

# Stakes and Beliefs<sup>1</sup>

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The idea that beliefs may be *stake-sensitive* is explored. This is the idea that the strength with which a single, persistent belief is held may vary and depend upon what the believer takes to be at stake. The stakes in question are tied to the truth of the belief—not, as in Pascal’s wager and other cases, to the belief’s presence. Categorical beliefs and degrees of belief are considered; both kinds of account typically exclude the idea and treat belief as *stake-invariant*, though an exception is briefly described. The role of the assumption of stake-invariance in familiar accounts of degrees of belief is also discussed, and morals are drawn concerning finite and countable Dutch book arguments.

“Having any definite degree of belief implies a certain measure of consistency, namely willingness to bet on a given proposition at the same odds for any stake...”

F.P. Ramsey, “Truth and probability,” 1926

Suppose that Ramsey was mistaken about this. What would that mean? Perhaps that there just are no ‘definite degrees of belief’ of the sort he had in mind. Or if there are, that a person might have one, yet have inconstant willingness to bet on its content, depending on what is at stake. The first alternative, that definite degrees of belief do not exist, is the point or a consequence of any number of critiques of the family of accounts of partial belief that descend from Ramsey’s and de Finetti’s work.<sup>2</sup> The second alternative is the one I explore in this paper.

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<sup>2</sup> I do not offer a taxonomy of accounts of degrees of belief here. We can distinguish multiple accounts descending from Ramsey (1926) and de Finetti (1937); some of the criteria that Good (1971) employs are relevant, as are others. But I will lapse into using singular phrases like ‘the Ramsey-de Finetti model of action-guiding belief’ without attempting to explicitly specify which variants count.

Let us label the sort of consistency Ramsey has in mind *stake-invariance*; the second alternative is that partial beliefs are not stake-invariant, that instead they are *stake-sensitive*. In that case, their strength is indicated by their influence on choice, yet that influence is inconstant and variable, depending on what is at stake. There are at least two reasons for giving some thought to this possibility: First, the assertion that degrees of belief are stake-invariant is an integral part of the familiar accounts of partial belief that Ramsey and de Finetti pioneered, and such accounts will be considerably changed if that idea is dropped. I will later sketch a connection between stake-invariance and grounds for regarding probability as a logic for partial belief.

A second reason for thinking about stake-invariance is that the idea has broader import than its application to degrees of belief alone. We can raise the same question about categorical belief (beliefs understood as coming in various categories of strength): Is it a feature of a categorical belief that  $p$ , a sort of consistency about having it, that it is insensitive to what is at stake? That question, and the issues I will discuss, need clarification. In what follows, I will try to be more precise about the sort of stake-sensitivity I have in mind, by pointing toward what it would be, and by distinguishing it from other phenomena that may be near by, but that I take to be different.

To clarify the following exploration, here is a preview of several points: First, the idea that beliefs are stake-sensitive is an interesting one, but there is little evidence to indicate that philosophers (especially epistemologists and logicians) have considered it or taken it very seriously. It is an idea that I will be entertaining rather than recommending. Second, in undertaking to clarify the idea, I will at first consider it mainly in connection with categorical belief, although (third) a motivation for raising the idea at all comes from thinking about degrees of belief, and about efforts to defend probabilism as a logic of partial belief. In the latter part of this paper I will turn the discussion to additivity and countable additivity as constraints on rational degrees of belief.

### 1. What would stake-sensitivity be?

Consider John, who believes that he is a skillful poker player ( $p$ ), and distinguish two sorts of stakes. Whether or not he is a skillful poker player, whether or not  $p$  is true, may matter to him a little or a lot. Something, large or small, is at stake on the truth of  $p$ . He may take opportunities to enter high-stakes or low-stakes games, for example. This is the sort of stake that Ramsey has in mind: the payoff of a bet on  $p$  depends on  $p$ 's truth or falsehood, and such bets may be high-stake or low-stake. A second sort of stake may also be at hand: John's *believing* that  $p$  may itself carry consequences that matter to him, and so may the absence of that belief. Confidence in his ability may contribute to his success, while self-doubt may undermine it. Or self-confidence, overconfidence may contribute to losses; the same point applies. The consequences ground significant pragmatic reasons for holding the belief, or not; what's at stake on having the belief may influence what one believes. It is a familiar and well-explored idea that belief may be held or withheld for pragmatic reasons.<sup>3</sup> How that may be accomplished and how pragmatic norms may govern believing is debatable, but it is not my present concern. I am

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<sup>3</sup> A classic example is Pascal's wager. For other examples and discussions see Heil (1992), Foley (1993), Kelly (2002; 2003); there are many others.

instead interested in logical and epistemic norms for belief, whether for categorical belief or for belief that comes in degrees. So I am here less interested in the influence of stakes that are tied to the presence of belief-states—stakes that contribute to pragmatic reasons for believing, for choosing to believe, for bringing it about that one believes—and more interested in influences by stakes attached to whether belief-states are true. Henceforth, in discussing the stake-sensitivity of a belief that  $p$ , let us understand ‘what is at stake’ to be ‘what is at stake on the truth or falsehood of  $p$ ,’ rather than on the believing of it.

