particle distribution function. As a result the approximation of linear Landau damping breaks down. In the next subsection, we will modify the KNLS equation to include the effects arising from the nonlinear Landau damping of the ion-acoustic mode (Alfvén waves).

2.3 Modification of the KNLS equation to include nonlinear Landau damping effects

In this subsection, we will modify the coefficient $\chi_{//}$ to incorporate the effects arising from nonlinear Landau damping. The nonlinear damping effects play an important role, when the parameter $\frac{\tau}{\tau_l}$ is much less than one (Lee and Diamond, 1986; Medvedev and Diamond, 1996; Medvedev et al., 1997a). In the presence of nonlinear damping the coefficient $\chi_{//}$ is redefined as:

$$\chi_{//} = \chi_{//}\frac{\gamma_{nl}}{\gamma_l}. \quad (15)$$

Here, γ_l and γ_{nl} are damping rates due to the linear and nonlinear Landau damping effects, respectively. The damping rate γ_{nl} consists of two terms:

$$\gamma_{nl} = \gamma_{untr} + \gamma_{tr}. \quad (16)$$

Here, γ_{untr} is the damping rate due to the untrapped species and γ_{tr} is that due to the trapped particles, respectively. O'Neil (1965) has calculated the nonlinear damping rates using the wave amplitudes that are perturbed by the modified particle distribution function (resulting from nonlinear damping). These damping rates are given as follows (O'Neil, 1965; Sagdeev and Galeev, 1969):

$$\gamma_{untr} = \gamma_l \Sigma_{n=0}^{n=\infty}\frac{64}{\pi}\int d\kappa \frac{2n\pi^2 sin(\frac{\pi n t}{\kappa F \tau})}{\kappa^5 F^2(1+q^{2n})(1+q^{-2n})}. \quad (17)$$

$$\gamma_{tr} = \gamma_l \Sigma_{n=0}^{n=\infty}\frac{64}{\pi}\int d\kappa \left(\frac{(2n+1)\pi^2 \kappa sin[\frac{(2n+1)\pi t}{2F\tau}]}{F^2(1+q^{2n+1})(1+q^{-2n-1})}\right). \quad (18)$$

Here, $q = exp\left(\frac{\pi F'}{F}\right)$; $F' = F[(1-\kappa^2)^{1/2}, \pi/2]$, and $F(\kappa) = \int_0^{\pi/2} d\zeta(1-\kappa^2 sin^2\zeta)^{-1/2}$. This is a complete elliptic integral of the first kind. The quantity $\zeta = \frac{kz}{2}$ and τ is period of oscillation of a proton in the trough of the wave. The variable κ is given by

$$\kappa^2 = \frac{|2e\phi_0|}{W + |e\phi_0|}. \quad (19)$$

The ratio κ depends on the wave potential and the total energy W of the trapped particles.

The modified $\chi_{//}$ (Eq. 15) is inserted in Eqs. (4-6) that define the coefficients m_1 and m_2. The resulting modified KNLS equation can be used to study the nonlinear evolution of Alfvénic structures in the presence of nonlinear Landau damping effects. The expressions (17-18) assume that ϕ_0 is independent of time. These expressions are valid to a leading order in the expansion parameter $\frac{\tau}{\tau_l}$. Corrections to these expressions have been computed by Bailey and Denavit (1970) who take into account the influence of time variation in ϕ_0 on the particle orbit.

We note that the nonlinear damping rate (γ_{nl}) oscillates with time and the time-dependent behaviour resembles that of a damped oscillation, asymptotically approaching γ_l=0. The time-dependent damping coefficient results in a time varying heat flux coefficient $\chi_{//}$. We define an average coefficient by

$$<\chi_{//}> = \frac{\int_0^\tau \chi_{//} dt}{\tau}. \quad (20)$$

We emphasize again that our approach is simple and enables us to follow the evolution of nonlinear structures at all times, while the earlier approach by Medvedev et al. (1998) applies only in the limit of asymptotically large times.

We note that the relations (15-20) are valid for a sinusoidal wave form given in Eq. (8). The generalization to more than one wave has been discussed in the literature (Sagdeev and Galeev, 1969). In the presence of a spectrum of waves we define the phase mixing time t_{pm}:

$$t_{pm} = \frac{2\pi}{\Delta(\omega - kv)}. \quad (21)$$

Here, Δ represents the characteristic spread in the wave spectrum. We list the solar wind parameters at 1 AU as follows (Hundhausen, 1995): T_e =12eV; T_i =10eV; ρ_i =100km; C_s =60 kms^{-1}; V_A =40kms^{-1}; B_0 =7γ; ω_{ci} = 1 s^{-1}. Here, γ=10^{-5}Gauss.

