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Testing for nonlinearity in radiocarbon data

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Abstract. The radiocarbon record that has been extended from 7199 BC to 1891 AD is of fundamental importance to understand century-scale variations of solar activity. We have, therefore, studied how to extract information from dynamic reconstructions of this observational record.
Using some rather unusual methods of nonlinear dynamics, we have found that the data are significantly different from linear colored noise and that there is some evidence of nonlinear behavior. The method of recurrence plots exhibits that the grand minima of solar activity are quite different in their recurrence. Most remarkably, it suggests that the recent epoch seems to be similar to the Medieval maximum.

1 Introduction

To study possible climate changes, several aspects of solar-terrestrial relationships have recently become of some importance. This has also challenged for a rather detailed knowledge of the history of solar activity. Due to the complexity of variations in solar activity, a long record is necessary for understanding the nature of the underlying processes. It was shown that changes in atmospheric ^{14}C, as given by the decay-corrected $\Delta^{14}C$ activity in tree rings, can provide essential information on solar variations (Stuiver and Quay, 1980). From $\Delta^{14}C$ records the 208-yr period, the Suess wiggle, was found as an outstanding component that can be related to grand minima and maxima of solar activity (cf. Damon and Sonett, 1991). However, these century-scale variations look rather complicated. Moreover, recent findings in dynamo theory for solar activity suggest that such oscillations are of chaotic nature (cf. Weiss, 1985, Feudel *et al.*, 1993).
The purpose of this letter is to test a $\Delta^{14}C$ record that has been extended from 7199 BC to 1891 AD for nonlinearity. The usual methods applied in nonlinear time series analysis for classifying an irregular signal (e.g. estimation of Lyapunov exponents or fractal dimensions) do not give reliable results for our data. Therefore, we use here two different methods: the first is based on a particular statistical test for white noise and the second uses recurrence plots to visualize hidden structure inherent the data. The results obtained from the $\Delta^{14}C$ record are compared with those from different kinds of surrogate data.

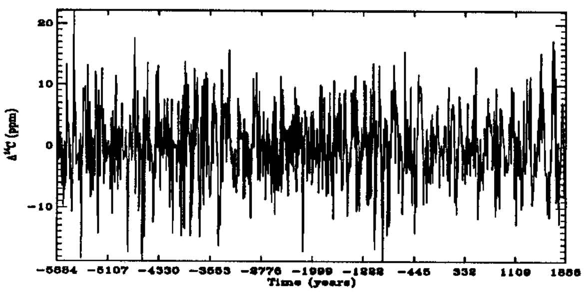

Fig. 1. Stationary part of the detrended Δ^{14} C record.

2 Data

The radiocarbon sequence used here is a composite of segments, some overlapping, from different records (cf. Sonett, 1992). To construct a continuous sequence, all records are interleaved in ascending time (Fig. 1). The major trend in the radiocarbon data is attributed to secular changes in the intensity of the geomagnetic dipole moment.
These properties make some pre-processing of the data necessary: Because the data are not equidistant, we bin them to $\Delta t = 5$ yr. The rather small gaps inherent the record are removed via linear interpolation. To detrend the long-scaled variations, we fit locally linear polynomials.