Think of stake-sensitivity as a specific kind of context-sensitivity. John’s present *expectations* about the significance of  $p$ ’s being true, including the pragmatic significance of  $p$ ’s being true, depend on elements of John’s present context.<sup>4</sup> Might his *belief-state* concerning  $p$  also be affected by those elements and expectations, and so be context-sensitive in a way we can label ‘stake-sensitive’? Put another way, might John be in a kind of mixed state  $M$ , so that in the presence of some circumstances with stakes of magnitude  $S$ , his state is, or yields, a believing that  $p$  to one extent, yet in the presence of other circumstances with stakes of magnitude  $S^*$ , a believing that  $p$  to a different extent?

The sensitivity we are imagining concerns alterations not of the content,  $p$ , but of the strength of the belief-state, the extent to which John believes that  $p$ . We can consider it both for belief-states that are taken to be categorical, and for belief-states that are taken to come in more fine-grained degrees. In the former case, believing that  $p$  to a particular extent might be believing, or disbelieving, or withholding. Or if our taxonomy of categorical belief admits more categories like being morally certain, doubting, and so on, then to believe to a particular extent would be to be in one of those categories. For belief-states that are taken to come in degrees, believing to a certain extent is believing to some particular degree. There are a variety of things that might mean, but we will focus here on the familiar account that derives from Ramsey and de Finetti.

The issue is whether some beliefs are stake-sensitive, not whether all beliefs are. If it is correct that there has been little attention and support for the idea that beliefs are stake-sensitive, then the view that beliefs can be stake-sensitive is already interesting. And if Ramsey’s view is in error, even if only about some beliefs, that too has interesting implications for well-known accounts of degrees of belief.

John’s having a belief is something that typically occurs in conjunction with other things that happen to him, or other things he does. Those things may be influenced by what John takes to be at stake, and in the interest of clarifying the idea of stake-sensitive belief, they are worth noticing.

The *content* of a belief, and attributions of content to a belief are sensitive to the context in which it is present. Whether or not a belief is *occurrent* is surely sensitive to context, and no doubt sometimes to the stakes.

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<sup>4</sup> And no doubt also on his background and other beliefs. Beliefs are influenced by other beliefs, and their historical backgrounds influence our doxastic states. On a given occasion of believing that  $p$ , what the believer takes to be at stake on the truth of  $p$  is a fact about the believer. But it is not necessarily a fact about what the believer takes to be evidence for  $p$ , and typically it *is* a fact that is connected with features of the circumstances in which the believer finds himself.

Our *decisions* are sensitive to the stakes at hand. Obviously so, when our reasons for choosing involve the relative values of the possible consequences of the acts. But also in the further sense that we do not treat weighty high-stakes decisions in the same way we treat trivial ones. We typically put more time and effort into weighty deliberations, and we usually choose more conservatively when stakes are high than when they are not. But the fact that deliberations are stake-sensitive does not entail that the beliefs on which they are based are themselves stake-sensitive. They might be (that's the question we are raising), but the stake-sensitivity of deliberation and choice may arise elsewhere: in the particular values to which we attend, in our regard for those values and risks, and in the assessments of risky choices that we derive from them.

The *attentive effort* that we devote to our beliefs may also be stake-sensitive. In a deliberative context where the decision is relatively trivial, John may tend to neglect some beliefs that he would recognize as relevant, were he to try to think of them, whereas he would pay more attention, and look more carefully for (beliefs about) factors relevant to weighty matters. This stake-sensitivity of attentive effort is probably not confined to deliberation. Even when no particular choice is at hand, if one is reasoning about a weighty matter, one may well give consideration to more (beliefs about) relevant reasons than one does if one is engaged in inference about a subject of trivial significance.

What we are willing to *say* is stake-sensitive. The importance of matters at hand may influence, *via* expectations of conversational implicature, what John thinks it is worth bothering to say. The weightiness of matters at hand makes us more or less careful about making or withholding *assertions* to others. And how we *describe* our doxastic state to others (how sure John says he is about *p*) may be influenced by the importance of matters to which the content of the doxastic state (*p*), or perhaps the presence of the doxastic state, is relevant. As with deliberation and decision-making, these influences might involve a sensitivity of belief itself to the stakes, but the stake-sensitivity of such sayings might instead arise elsewhere: in conventions of conversational implicature, in patterns of expectations tied to assertion, in introspective assessments of our doxastic states.

What about what we *accept*? 'Acceptance' is used in many ways, and whether its stake-sensitivity or stake-invariance differs from the stake-sensitivity or stake-invariance of belief depends upon what sort of acceptance we have in mind, and how close or distant it is from belief. If acceptance is no different from belief, we can expect the stakes to influence them, or not, to the same extent. But in many uses, 'acceptance' is understood to differ from belief, and usually in ways that suggest context- and stake-sensitivity.<sup>5</sup> For present purposes, we are less interested in this than in whether stake-sensitivity is a feature of belief.

