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Expected discounted Penalty Function for a Thinning Risk Model *

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Abstract

In this paper, we consider a risk model in which the premium arriving number process is a Poisson process with parameter $\lambda > 0$ while the claim number process is the thinning of the premium arriving number process. Under such a model, we obtain the integral equation, the integro-differential equation and the recursive formula for the expected discounted penalty function. Using the integro-differential equation we get the closed form expressions for the Laplace transform of the time of ruin and the deficit at ruin when the premium and the claim sizes are exponentially distributed.

Keywords: Thinning, Poisson process, expected discounted penalty function.

AMS Subject Classification: 91B30.

§1. Introduction

Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a complete probability space, all random variables in this paper are defined on this space. Boikov[1] introduced a risk model given by

$$U_t^0 = u + \sum_{i=1}^{N(t)} X_i - \sum_{i=1}^{N_1(t)} Y_i, \qquad t \ge 0, \tag{1.1}$$

where $u \ge 0$ is the initial capital of an insurer; $\{N(t), t \ge 0\}$ is a Poisson process with parameter λ>0 and $N(t)$ is the total number of the contracts sold by insurer in $(0, t]$; $\{X_i, i \ge 1\}$ is a sequence of independent and identically distributed (i.i.d.) nonnegative random variables with a common distribution function $G(x)$, $G(0) = 0$, which is independent of $N(t)$; X_i indicates the i-th premium size; $\{N_1(t), t \ge 0\}$ is a Poisson process with parameter $\lambda_1 > 0$, which is the number of claims in $(0, t]$; let $\{Y_i, i \ge 1\}$ is a nonnegative sequence of i.i.d. random variables with common distribution function $F(y)$, $F(0) = 0$,

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which is independent of $N_1(t)$; Y_i denotes the amount of the i-th claim; U_t^0 is the surplus of the insurer at time t. In (1.1), it is assumed that $\sum\limits_{i}^{N(t)} X_i$ and $\sum\limits_{i}^{N_1(t)} Y_i$ are independent.

We now generalize risk process (1.1) as follows:

$$U_t = u + \sum_{i=1}^{N_t} X_i - \sum_{i=1}^{N_t^p} Y_i, \qquad t \geq 0, \tag{1.2}$$

where N_t^p is the P-thinning process of N_t, i.e. N_t^p is a Poisson process with parameter $\lambda p > 0$, $0 < p < 1$ (see Chen[2]). Note that we don't distinguish between the symbols such as $N(t)$ and N_t. Let

$$S_t^p = \sum_{i=1}^{N_t} X_i - \sum_{i=1}^{N_t^p} Y_i.$$

The net profit condition is $\mathsf{E}S_t^p = (\lambda \mathsf{E}X_i - \lambda p \mathsf{E}Y_i)t > 0$, or, equivalently, $\mathsf{E}X_i > p\mathsf{E}Y_i$.

The time of ruin T is defined as $T = \inf\{t : U_t < 0\}$ with $\inf\{\emptyset\} = \infty$. The expected discounted penalty function under risk process (1.2) is:

$$\begin{aligned} \phi_\delta(u) &= \mathsf{E}[w(U_{T-}, |U_T|)\mathrm{e}^{-\delta T} I(T < \infty)|U_0 = u] \\ &\triangleq \mathsf{E}^u[w(U_{T-}, |U_T|)\mathrm{e}^{-\delta T} I(T < \infty)], \end{aligned} \tag{1.3}$$

where $w(x, y)$, $x \geq 0$, $y \geq 0$ is a nonnegative function such that $\phi_\delta(u)$ exist; $\delta \geq 0$; and $I(C)$ is the indicator function of a set C.

Boikov[1] derived the integral equations and exponential bounds of survival probability for risk process (1.1). Furthermore, Yao and Xu[3] obtained the integral equation for the expected discounted penalty function.

In Section 2, we use the strong Markov property of U_t to derive the integral equation and the recursive formula for the expected discounted penalty function. In Section 3, we obtain the explicit expressions for the Laplace transform of the time of ruin, the deficit at ruin under some certain assumptions. Finally, we give the numerical analysis for ruin probability.

§2. Integral and Recursive Equations

It is easy to see that the risk process (1.2) has stationary and independent increments and that it is right continuous with left limit. Thus, it is a Lévy process with the strong Markov property.