We calculate τ with wave electric field of 10 mV/m or higher as observed at 1 AU (Hasegawa, 1975). Using the above parameters, we compare the phase mixing time t_{pm} with the particle trapping time τ. At wave-particle resonance, the effects due to trapped particles will dominate over the phase mixing time. At smaller amplitudes, the effects due to phase mixing can dominate.

The KNLS equation with modified coefficients m_1 and m_2 can be studied numerically to examine the role of nonlinear Landau damping in the formation of dissipative structures. The detailed numerical studies will be reported in a separate publication.

3 Role of mirror geometry in the linear Landau damping of ion-acoustic mode

The studies on linear Landau damping of the ion-acoustic mode (excited nonlinearly by the Alfvénic fluctuations) are based on the assumption that the magnetic field (parallel to the direction of propagation of ion-acoustic mode) is a straight line. This assumption breaks down in the mirror geometry of the geomagnetic field, when the frequency of the wave is comparable to the bounce frequency of protons. For mirroring protons with velocity v_0 and mirroring distance z_0 , the bounce frequency ω_{bm} is given as $\omega_{bm} \sim \frac{v_0}{z_0}$. In the magnetosphere near 1 AU, the wavelength of the mode is assumed to be of the order of the scale length of the mirror. Based on the mirror size of $\frac{2R_E}{3}$, the wave period of the ion-acoustic mode is estimated (using the parameters listed in sec. 2.3). The wave period is found to be comparable to the bounce period ($\sim 100s$) of a few keV protons (Nishihara et al., 1969; Prakash, 1989). As a result, at 1 AU the effects due to the mirror geometry should be examined in the nonlinear evolution of the Alfvénic structure in the solar wind plasma. This section of the paper pertains to these studies.

Under the paraxial approximation of the geomagnetic field is given as (Hasegawa, 1975; Nishihara et al., 1969):

$$B_0(z) = B_0(1 + bz^2). \tag{22}$$

The unperturbed trajectory of a proton (in the absence of the ion-acoustic mode) in this field is given as:

$$\ddot{z}' = -\frac{\mu}{m}\frac{\partial B_0(z')}{\partial z'}. \tag{23}$$

The equation can be integrated immediately to yield:

$$z' = z_0 sin(\omega_{bm}t' + \psi). \tag{24}$$

$$v_{//}' = z_0\omega_{bm}cos(\omega_b t' + \psi). \tag{25}$$

Here, the bounce frequency is given by:

$$\omega_{bm} = \left(\frac{2\mu b B_0}{m}\right)^{1/2}. \tag{26}$$

$$\mu = \frac{mv_\perp^2}{2B_0(z)}. \tag{27}$$

The energy of the particle is given as:

$$w = \frac{m}{2}(v_{//}^2 + v_\perp^2). \tag{28}$$

The protons executing bounce motion can interact with the electric field of the ion-acoustic mode and modify its Landau damping rate. Because of the inhomogeneity along the z direction the electric field of the ion-acoustic mode is assumed of the following form:

$$\mathbf{E}(z,t) = e^{-i\omega t}\Sigma_{n=-\infty}^{\infty} E_n e^{inkz}\mathbf{e_z}. \tag{29}$$

Here E_n is given as:

$$E_n = \frac{1}{2L}\int_{-L}^{L} E(z)e^{i\omega t - inkz}dz, \tag{30}$$

where $k = \frac{\pi}{L}$. L may be taken to be the distance z at which most of the beam particles are reflected. The perturbed distribution function of protons in the presence of the ion-acoustic mode is given by linearized one-dimensional Vlasov equation (Hasegawa, 1975):

$$\frac{\partial f_1}{\partial t} + v_{//}\frac{\partial f_1}{\partial z} + \frac{e}{m}(\mathbf{v}\times\mathbf{B_0})\cdot\frac{\partial f_1}{\partial \mathbf{v}} = -\frac{e}{m}E\frac{\partial f_0}{\partial v_{//}}. \tag{31}$$

The perturbed distribution function f_1 can be obtained by integrating the right hand side along the unperturbed trajectory.

$$f_1(\mathbf{v},z)e^{-i\omega t} = -e\frac{\partial f_0}{\partial w}\int_{-\infty}^{t} dt' E(z')v_{//}' e^{-i\omega t'}. \tag{32}$$

The unperturbed distribution function for proton beam is assumed to be:

$$f_0(w,\mu) = \delta(w - w_0)\delta(\mu - \mu_0)(w_0 - \mu_0 B_0)^{1/2}. \tag{33}$$