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<sup>5</sup> For examples: Stalnaker (1984, Chapter 5) regards belief as one sort of acceptance concept; other sorts of acceptance involve acting as if one believes, in a variety of ways that may be for a purpose, compartmentalized, and direct products of decision. Similarly Bratman (1992). Note that Stalnaker distinguishes a belief from a belief state; the latter is a state of having a total set of beliefs. Van Fraassen (1980) takes acceptance of *p* (a theory *T*) to involve belief in a related *p*\* (that *T* is empirically adequate), together with pragmatic commitments. Kaplan (1996) ties categorical belief that *p* to preference for asserting *p* in a particular sort of circumstance, a context of inquiry; this account is a descendent of one in which accepting that *p*

A belief's *epistemic justification* may depend on what is at stake. And so may whether or not it is *knowledge*. Or perhaps the stakes matter to how we *judge* whether a belief is justified, or is known.<sup>6</sup> On some views the stake-sensitivity of justification or knowledge is very closely tied to the stake-sensitivity of assertion. But whether we think the connection between justification or knowledge and assertability is immediate, or is just often strong, what is at stake seems to matter to justification and to knowledge. Here again, the stake-sensitivity might be a stake-sensitivity of belief itself, but it is possible that the stake-sensitivity lies elsewhere: in our expectations concerning epistemic standards that a belief must meet in order to be justified (to a particular extent, perhaps), or to be known.

The *access* we have to our beliefs seems context-sensitive, and so might be stake-sensitive, what is at stake being a part of context. This will depend upon what we mean by 'access,' and it may be not clear whether this would imply a stake-sensitivity of belief. Perhaps a belief is variably accessible in different contexts—so is present, perhaps to different extents, in some contexts yet not in others—and so it is context-sensitive. Or perhaps the different contexts elicit distinct beliefs, none of which is context-sensitive in the way we are considering. It may be hard to see a significant difference between those possibilities, but one reason for suspecting there is a difference is the familiar phenomenon of overcoming imperfect access, as when one remembers something with an effort. This typically seems less like belief acquisition or belief change and more like a renewed engagement with a persistent unchanged belief. So if the stakes can be the contextual element that matters in this way, then stake-sensitive accessibility might amount to stake-sensitive belief.

John provides dinner for his fellow poker players. There are no peanut products in it, he believes. But when he learns that one of the players is very allergic to peanuts, he's not so sure, the high stakes make him think twice. Our distinctions enable us to see some things that may be going on. Perhaps learning that a player is allergic makes John think twice about whether to *serve* the dinner; it influences his decision-making. Or it influences what he is willing to *say* about the dinner. Or it influences whether he *knows* the dinner is peanut-free (or whether others would judge that he knows.) In these cases the stakes are influential, but their influence need not be on *belief*.

Or perhaps after learning about the allergy John does not attend to precisely the same belief. His attention shifts from 'There aren't any significant peanut products in the dinner,' to 'The dinner is completely free of any tiny traces of peanuts.' Or perhaps the learning experience about the stakes provokes him to change his mind about the dinner: as often happens, learning provokes a revision in beliefs.<sup>7</sup> In these cases the stakes are influential, but their influence need not be on a *single persistent* belief.

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was treated in a similar way. In both accounts Kaplan contrasts acceptance/belief with degrees of confidence. Many other accounts make use of an idea of acceptance.

<sup>6</sup> For examples, Lewis (1996); Cohen (1999); Fantl & McGrath (2002). There is an extensive literature on the ways context influences our standards and judgments about justification and knowledge. The stakes can matter—directly, or by influencing the salience of possible error, or perhaps in other ways.

<sup>7</sup> As might happen when the stakes are raised by John's grocer Daniel, who offers a large wager that there are peanut products in the dinner.

This raises a difficulty. If beliefs are stake-sensitive, how can we tell? Consider two possibilities: a) The character of John's persistent belief about the dinner is affected and shifted by altered stakes; his persistent belief is stake-sensitive. b) John changes his mind from one belief to another in response to learning new information (about what is at stake); one belief is abandoned for another. A natural sort of evidence to seek when looking for stake-sensitivity is *responsiveness to changes* in what is at stake. But we find that in either of the two possibilities just mentioned. We change our minds often and for all sorts of reasons, and the responsiveness-to-altered-stakes approach to judging stake-sensitivity may always bring with it the question of whether there is a persistent stake-sensitive belief, or a change of mind.<sup>8</sup>

We may be stuck with this uncertainty about what responsiveness to altered stakes indicates. John's diachronic response to altered stakes may be evidence about his synchronic doxastic states, and that evidence may be ambiguous. The question of how learning the stakes may lead to diachronic belief change differs, though, from the question of whether the synchronic belief depends on what is at stake. The latter question is better indicated by the mixed-state metaphor used earlier; other ways of getting at it are: Is the extent (understood either categorically, or in degree) to which a belief-that- $p$  is held partly determined by a parameter that reflects the stakes that depend on  $p$ ? Is the extent to which  $p$  is believed *indeterminate* in the absence of information about the relevant stakes? May John, at  $t$ , simultaneously believe that  $p$ , and also not believe that  $p$ , depending on what's at stake on the truth of  $p$ ? (Or, may he be simultaneously disposed to believe that  $p$ , and also disposed otherwise, depending on what's at stake?) In order to accurately say what it is that John believes concerning  $p$ , do we need to specify what he takes to be at stake on  $p$ ? Is that a natural and fruitful way to model belief?