Theorem 2.1 Let $u \geq 0$, the $\phi_\delta(u)$ satisfies the integral equation

$$\begin{aligned}(\lambda+\delta)\phi_\delta(u) &= (1-p)\lambda\int_0^\infty \phi_\delta(u+x)\mathrm{d}G(x) \\ &\quad +p\lambda\Big[\int_0^\infty\int_0^{u+x}\phi_\delta(u+x-y)\mathrm{d}F(y)\mathrm{d}G(x) \\ &\quad +\int_0^\infty\int_{u+x}^\infty w(u,y-u-x)\mathrm{d}F(y)\mathrm{d}G(x)\Big]. \qquad (2.1)\end{aligned}$$

Proof Let T_1 denotes the first jump time of N_t. Then T_1 is an exponential random variable with parameter λ. Thus, $\mathsf{P}(T_1<\infty)=1$. Since $U_t \geq 0$ for $0\leq t<T_1$, ruin does not occur before time T_1. Using the strong Markov property of U_t, we get

$$\phi_\delta(u)=\mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})]. \qquad (2.2)$$

We can decompose (2.2) into two parts:

$$\begin{aligned}\phi_\delta(u) &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N_{T_1}^p=0)]+\mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N_{T_1}^p=1)] \\ &\triangleq E_1+E_2.\end{aligned}$$

Using the independent assumption, we get

$$\begin{aligned}E_1 &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N_{T_1}^p=0)]=\mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(u+X_1)I(N_{T_1}^p=0)] \\ &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N_{T_1}^p=0)]\mathsf{E}[\phi_\delta(u+X_1)].\end{aligned}$$

Since N_t^p is the P-thinning of N_t, we have

$$\begin{aligned}\mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N_{T_1}^p=0)] &= \int_0^\infty \lambda\mathrm{e}^{-\lambda t}\mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N_{T_1}^p=0)|T_1=t]\mathrm{d}t \\ &= \int_0^\infty \lambda\mathrm{e}^{-(\lambda+\delta)t}\mathsf{P}(N_{T_1}^p=0|T_1=t)\mathrm{d}t \\ &= \lambda(1-p)\int_0^\infty \mathrm{e}^{-(\lambda+\delta)t}\mathrm{d}t=\frac{\lambda(1-p)}{\lambda+\delta}. \qquad (2.3)\end{aligned}$$

Thus, we have

$$E_1=\frac{\lambda(1-p)}{\lambda+\delta}\int_0^\infty\phi_\delta(u+x)\mathrm{d}G(x). \qquad (2.4)$$

Let $u\geq 0$. Assume that $u+X_1-Y_1<0$, that is $U_{T_1}<0$, from the strong Markov property of U_t, one concludes that

$$\phi_\delta(U_{T_1})=\mathsf{E}^{U_{T_1}}[w(U_{T-},|U_T|)\mathrm{e}^{-\delta T}I(T<\infty)]=w(u,Y_1-u-X_1).$$

It follows that

$$\begin{aligned} & \mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N^p_{T_1}=1)I(u+X_1-Y_1<0)] \\ = & \mathsf{E}[\mathrm{e}^{-\delta T_1}I(N^p_{T_1}=1)I(u+X_1-Y_1<0)w(u,Y_1-u-X_1)]. \end{aligned}$$

Thus, from the assumption of independence, we get

$$\begin{aligned} E_2 &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N^p_{T_1}=1)I(u+X_1-Y_1\geq 0)] \\ &\quad +\mathsf{E}^u[\mathrm{e}^{-\delta T_1}\phi_\delta(U_{T_1})I(N^p_{T_1}=1)I(u+X_1-Y_1<0)] \\ &= \mathsf{E}[\mathrm{e}^{-\delta T_1}I(N^p_{T_1}=1)][\mathsf{E}(\phi_\delta(u+X_1-Y_1)I(u+X_1-Y_1\geq 0)) \\ &\quad +\mathsf{E}(w(u,Y_1-u-X_1)I(u+X_1-Y_1<0))]. \end{aligned} \tag{2.5}$$

Note that $\mathsf{P}(N^p_{T_1}=1|T_1=t)=\mathsf{P}(N^p_t=1|T_1=t)=p$, similar to (2.3), we obtain

$$\mathsf{E}[\mathrm{e}^{-\delta T_1}I(N^p_{T_1}=1)]=\frac{\lambda p}{\lambda+\delta}. \tag{2.6}$$

Thus from (2.5) we get

$$\begin{aligned} E_2 &= \frac{\lambda p}{\lambda+\delta}\Big[\int_0^\infty \mathrm{d}G(x)\int_0^{u+x}\phi_\delta(u+x-y)\mathrm{d}F(y) \\ &\quad +\int_0^\infty \mathrm{d}G(x)\int_{u+x}^\infty w(u,y-u-x)\mathrm{d}F(y)\Big]. \end{aligned} \tag{2.7}$$

from (2.4) and (2.7) we obtain the integral equation (2.1). □

Let T_n denotes the n-th time of receiving premium, define

$$\phi_{\delta,n}(u)=\mathsf{E}^u[\mathrm{e}^{-\delta T}w(U_{T^-},|U_T|)I(T=T_n)].$$

Following Wu et al.[4] and Yuen et al.[5], we now derive recursive integral equations for $\phi_{\delta,n}(u)$.