The perturbed distribution function (Eq. 32) is obtained after integrating over the unperturbed particle trajectories as given by Eqs. (24;25). The perturbed distribution function is inserted into the Poisson's equation (Nishihara et al., 1969):

$$\nabla\cdot\mathbf{E} = \frac{\rho_1}{\epsilon_0}. \tag{34}$$

The resulting equation is simplified to obtain dielectric function (Nishihara et al., 1969):

$$\epsilon_{//} = \Sigma_{l=1}^{\infty}\frac{l^2\omega_{b0}^2\alpha}{(\omega^2 - l^2\omega_{b0}^2)}. \tag{35}$$

Here ω_{b0} is the proton bounce frequency at z=0 and α is given by

$$\alpha \sim \frac{1}{2\pi^2}\left(1+\frac{v_{//0}^2}{v_{\perp 0}^2}\right). \tag{36}$$

Here, α depends on the ratio of parallel to perpendicular energy of the bouncing proton at z=0. The total dielectric function of the perturbed bounced motion of the protons in the ambient plasma is given as:

$$\epsilon = \epsilon_b + \epsilon_{//}. \tag{37}$$

Here ϵ_b is given by

$$\epsilon_b = \frac{\omega_{pe}^2}{k^2 v_{Te}^2} + \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} + i(\frac{\pi}{2})^{1/2}\frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\frac{\omega}{k v_{Ti}}. \tag{38}$$

Using Eqs. (35-38) we obtain the dispersion relation of the ion-acoustic mode ($\omega \sim \omega_{b0}$) in the presence of mirroring protons in the ambient plasma (by using ϵ=0) as follows:

$$\omega = \omega_{b0}\left(1-\frac{\alpha\omega_{p0}^2}{2\omega_{b0}^2}S\right) + i\alpha(\frac{\pi}{2})^{1/2}\frac{\omega_{p0}^2}{k v_{Ti}}\left(\frac{T_e}{T_i+T_e}\right), \tag{39}$$

where S is given by

$$S = \frac{1}{\frac{\omega_{pe}^2}{k^2 v_{Te}^2}+\frac{\omega_{pi}^2}{k^2 V_{Ti}^2}}. \tag{40}$$

Here, ω_{p0} is the ion plasma frequency at z=0.

The interaction of the ion-acoustic mode with the bounced motion of protons can lead to instability of the mode. We emphasize that the ion-acoustic mode in the mirror geometry is not Landau damped, but grows. The growth instead of Landau damping of the ion-acoustic mode (in the mirror geometry) can modify the dissipation characteristics of Alfvén waves observed in solar wind plasma. These effects can play a significant role in modifying the nonlinear evolution of the dissipative structures observed in solar wind plasma at distance of 1 AU. It is important to note that the conclusions of this section are valid only to a leading order in the parameter $\frac{\omega}{\omega_{bm}}$.

4 Conclusions

Nonlinear damping processes play a key role in the formation of nonlinear structures excited in the solar wind plasma. Better understanding of these processes is crucial to determine the nature of modes of energy propagation within solar wind plasma. The forms of these energy modes determine the interaction of the solar wind with the terrestrial magnetosphere. Hence, these studies are important to improve current understanding of mechanism of the onset of magnetospheric substorms.

Based on the analytic formulation (Eqs. 15-20) presented here, numerical studies will be carried out to examine the nature of discontinuous wave forms observed in solar wind plasma. These studies will advance current knowledge of the mechanism of formation of Alfvénic structures in solar wind plasma. The work also provides an impetus for future analytical work to rederive KNLS equation with full nonlinear wave-particle interactions. The future work in this direction will address these issues.

The studies on the role of mirror geometry on the linear Landau damping of the ion-acoustic modes are important. The growth instead of damping of the ion-acoustic mode in the mirror geometry can have significant impact on the mechanism of formation of S-type and arc-polarized Alfvénic structures. The important results of the paper (Eqs. (15-20); Eqs. (39-40)) can be used to study the role of nonlinear Landau damping and the bounced motion of protons in the formation of S-type and arc-polarized wave forms. It is our hope that a better understanding of these effects will improve our capability to interpret the spacecraft observations at distances 1-5 AU. This is the main thrust of this paper.

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References

Bailey, V. L., Jr., and Denavit, J., Nonlinear oscillations in a collisionless plasma, *Phys. Fluids*, *13*, 451-458, 1970.

Belcher, J., and Davis, L., Jr., Large-amplitude Alfvén waves in the interplanetary medium, *J. Geophys. Res.*, *76*, 3534-3563, 1971.

Galinsky, V. L., Shevchenko, V. I., Medvedev, M. V., and Diamond, P. H., Emergence of dissipative structures in coherent nonlinear Alfvén waves, *Comments Plasma Phys. Controlled Fusion*, *18*, 249-261, 1997.