We typically do not learn about another person's beliefs without also learning something about his or her interests. But interests can be hidden, and they often are. There is no reason to think that we are always well informed about what another person takes to be at stake on the truth of each his beliefs. How strongly does that curtail our understanding of the beliefs of others? If belief is stake-sensitive, and we do not know what he takes the stakes to be, do we lose our understanding of what John believes? Are we in the dark about it more often than we would otherwise think?

## 2. Belief: models and norms

Any account of what beliefs are and what they do (or what we do with them) that we can actually construct and present will be an incomplete one; it will be a model, more or less successful, of some feature(s) of beliefs. This is true of both categorical models and degree-of-belief models of our doxastic states. The account of degrees-of-belief that derives from the work of Ramsey and de Finetti takes them to be judgmental weights in deliberation about actions. This

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<sup>8</sup> I take responsiveness-to-altered-stakes to be the believer's responsiveness to his revised assessment of stakes (not to their mere presence, of which he might be unaware); this includes such cases as when one gives little or no occurrent thought to the stakes and then suddenly understands them to be high. The sudden understanding might be taken to be a revision of a dispositional doxastic state that differed from the new belief that the stakes are high, provoking a change of mind.

is a model of *choice-guiding* degrees of belief (Ramsey called them belief *qua* basis for action).<sup>9</sup> Associated with this model of choice-guiding degrees of belief is a *betting model* of degrees of belief, according to which *S*'s degree of belief that *p* is indicated by the precise betting quotient corresponding to the least favorable rate at which he will bet on *p*.<sup>10</sup> This idealized betting model is wholly motivated by the idea that it sheds light on choice-guiding degrees of belief. (That is, on the denizens of the choice-guiding model of degrees of belief.) The models have much in common, but it is worth noticing that the models are different: the choice-guiding model reflects the more general idea that degrees of belief are influences on deliberations about choices of all sorts. The betting-model simplifies things by using a bet on a proposition to represent an action whose outcomes depend upon whether or not that proposition is true, and by making idealized assumptions about bets (for example, that their values lie entirely in their payoffs).

Neither the choice-guiding model nor the betting model of degrees of belief depicts all the features that we regard beliefs as having. This is so in multiple ways, and one of them is that, in some of their roles or guises, we treat beliefs as categorical rather than as coming in fine-grained degrees. The connection, tenuous or close, between categorical beliefs and degrees of belief is at least in part a matter of the connection, tenuous or close, between assertion and action. We will not thoroughly investigate that connection here. Such an investigation might seek to identify, distinguish, and count varieties of states that are belief-states, or that are in the neighborhood of belief-states. Are categorical beliefs and degrees of belief different aspects of a single kind of state, or are they different but related states? If the latter, are there other underlying states on which, *qua* belief, each depends and to which each contributes? Let us set these questions aside here, acknowledging that a choice-guiding model of degrees of belief neglects some categorical features apparent in belief's guidance of assertion, and that models of belief that attend only to its categorical features neglect some properties apparent in belief's influence on deliberation.

So is it a fruitful way of modeling belief-states to regard them as stake-sensitive, or not? In general, whether it is fruitful to model *x* as being *p* depends in part on the point of the model. A model of belief employed in logic or epistemology is one thing, a model used to develop techniques for manipulating voters and consumers is likely something else. A model used to develop artificial reasoners, voters, and consumers might differ from either. My interest is in treatments of belief that are appropriate and useful in epistemology and logic.

Epistemology and logic are normative subjects, and it is appropriate to say something about normativity here. The question, 'what is a fruitful way of modeling belief?' is not identical to the question, 'what is a fruitful way of modeling *rational* belief?' since fruitful ways of modeling beliefs might address beliefs both rational and irrational. And I am interested in how to model beliefs that, with respect to many norms of rationality, may be rational or not. Yet fruitful ways of modeling belief may employ constraints that have some normative force.

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<sup>9</sup> Ramsey (1926), p.67; from here on when I speak of choice-guiding degrees of belief, it is the Ramsey-de Finetti model that I have in mind.

<sup>10</sup> Here I focus on models of *precise* degrees of belief. About vague degrees of belief we can also raise questions of stake-sensitivity. Does what is at stake shift the range that characterizes a vague degree of belief? Does it make vague degrees of belief sharper or vaguer? These are models and questions that I set aside in this paper, except to suggest that the first sort of influence is at odds with usual ways of thinking of categorical belief in epistemology.

Epistemologists and logicians typically do not rely entirely on purely descriptive psychology, if there is such a thing, to distinguish states that are beliefs from states that are something else. So while I do not want to presuppose some rich account of rational belief and in effect narrow the question to whether *justified* beliefs (for example) are stake-sensitive, we can allow that a fruitful way of modeling belief may include some normative constraints on what it takes to have a belief. In asking about belief as treated in epistemology and logic, I am asking whether stake-sensitivity is compatible with assumptions of those treatments, some of which may be normative assumptions.