Theorem 2.2 For $n=1,2,\cdots$, and $u\geq 0$,

$$\phi_{\delta,1}(u) = \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_{u+x}^\infty w(u,y-u-x)\mathrm{d}F(y)\mathrm{d}G(x), \tag{2.8}$$

$$\begin{aligned} \phi_{\delta,n}(u) &= \frac{\lambda(1-p)}{\lambda+\delta}\int_0^\infty \phi_{\delta,n-1}(u+x)\mathrm{d}G(x) \\ &\quad +\frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_0^{u+x}\phi_{\delta,n-1}(u+x-y)\mathrm{d}F(y)\mathrm{d}G(x), \qquad n=2,3,\cdots. \end{aligned} \tag{2.9}$$

Proof Since $\{T=T_n\}=\{U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0\}$, and ruin can occur only at claim times, we have

$$\begin{aligned}\phi_{\delta,1}(u) &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}w(U_{T_1^-},|U_{T_1}|)I(U_{T_1^-}\geq 0,U_{T_1}<0)]\\ &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}w(U_{T_1^-},|U_{T_1}|)I(U_{T_1^-}\geq 0,U_{T_1}<0)I(N^p_{T_1}=1)]\\ &= \mathsf{E}^u[\mathrm{e}^{-\delta T_1}w(u,|u+X_1-Y_1|)I(U_{T_1^-}\geq 0,U_{T_1}<0)I(N^p_{T_1}=1)].\end{aligned}$$

Using the independent assumption, we conclude

$$\phi_{\delta,1}(u)=\int_0^\infty\int_{u+x}^\infty \mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N^p_{T_1}=1)]w(u,y-u-x)\mathrm{d}F(y)\mathrm{d}G(x).$$

From this, together with (2.6), we obtain (2.8).

For $n\geq 2$, we get

$$\phi_{\delta,n}(u)=\mathsf{E}^u[\mathrm{e}^{-\delta T_n}w(U_{T_n^-},|U_{T_n}|)I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)]. \tag{2.10}$$

Similar to theorem 2.1, we decompose (2.10) into two parts

$$\begin{aligned}\phi_{\delta,n}(u) &= \mathsf{E}^u[\mathrm{e}^{-\delta T_n}w(U_{T_n^-},|U_{T_n}|)\\ &\quad\times I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)I(N^p_{T_1}=0)]\\ &\quad+\mathsf{E}^u[\mathrm{e}^{-\delta T_n}w(U_{T_n^-},|U_{T_n}|)\\ &\quad\times I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)I(N^p_{T_1}=1)]\\ &\triangleq E_3+E_4.\end{aligned} \tag{2.11}$$

We may rewrite E_3 as

$$\begin{aligned}E_3 &= \int_0^\infty \mathsf{E}^u[\mathrm{e}^{-\delta T_n}w(U_{T_n^-},|U_{T_n}|)I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)\\ &\quad\times I(N^p_{T_1}=0)|X_1=x]\mathrm{d}G(x).\end{aligned} \tag{2.12}$$

Using the strong Markov property of U_t, the independent assumption and (2.3), we conclude that the expectation in the integrand on the right-hand side of (2.12) is

$$\begin{aligned}&\mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N^p_{T_1}=0)]\mathsf{E}^u[\mathrm{e}^{-\delta(T_n-T_1)}w(U_{T_n^-},|U_{T_n}|)\\ &\times I(U_{T_2^-}>0,U_{T_2}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)|X_1=x]\mathrm{d}G(x)\\ =\ &\frac{\lambda(1-p)}{\lambda+\delta}\mathsf{E}^{u+x}[\mathrm{e}^{-\delta T_{n-1}}w(U_{T_{n-1}^-},|U_{T_{n-1}}|)\\ &\times I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_{n-1}^-}>0,U_{T_{n-1}}<0)]\mathrm{d}G(x)\\ =\ &\frac{\lambda(1-p)}{\lambda+\delta}\phi_{\delta,n-1}(u+x)\mathrm{d}G(x).\end{aligned} \tag{2.13}$$