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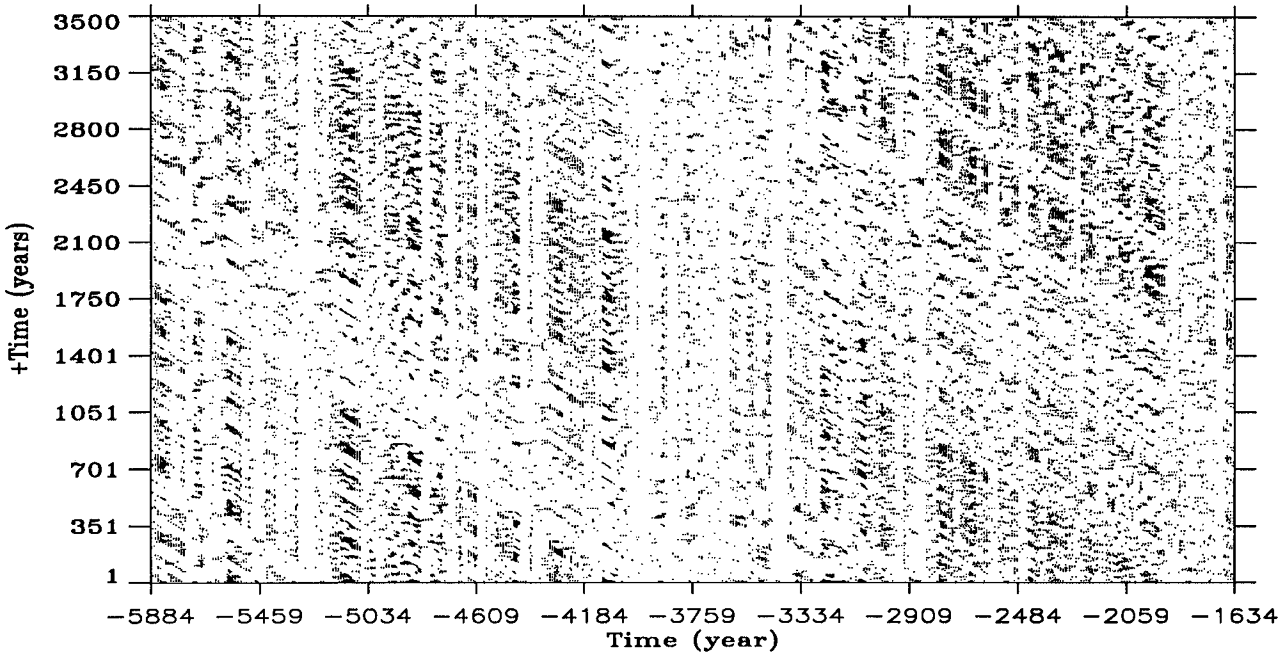

Fig. 2. Recurrence plot of the detrended Δ^{14} C record (embedding parameters are $\tau = 2$ and $m = 3$). If the distance of two points in phase space is less than about 10 % of the phase space diameter, it is coded black; otherwise, it is coded white. The horizontal axis represents the reference time interval from 5884 BC to 1634 BC . The vertical axis denotes the years which follow the respective reference time of the abscissa.