### 3. Stake-sensitivity neglected.

Let us begin with categorical belief. *Are* categorical beliefs modeled as being stake-sensitive? Epistemologists tend to dwell on the context-sensitivity of justification and knowledge, but have no trouble recognizing the various sorts of context-sensitivities listed in section 1. But it is hard to find stake-sensitive beliefs treated in work in epistemology. In work on reasoning and AI the idea that belief stake-sensitive may be more prevalent; there are at least suggestions of a need for such models (Thomason 1986, 2007). *Should* beliefs be modeled as being (sometimes) stake-sensitive? That surely depends on the aims of the modeler, and on the chances that such a model would be fruitful. In this section I will briefly explore ideas that may underlie philosophical disinterest in stake-sensitive belief, and then by way of contrast mention Richmond Thomason's recommendations that belief models allowing stake-sensitivity, among other things, are worth developing.<sup>11</sup>

One line of thought builds on the idea that the point of believing that  $p$  has something to do with  $p$ 's truth. Truth is the condition of *correctness* for (categorical) belief (Shah 2003, Gibbard 2005), something belief shares with other states (Stalnaker 1984). Belief *aims* at the truth (Velleman 2000, Wedgwood 2002). Now truth's being the condition of correctness does not, in itself, preclude a sensitivity of belief to stakes; many beliefs are false, and are no less beliefs for that. And false beliefs may be reasonable by any standards, including epistemic ones. If a tension can be found between truth and stake-sensitive belief, it will instead depend on taking truth to be the aim of belief. If truth is our aim, and for epistemic purposes perhaps the sole aim of belief, what follows concerning stake-sensitivity? Suppose a stake-sensitive belief is a doxastic commitment of mixed character, resolvable in a variety of ways in the presence of different stakes. Is its pursuit of truth thereby undermined, is it hampered to an extent that disqualifies the commitment as belief? The doxastic state carries capacities to produce definite beliefs achieving scattered degrees of success. Some of the resolutions would yield beliefs that fall farther from their aim than others do. But equally, some of the resolutions would yield beliefs that come nearer to achieving their aim than others do, so it is hard to see that the problem, if there is one, lies here.

Here is a more likely source of qualms: Stake-sensitive belief is *unstable* in its pursuit of the aim of truth, and the instability is induced by pragmatic influences, the stakes. What is troubling is not the variety of potential resolutions, it is the pragmatic influence that produces

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<sup>11</sup> In Armendt (2008) several examples of relevant views expressed in recent epistemological work are briefly raised and discussed. Though interesting, they do not amount to endorsements of epistemic accounts involving stake-sensitive belief.



them. Pragmatic influences of this kind arise from sources that lack epistemic legitimacy (e.g. they are not evidential; they are not truth-tracking). This line of thought discounts stake-sensitive beliefs on grounds of (epistemic) irrationality, traceable to an influence regarded as beyond the epistemic pale.<sup>12</sup> The crucial assumption is that this pragmatically-induced instability in pursuit of the truth is a disqualifier for belief-states. Is that right? Is belief itself among the states that are incompatible with it?

Another line of thought notices that belief is a member of a family of states, each of which answers to truth as a standard of correctness: pretending, imagining, accepting for the sake of argument, accepting for the sake of making plans. These are all valuable things we do, and among them belief is special. The members of this family are states that exhibit varying degrees of compartmentalization (Stalnaker 1984, 1999; Bratman 1992, Velleman 2000). Belief is a state that engages our reasoning and deliberation in all contexts, or at least a wide range of contexts; it is a more general, less compartmentalized commitment than the others. Its presence across contexts is not just a matter of exerting some influence or other on reasoning and action in different contexts, it involves a constancy of such influence. This is a conception that leaves little room for stake-sensitivity.

“Reasonable belief is, in an important way, context independent: at any one time a reasonable agent normally either believes something (to degree  $n$ ) or does not believe it (to that degree). She does not at the same time believe that  $p$  relative to one context but not relative to another. ...

An agent’s beliefs are subject to an ideal of integration. Other things equal one should be able to agglomerate one’s beliefs into a larger, overall view; and this larger view should satisfy demands for consistency and coherence.” (Michael Bratman 1992)

Robert Stalnaker (1999) discussed two ways of thinking about belief that are in tension. *Available belief* is appropriately calibrated to motivational states that determine how the information is to be used. However,

“We think of our ordinary concepts of knowledge and belief as concepts of available knowledge and belief... but *we are also inclined to think of knowledge and belief as states that at least ought to be independent of what we want or are inclined to try to get.* ... Rational creatures, or cognitive beings, are organisms or machines that are capable of making their behavior sensitive to facts about their environments. Our folk theory, as well as our attempts at more scientific theories of cognition and rationality, assume, in their explanations of this capacity, that cognitive beings have two independent but interacting components—one that is a representation of the way the world is...and another that represents the goals, purposes, ends, desires that its actions are aiming to achieve or satisfy....*Our belief states are supposed to be generalized capacities that would serve our ends whatever they happened to be.*” (p. 270, my emphasis)

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<sup>12</sup> And it applies equally to Ramsey’s degrees of belief.

These remarks occur in a discussion of the problem of logical omniscience, and Stalnaker's larger point is to suggest that we need more subtle ways of thinking about these two sides of doxastic states. In any case, my suggestion is that the quoted remarks express a standard approach in epistemology, as well as in our folk theory.