Similar to (2.13) and using (2.6), we get

$$\begin{aligned} E_4 &= \int_0^\infty\int_0^{u+x} \mathsf{E}^u[\mathrm{e}^{-\delta T_n} w(U_{T_n^-},|U_{T_n}|)I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0) \\ &\quad \times I(N_{T_1}^p=1)|X_1=x,Y_1=y]\mathrm{d}F(y)\mathrm{d}G(x) \\ &= \int_0^\infty\int_0^{u+x} \mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N_{T_1}^p=1)]\mathsf{E}^u[\mathrm{e}^{-\delta(T_n-T_1)}w(U_{T_n^-},|U_{T_n}|) \\ &\quad \times I(U_{T_2^-}>0,U_{T_2}>0,\cdots,U_{T_n^-}>0,U_{T_n}<0)|X_1=x,Y_1=y]\mathrm{d}F(y)\mathrm{d}G(x) \\ &= \int_0^\infty\int_0^{u+x} \mathsf{E}^u[\mathrm{e}^{-\delta T_1}I(N_{T_1}^p=1)]\mathsf{E}^{u+x-y}[\mathrm{e}^{-\delta T_{n-1}}w(U_{T_{n-1}^-},|U_{T_{n-1}}|) \\ &\quad \times I(U_{T_1^-}>0,U_{T_1}>0,\cdots,U_{T_{n-1}^-}>0,U_{T_{n-1}}<0)]\mathrm{d}F(y)\mathrm{d}G(x) \\ &= \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_0^{u+x}\phi_{\delta,n-1}(u+x-y)\mathrm{d}F(y)\mathrm{d}G(x). \end{aligned} \tag{2.14}$$

Therefore, from (2.11)–(2.14),we conclude (2.9). □

From the definition of $\phi_{\delta,n}(u)$, $n=1,2,\cdots$, we get (see Wu et al.[4])

$$\begin{aligned} \phi_\delta(u) &= \sum_{n=1}^\infty \phi_{\delta,n}(u) = \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_{u+x}^\infty w(u,y-u-x)\mathrm{d}F(y)\mathrm{d}G(x) \\ &\quad + \frac{\lambda(1-p)}{\lambda+\delta}\int_0^\infty \sum_{n=2}^\infty \phi_{\delta,n-1}(u+x)\mathrm{d}G(x) \\ &\quad + \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_0^{u+x}\sum_{n=2}^\infty \phi_{\delta,n-1}(u+x-y)\mathrm{d}F(y)\mathrm{d}G(x) \\ &= \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_{u+x}^\infty w(u,y-u-x)\mathrm{d}F(y)\mathrm{d}G(x) \\ &\quad + \frac{\lambda(1-p)}{\lambda+\delta}\int_0^\infty \phi_\delta(u+x)\mathrm{d}G(x) \\ &\quad + \frac{\lambda p}{\lambda+\delta}\int_0^\infty\int_0^{u+x}\phi_\delta(u+x-y)\mathrm{d}F(y)\mathrm{d}G(x), \end{aligned} \tag{2.15}$$

that is (2.1) in theorem 2.1.

The equation (2.15) gives us an recursive method to calculate $\phi_\delta(u)$.

§3. Some Closed form Expressions

In this section, we give explicit expressions for some ruin quantities when the premium and the claim sizes are exponentially distributed.

It is easy to verify the following theorem 3.1.

Theorem 3.1 Let $G(x)=1-\mathrm{e}^{-ax}$, $F(x)=1-\mathrm{e}^{-by}$, and $p<b/a$, $x,y,a,b>0$, $w(x,y)$ has twice continuously partial derivative in $x>0$, $\int_0^\infty w'_x(x,y)\mathrm{d}y$ and $\int_0^\infty w''_{xx}(x,y)\mathrm{d}y$ are uniformly convergent, then $\phi_\delta(u)$ is satisfies the integro-differential equation

$$\begin{aligned}&(\lambda+\delta)\phi''_\delta(u)+(b\lambda+b\delta-a\delta-ap\lambda)\phi'_\delta(u)-ab\delta\phi_\delta(u)\\ =\ & -abp\lambda e^{-bu}\int_0^\infty w'_u(u,z)e^{-bz}dz+\frac{ab}{a+b}p\lambda e^{-bu}\int_0^\infty w''_{uu}(u,z)e^{-bz}dz. \qquad (3.1)\end{aligned}$$