Hasegawa, A., *Plasma instabilities and nonlinear effects*, 52-62, Springer Verlag, New York Heidelberg Berlin, 1975.

Hammett, G. W., and Perkins, F. W., Fluid moment models for Landau damping with applications to the ion-temperature - gradient instability, *Phys. Rev. Lett.*, *64*, 3019-3022, 1990.

Hollweg, J. H., Nonlinear Landau damping of Alfvén waves, *Phys. Rev. Lett.*, *27*, 1349-1352, 1971.

Hundhausen, A. J., *Introduction to Space Plasma Physics*, eds. M. G. Kivelson, and C. T. Russell, Cambridge University, 52, 1995.

Isichenko, M. B., Nonlinear Landau damping in collisionless plasma and inviscid fluid, *Phys. Rev. Lett.*, *78*, 2369-2372, 1997.

Lee, G. S., and P. H. Diamond, Theory of nonlinear ion-temperature gradient driven by turbulence in tokamaks, *Phys. Fluids*, *29*, 3291-3313, 1986.

Manfredi, G., Long time behaviour of nonlinear Landau damping, *Phys. Rev. Lett.*, *79*, 2815-2818, 1997.

Medvedev, M. V., and Diamond, P. H., Fluid models, for kinetic effects on coherent nonlinear Alfvén waves. I. Fundamental theory, *Phys. Plasmas*, *3*, 863-873, 1996.

Medvedev, M. V., Shevchenko, V. I., Diamond, P. H., Galinsky, V. L., Fluid models for kinetic effects on coherent nonlinear Alfvén waves. II. Numerical Solutions, *Phys. Plasmas, 4,* 1257-1285, 1997a

Medvedev, M. V., Diamond, P. H., Shevchenko, V. I., and Galinsky, V. L., Dissipative dynamics of collisionless nonlinear Alfvén wave trains, *Phys. Rev. Lett., 78,* 4934-4937, 1997b.

Medvedev, M. V., and Diamond, P. H., Toward a simple model of Alfvénic compressible turbulence, *Phys. Rev. E, 56,* 2371-2374, 1997.

Medvedev, M. V., Diamond, P. H., and Rosenbluth, M. N., Asymptotic theory of nonlinear Landau damping and particle trapping in waves of finite amplitude, *Phys. Rev. Lett., 81,* 5824-5827, 1998.

Mjϕlhus E., and Wyller, T., Nonlinear Alfvén waves in a finite beta plasma, *J. Plasma Physics, 40,* 299-318, 1988.

Nishihara , K., Hasegawa, A., Maclennan, and C. G. Lanzerotti, L. J., Electrostatic instability excited by an electron beam trapped in the magnetic mirror of the magnetosphere , *Planet. Space Sci., 20,* 747-759,1969.

O'Neil, T. O., Collisionless damping of nonlinear plasma oscillations, *Phys. Fluids, 8,* 2255-2262, 1965.

Prakash, M., Particle precipitation due to nonlinear wave-particle interactions in the magnetosphere, *J. Geophys. Res., 94,* 2497-2504, 1989.

Rogister, A., Parallel propagation of nonlinear low frequency waves in high β plasma, *Phys. Fluids, 14,* 2733, 1971.

Sagdeev R. Z., and A. A. Galeev, *Nonlinear plasma theory,* 42-54, W. A. Benjamin, inc., New York 1969.

Spangler, S. R., Kinetic effects of Alfvén wave nonlinearity, I. Ponderomotive density fluctuations, *Phys. Fluids, B1,* 1738-1746, 1989.

Spangler, S. R., Kinetic effects on Alfvén wave nonlinearity, II. The modified nonlinear wave equation, *Phys. Fluids, B2,* 407-418, 1990.

Tsurutani, B. T., and W. D. Gonzalez, The cause of high-intensity long duration continuous AE activity (HILDCASS): Interplanetary Alfvén wave trains, *Planet. Space Sci., 35,* 405-412, 1987.

Tsurutani, B. T., Smith, E. J., Neugebauer, M., Goldstein, B. E., Mok, J. S., Arballo, J. K., Balogh, A., Southwood, D. J., and Feldman, W. C., The relationship between interplanetary discontinuities and Alfvén waves : Ulysses observations, *Geophys. Res. Lett., 21,* 2267-2270, 1994.

Vasquez, B. J., and Hollweg, J., Formation of arc-shaped Alfvén waves and rotational discontinuities from oblique linearly polarized wave trains, *J. Geophys. Res., 101,* 13,527-13,540, 1996.

Wharton, C. B., Malmberg, J. H., and O'Neil, T. M., Nonlinear effects of large amplitude plasma waves, *Phys. Fluids, 11,* 1761-1763, 1968.