Prevalent though that standard approach may be in epistemology and other areas, other views can be found. By way of contrast, in work more closely aligned with research on reasoning and AI than with traditional epistemology, Richmond Thomason writes:

“There are occasions when we can't act without a belief, and in which high standards for belief prevent us from having an appropriate belief. In these cases, an urgent need to act can cause us to lower our standards. ... There are occasions when we have a belief that is well justified, but the consequences of acting on this belief if we are wrong are very harmful. In these cases, we can destroy the belief by changing our standards. ...

We can model these effects by supposing that beliefs are local. Rather than appealing to a global, monolithic attitude, we construct belief-like attitudes for the occasion at hand out of a large stock of potential beliefs that can be combined much as we might select and combine propositional axioms for some ad hoc purpose. Potential beliefs come with features indicating their provenance and, for instance, the circumstances under which we learned them, and they are sorted according to their entrenchment or plausibility. When we combine potential beliefs into a modality that will guide our actions in a given situation, we can manipulate the beliefs by filtering out less plausible proto-beliefs *in the presence of risk*, or allowing them in when it is urgent to have a belief of some sort.” (Thomason 2007, section 3, my emphasis)

In earlier papers, Thomason sketches a framework on which to build the sort of model he has in mind (1986, 2000). The belief states and associated mechanisms are complex; the account employs modular sub-agents and relations of limited accessibility between belief-holding modules, to model an individual's beliefs and the roles of belief in social interactions. Beliefs that are so modeled may be highly sensitive to context, or at least their influences may be. To the extent that such accounts dwell on the complex ways in which highly specific beliefs are *activated* in different contexts (as opposed to being *characterized* by contextual features), this is not quite context-sensitivity as I described it. And to the extent that the activations are sensitive to *epistemic* stimuli, the source of the context-sensitivity is generally different from the sort of stake-sensitivity about which I am inquiring. Nevertheless, Thomason clearly seeks to model belief as, among other things, stake-sensitive: “Part of the context, then, that sustains a belief is the current assessment of risk that attaches to acting on that belief; and by reassessing risk, we are able to undermine beliefs by increasing our threshold of credulity” (1986). Thomason's approach is noteworthy for its potential to yield rich, complex models of our doxastic states, should we be able to fill in the framework. It is also noteworthy in its departure from much more common accounts of belief used in epistemology and other philosophical work.

Let us return to the Ramsey-de Finetti model of choice-guiding degrees of belief and its associated betting model. As the opening quotation from Ramsey indicates, they take a clear stand on whether stakes influence belief, and the answer is negative: degrees of belief are stake-

invariant. The idea is that at a given time an agent has *a* degree of belief that *p*, not lots of them that depend on the stakes. There are other respects in which this account of belief differs from treatments of categorical belief implicit or explicit in epistemology, but on the idea of stake-invariance the two approaches are in agreement.

#### 4. Motivating probabilism.

Ramsey-de Finetti degrees of belief have a logic, and that logic is, at least in part, probability. Preference-theoretic representation arguments, Dutch book arguments, and error-minimization arguments can be offered in support of that claim. The stake-invariance of degrees of belief is important to each of these arguments for probabilism. Here I will focus on a pair of them—the standard synchronic Dutch Book argument (*DBA*) that rational degrees of belief should satisfy the rules of finitely additive probability, and more briefly, the similar synchronic argument for countably additive probability.

In philosophical literature and elsewhere most critiques of the *DBA* are directed toward extraneous matters rather than toward the heart of the argument. This is old news. Brian Skyrms knew it a long time ago, and pointed to a better way of understanding the argument and what is extraneous (Skyrms 1980, 1984). It's a way suggested by Ramsey, and based on the ideas that the key issue is consistency, that the story of a cunning better who imposes a sure loss is dramatic window-dressing.

But not all critiques miss the heart of the argument (Hajek 2008). Some that matter are directed toward the *package principle*, as it's come to be called, concerning the additivity of the values of bets. Yet even among objections to the package principle are critiques that miss the epistemic point of the *DBA*. This is something we will see in the next section, when we consider the implications of stake-invariance for objections to the package principle.

The *DBA* defends probabilism by showing that if a system of degrees of belief violates any one of a set of sufficient conditions for probabilism, the holder of the system of beliefs is vulnerable to a Dutch Book, a collection of bets based on the believer's own degrees of belief such that every possible outcome yields a net loss to the believer. Vulnerability to a Dutch Book is a flawed state of affairs, and the converse *DBA* shows that it is avoidable when the system of beliefs conforms to probabilism; that is, when the quantitative degrees of belief are probabilities. The addition rule usually gets the most attention,<sup>13</sup> and the idea used in the argument is this: a violation of the additivity of degrees of belief is an indication that the believer exhibits an inconsistency in assessing his options. As Ramsey expresses it, his choices would depend upon the forms in which the options are presented, which, Ramsey goes on to say, would be absurd. The plausibility of this judgment comes from the nature of the additivity violation: Depending on the direction of the violation, the agent is willing to pay more (less) for a pair of bets on *p* and on *q*, when they are incompatible, than he is willing to pay for a bet on the disjunction  $p \vee q$ . (Recall that in the betting model, degrees of belief are indicated by the odds at which one would

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<sup>13</sup> When *p*, *q* are incompatible,  $dob(p) + dob(q) = dob(p \vee q)$ . The *dobs* represent what the believer would pay for a bet that pays \$1. If the rule is violated, the cunning bettor makes money from the believer by selling the high side to him and buying back the low side.

bet, or by the price one would pay for a bet-ticket that pays a fixed amount.) Since holding the pair of bets is exactly equivalent to holding a bet on the disjunction, there is an inconsistency in the agent's evaluations (and it can be argued that this is traceable to an inconsistency in his beliefs' influence in deliberation).