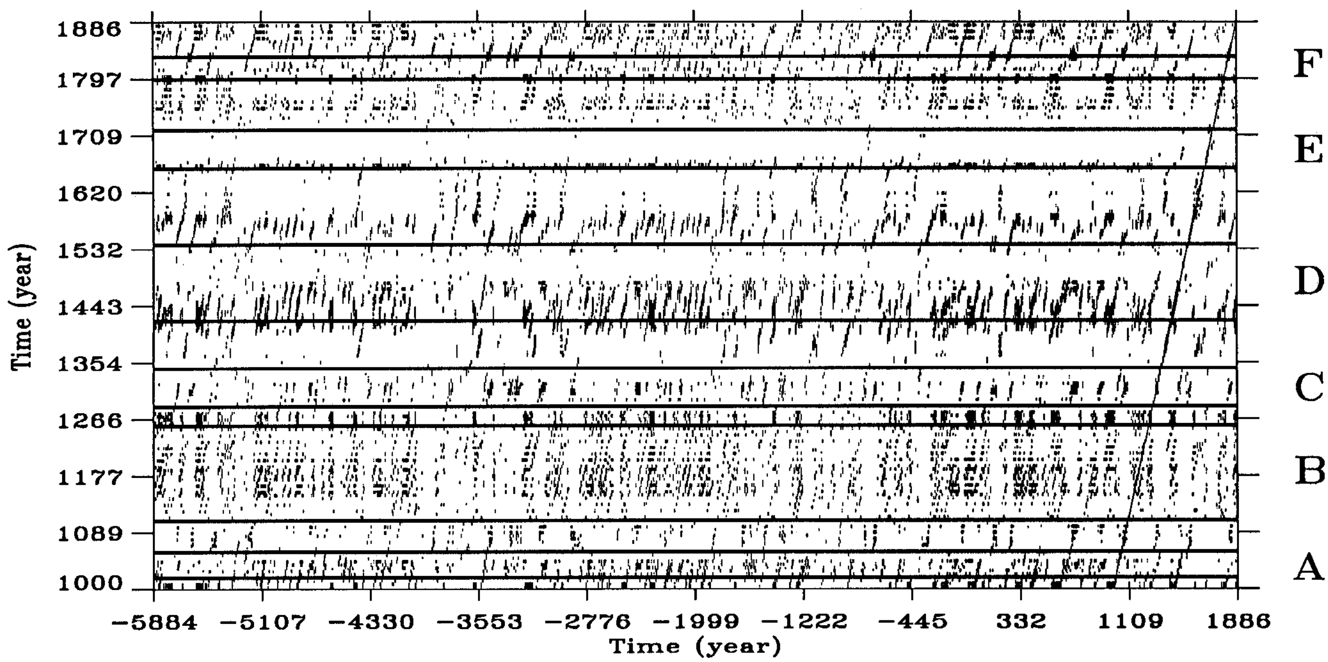

Fig. 3. Recurrence plot of the same data as in Fig. 2 ($\tau = 2$ and $m = 3$). The color coding gives the distance between phase space points x_t and x_s where t runs from 1000 AD to 1896 AD (vertical axis) and s runs over the entire data set. Similar to Fig. 2 black corresponds to distances less than 10 %. The horizontal lines assigned by the capital letters denote the grand events of solar activity: A – Oort, B – Medieval, C – Wolf, D – Spörer, E – Maunder, F – Dalton. The diagonal line in the right part of the figure is due to the self-recurrence.

Since most techniques of data analysis assume stationary records, we test our data for strong stationarity (Isliker and Kurths, 1993). This leads to neglect the oldest 1300 yr, i.e. in the following data beginning at 5884 BC will be studied. It is interesting to note that the power spectrum analysis after this filtering yields the same significant periods as reported in the literature (cf. Damon and Sonett, 1992).

3 Data Analysis

To avoid some kinds of artifacts due to filtering, noise or data length, the following types of surrogate data are analyzed in parallel to the Δ^{14} C record:

- data obtained from shuffling the considered record, which removes all correlations
- data generated by phase randomization in the Fourier space, which destroys nonlinear dependences (Theiler *et al.*, 1992)
- autoregressive processes as linear models of the record (Kurths and Herzel, 1987).

Firstly, we apply the BDS (**B**rock-**D**echert-**S**cheinkman) test (Brock *et al.*, 1987; Scheinkman and LeBaron, 1989) based on the correlation sum $C(m,\epsilon)$ which was introduced to calculate the correlation dimension (Grassberger and Procaccia, 1983). This $C(m,\epsilon)$ measures the probability that two vectors in the m-dimensional phase space are within ϵ of each other in all their Cartesian coordinates.

This BDS test is a statistical test against the null hypothesis that the numbers in a series are independent and drawn from a probability distribution which is identical for each number. In this case we have $C(m,\epsilon) \approx (C(1,\epsilon))^m$. It is important to note that the BDS statistics has been shown to have power against many sequences generated by deterministic maps, but which pass standard tests of randomness based on autocorrelation functions (Brock *et al.*, 1987). This test is applied here in two ways. Firstly, it is used for distinguishing the given data from white noise, that is exactly the purpose for which it was developed. Secondly, we use the BDS statistics as a discriminating statistics in combination with our surrogate data. Applying these tests, we obtain the following results:

- For both, the considered data as well as the phase randomized data the null hypothesis – white noise – has to be rejected, but not for the shuffled data, as expected
- The differences found in the BDS statistics between the considered data and the phase randomized data is an indication to some nonlinearities within the considered data.

To study rather short erratic time series, which are typical in geophysical or biological measurements, a new graphical tool has been recently introduced (Koebbe and Mayer-Kress, 1992); it is called recurrence plot. This plot represents characteristic elements of recurrence patterns in the following manner: Firstly, the given 1-dimensional record x_t is used to reconstruct an m-dimensional phase space by the well-known embedding technique (Takens, 1981) yielding vectors $\underline{x}_t$. Secondly, the distance between a vector $\underline{x}_t$ and a vector $\underline{x}_s$ is calculated

$$d(t,s) = ||\underline{x}_t - \underline{x}_s||. \tag{1}$$

Next, we define a color function from this distance in dependence on a cutoff value ϵ

$$p(t,s) = \begin{cases} \text{black} & \text{if } d(t,s) < \epsilon \\ \text{white} & \text{otherwise.} \end{cases} \tag{2}$$

This $p(t,s)$ is the recurrence function that generates the recurrence plot in the $t \times s$ array. The $(t,s)^{th}$ element in this array becomes black if the two vectors $\underline{x}_t$ and $\underline{x}_s$ are close, in other words recurrent, and remains white otherwise.

It is easy to check that such a recurrence plot exhibits an unstructured feature in the case of white noise, but forms well-ordered diagonal stripes for periodic behavior.

We use here an embedding with delay times $\tau = 2 - 9$ and embedding dimensions $m = 3 - 7$. As mentioned before, these plots are calculated from the filtered radiocarbon record as well as from the different types of surrogates. This leads to the following findings:

- There is a well-pronounced persistency of recurrence which is far from periodic (Fig. 2)
- The comparison with the recurrence plots obtained from the phase randomized surrogates shows a clear-cut difference of the Δ^{14}C data to linear stochastic processes
- A detailed analysis of the epoches, for which grand minima or maxima of solar activity has been reported (cf. Jirikowic and Damon, 1992), exhibits that there are different types of these large events (Fig. 3): the Maunder minimum (event E) is a unique feature during the time the radiocarbon record covers, whereas the Oort and the Dalton Minima as well as the Medieval Maximum (events A, F, B) show a pronounced tendency to recur. It is especially important to note that the recurrence plot of the final part of the data looks rather similar to that of the first part in the Medieval Maximum. Moreover, these two events seem to be in some resemblance to the epoches from about 2500 BC to 2000 BC and 5100 BC to 4300 BC.

We have also applied other techniques to pre-process and analyze our data, e. g. methods of noise reduction, estimation of Lyapunov exponents and dimensions, or local linear prediction. However, the results are difficult to interpret due to the (probably) high noise level and the relatively small amount of data points available.

4 Conclusions

To study the nature of century-scale variations of the solar activity, we have analyzed here a radiocarbon record, which dates back to 7199 BC. Our main findings are:

- These variations are significantly different from linear colored noise; there is some evidence of nonlinear behavior
- The grand minima of solar activity are quite different in their recurrence; only the Maunder minimum seems to be unique
- The structure of the recent data seems to be similar to the Medieval Maximum. This is in accordance with a suggestion in Jirikowic and Damon (1993).

These results are mainly based on some rather unusual techniques: a special test for white noise, recurrence plots, and the comparison of the observational record with several types of surrogate data. The method of recurrence plots is a useful tool for the study of the global structural behavior as well as for detailed analysis of special epoches. The usual techniques for testing nonlinear dynamics, which are mainly used in combination with low-dimensional systems, do not give reliable results.

In this letter we have presented the first application of different methods from nonlinear time series to a radiocarbon record. Of course, there remain several open questions such as further tests concerning possible artifacts caused by the pre-processing, a more quantitative description of the obtained recurrence plots, and a comparison with nonlinear models for solar activity.

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