But hold on, says the critic. An important and unargued assumption about the sum of the values of the bets on  $p$  and on  $q$  is being made. The violation of additivity occurs when that sum differs from the value of the bet on the disjunction. But the holdings that are said to be equivalent are on one hand, a bet on a disjunction, and on the other, a pair of bets on  $p$  and on  $q$ . Is the value of the pair of bets held together the same as the sum of the values of the individual bets that might each be held alone? This is the question of the value-additivity of bets; the required assumption has been dubbed the *package principle*. If it is legitimate, then it remains plausible that Dutch Book vulnerability (in this case, at least) is tied to inconsistency, but if not, there is a gap in the argument.

So what's wrong with the package principle? Well, all sorts of things readily come to mind. The principle says that when added together the separate values of two things held individually is the same as the value of the two things held together. When it comes to valuing things, it is easy to think of situations where this is not so. The values I judge the individuals and the pair to have (think of my willingness to offer value in exchange) may be affected by the size of my fortune or bankroll. Whatever are the goods in which the bets pay off, they may exhibit significant declining marginal value. The two things may be complementary goods (a hen and a rooster), or substitution goods (a coat and a sweater). These complications are all familiar to utility theorists, and they are hardly dismissible as entirely irrational.

Further issues arise concerning the *DBA* for countably additive probability: Should we expect a countably infinite package (book of bets) to obey a countable package principle? In general, sums of infinitely many values may not exist, or when they do, they may violate such a principle (Barrett & Arntzenius 1999), (Arntzenius, Elga & Hawthorne 2004). Arntzenius, Elga & Hawthorne say that a *DBA* that relies on a countable package principle relies on a fallacy, that no *DBA* for countable probability can be sound. Such an argument would rely on the

*Bet Agglomeration Principle:*

Suppose an infinite package of bets. If you judge each bet as favorable, you are committed to judging the whole package as favorable.

Yet infinite sums of positive values need not conform to such a principle, as is illustrated by

*Satan's Apple:*

A delicious apple cut into a countable infinity of pieces. Eve may take and eat whichever pieces she chooses. No penalty if she takes finitely many; expulsion from the Garden if she eats an infinite number. Her highest priority is to avoid expulsion.

There is no point at which Eve has an incentive to stop eating pieces of the apple, yet when she has eaten them all, or an infinite subset of them, the unfavorable outcome worst for her is the result.

Notice that the point Arntzenius, Elga & Hawthorne make is generic; it is simply that additivity may fail for some countable sets of quantities of value. A *DBA* against which this is a criticism must be an argument that incorrectly assumes otherwise.

In the next section, I suggest that objections to the package principle are of two kinds. One attends to generic interactions among quantities of value and notices that additivity may be violated by such interactions. I further suggest that these will not amount to genuine objections to the additivity of stake-invariant degrees of belief. But potential objections of a second kind remain, and they are not defused just by appeal to the stake-invariance of belief.

## 5. Stake-invariance and the package principle.

Most of the complications just raised arise from interactions among quantities of value; call objections to the package principle that are based on them *generic value-interaction* objections. There may be other sources of doubts about the additivity of degrees of belief, but these are prevalent in discussions of the package principle. And here is where stake-invariance bears some relevance. Generic value-interaction reasons for dissenting from the package principle appear to equally be reasons for dissenting from the stake-invariance of belief. If the quantities of value attached to a wager on  $p$  and to a wager on  $q$  interact with each other, or interact with other circumstances such as the size of my fortune, then why not quantities of value that are attached to a wager on  $p$  and to another wager on  $p$ ? But the combination of a wager on  $p$  and another wager on  $p$  just is a wager on  $p$  at enhanced stakes. And a countable book of wagers all on  $p$  (at diminishing stakes to keep the sum finite) is a wager on  $p$  at enhanced stakes. If the value of a wager at enhanced stakes is not proportional to the enhancement of the stakes, there is no single set of odds at which the wagers are made, and no single degree of belief in  $p$ . Stake-invariance of belief has been lost.

So a commitment to stake-invariant belief amounts to a commitment that generic value-interaction complications are excluded. The choice-guiding model of belief takes partial beliefs to be insensitive to the effects of interactions among quantities of value. This is not a rejection of the idea that our *deliberations* can be influenced by such interactions. It is a constraint on *beliefs*, built into a model that focuses on the role of belief in deliberation and choice. Is it a reasonable constraint? It is at least a common one, judging from our discussion of categorical belief.

If this is right, then one implication is that defenses of probabilism that rely on the stake-invariance of belief are thereby leaning on a widely accepted principle concerning belief itself (and on a principle even more widely accepted concerning rational belief). One of those defenses is the Dutch Book argument. When we understand that the argument is used in connection with a model of stake-invariant belief, we are in a position to see that some objections to it, the ones I have labeled generic value-interaction objections, can be defused. The objections focus on deliberative phenomena that the model, incorporating a standard presupposition about belief, discounts. Though the phenomena may not be unusual, if they occur they are treated by the model as arising from sources outside the doxastic inputs to deliberation, that is from sources other than belief.

Which specific objections that were mentioned above can be dismissed in this way? The generic value-interaction objections include those based on the interaction between the values of bets and the size of my bankroll, those based on a declining marginal value of goods in which the

bets pay off, and those based on the general failure of additivity for infinite sets of values. An objection to additivity that is not so readily dismissed is based on the possibility that bets on  $p$  and on  $q$  are complementary or supplementary goods; there is a disanalogy between those bets and a duplication of bets on  $p$ . When added to a bet on  $p$ , a bet on  $q$  increases the prospect of gain, compared to an additional bet on  $p$ , which increases the potential reward. It is not an obvious principle of rational deliberation that linearity of value with potential reward guarantees linearity of value with the prospect of gain. And this applies to the infinite case as well; the countable additivity of values of iterated bets on  $p$  does not in itself guarantee the countable additivity of other books.

So a commitment to the stake-invariance of belief still leaves room for objection to the package principle, as used in a *DBA*. And it makes relevant questions such as: In the context of an account of action-guiding belief, are there good (epistemic) reasons for doubting that the value of bets is linear with respect to the prospect or chance that they pay off? Are there better models of rational degrees of belief in which that idea is rejected? These questions and others I leave to another occasion.

In the quotation that opens this paper, Ramsey took stake-invariance to be a matter of consistency for degrees of belief. What did he mean by that? The quoted remark is one of a pair in which Ramsey mentioned consistency; he made them after he had presented his account of degrees of belief, and after he both mentioned the idea of the *DBA* and asserted that probability is the logic of consistency for partial belief. The first, quoted, remark is about what I have labeled stake-invariance. The second remark both asserts the idea of the converse *DBA* and links invulnerability to a *DB* to ‘a further measure of consistency’ that we usually call *coherence*. As I read them, in this pair of remarks Ramsey contrasted one kind of consistency that is *assumed* by his treatment (stake-invariance) with another kind that is an *outcome* of it (coherence).<sup>14</sup>

The assertions that stake-invariance and coherence qualify as kinds of consistency are both interesting. Ramsey made them in the context of his project to establish probability as the logic for partial belief, a project in which he was clearly willing to entertain conceptions of logic and consistency that extend beyond the setting of formal deductive logic. If stake-invariance is a kind of consistency, it seems first and foremost a consistency of practice, a consistency in decision-making. A single, stable betting quotient for  $p$  marks the boundary between bets on  $p$  that are acceptable and bets that are not, whatever their size. But when, as in Ramsey’s account, a belief’s strength is found in its influence on deliberation, it becomes natural to regard consistency in decision-making as reflecting consistency in degree of belief. Natural, that is, in

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<sup>14</sup> “We find, therefore, that a precise account of the nature of partial belief reveals that the laws of probability are laws of consistency, an extension to partial beliefs of formal logic, the logic of consistency. They do not depend for their meaning on any degree of belief in a proposition being uniquely determined as the rational one; they merely distinguish those sets of beliefs which obey them as consistent ones.

Having any definite degree of belief implies a certain measure of consistency, namely willingness to bet on a given proposition at the same odds for any stake, the stakes being measured in terms of ultimate values. Having degrees of belief obeying the laws of probability implies a further measure of consistency, namely such a consistency between the odds acceptable on different propositions as shall prevent a book being made against you.” (Ramsey 1926, pp. 78-79.)

the presence of assumptions or modeling constraints such as we have considered above. Suppose that Ramsey was in effect asserting that it is a consistency constraint on choice-guiding degrees of belief as his account develops them that they are stake-invariant. What defense did he offer? Ramsey had more to say about the logic of partial belief later in his paper, but I think his defense of the idea that stake-invariance is consistency is an implicit one, based on an idea of consistent practice, on the interpretation of strength of belief as influence in deliberation, and on the fruits of the belief model that he developed.

## 6. Conclusion.

I asked whether it makes sense to treat belief as stake-sensitive, having a particular sort of stake-sensitivity in mind. The question arises both for categorical belief and for degrees of belief. Treatments of categorical belief as stake-sensitive are rare in epistemology, perhaps less so in other work. The stake-invariance of belief is built into the choice-guiding model of degrees of belief that comes to us from Ramsey and de Finetti. Some commonly offered attacks on the package principle, and on Dutch Book arguments that employ it to motivate probabilism, rely on generic value-interaction phenomena that are ruled out by a commitment to stake-invariance. Other objections remain. Is an appeal to stake-invariance well-motivated, or is it just an *ad hoc* maneuver? The comparison with categorical belief indicates at least that the appeal is not *ad hoc*. No knock-down argument for stake-invariance has been offered, and fruitful models of stake-sensitive belief may be developed. But it is safe to say that when accounts of choice-guiding degrees of belief treat beliefs as stake-invariant, as they typically do, the accounts do not thereby become outliers in present-day epistemology.

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