Let $w(x,y)=1$ and $\delta>0$, we have

$$\phi_\delta(u)=\mathsf{E}^u[e^{-\delta T}I(T<\infty)]\triangleq\psi_\delta(u)$$

is the Laplace transform of the time of ruin with an initial surplus $u\ge0$, Let $w(x,y)=y$ and $\delta>0$, then

$$\phi_\delta(u)=\mathsf{E}^u[|U_T|e^{-\delta T}I(T<\infty)]\triangleq\xi_\delta(u)$$

is the discounted expectation of deficit at ruin with an initial surplus $u\ge0$, when G and F are exponential distributions with $G(x)=1-e^{-ax}$, $F(y)=1-e^{-by}$, $x,y,a,b>0$, $p<b/a$.

From theorem 3.1, we see that $\psi_\delta(u)$ satisfies the following equation:

$$(\lambda+\delta)y''+(b\lambda+b\delta-a\delta-ap\lambda)y'-ab\delta y=0, \qquad (3.2)$$

with boundary conditions:

$$\psi_\delta(+\infty)=0$$

and

$$\begin{aligned}(\lambda+\delta)\psi_\delta(0) &= (1-p)\lambda\int_0^\infty\psi_\delta(x)ae^{-ax}dx\\ &\quad+p\lambda\Big[\int_0^\infty\int_0^x\psi_\delta(x-y)be^{-by}ae^{-ax}dydx+\frac{a}{a+b}\Big].\end{aligned}$$

Solving this problem we get

$$\psi_\delta(u)=C_1e^{ru},$$

where

$$r=-\frac{b\lambda+b\delta-a\delta-ap\lambda}{2(\lambda+\delta)}-\frac12\sqrt{\Big(\frac{b\lambda+b\delta-a\delta-ap\lambda}{\lambda+\delta}\Big)^2+\frac{4ab\delta}{\lambda+\delta}}<0,$$

$$C_1=\frac{pa\lambda(r-a)}{(ar+br-a^2p)\lambda+(a+b)(r-a)\delta}>0.$$

Let $\delta=0$, we get the probability of ruin $\psi_0(u)$ as $\psi_0(u)=(ap/b)\cdot e^{(ap-b)u}$.

It is easy to see that $\xi_\delta(u)$ satisfies equation (3.2), together the following boundary conditions:

$$\xi_\delta(+\infty)=0$$

and

$$(\lambda+\delta)\xi_\delta(0) = (1-p)\lambda\int_0^\infty \xi_\delta(x)a\mathrm{e}^{-ax}\mathrm{d}x + p\lambda\Big[\int_0^\infty\int_0^x \xi_\delta(x-y)b\mathrm{e}^{-by}a\mathrm{e}^{-ax}\mathrm{d}y\mathrm{d}x + \frac{a}{b(a+b)}\Big].$$

Solving this problem, we get

$$\xi_\delta(u) = C_2\mathrm{e}^{ru},$$

where

$$C_2 = \frac{ap\lambda(r-a)}{(abr+b^2r-a^2bp)\lambda+(a+b)(r-a)b\delta} > 0.$$

Let $\delta=0$, we obtain the deficit at ruin $\xi_0(u)$ as $\xi_0(u)=(ap/b^2)\cdot\mathrm{e}^{(ap-b)u}$.

Now, we compare the ruin probability of risk process (1.1) with that of risk process (1.2) under the assumption $\mathsf{E}(U_t^0)=\mathsf{E}(U_t)$, i.e. $\lambda_1=p\lambda$. For convenience, we assume that the premium and the claim sizes are exponentially distributed, under the assumptions, Boikov[1] got the ruin probability of risk process (1.1)

$$\psi(u) = \frac{(a+b)\lambda_1}{b(\lambda+\lambda_1)}\exp\Big(\frac{a\lambda_1-b\lambda}{\lambda+\lambda_1}u\Big).$$

In the table, we present the values of $\psi(u)$, $\psi_0(u)$ and $\psi(u)/\psi_0(u)$, where $a=1/10$, $b=1/15$.

Table 1 Comparison of ruin probabilities

p	$\psi(u)$				$\psi_0(u)$				$\psi(u)/\psi_0(u)$			
	$u=0$	10	20	30	$u=0$	10	20	30	$u=0$	10	20	30
0.1	0.2273	0.1358	0.0811	0.0485	0.1500	0.0851	0.0483	0.0274	1.5152	1.5953	1.6796	1.7684
0.2	0.4167	0.2824	0.1914	0.1298	0.3000	0.1881	0.1180	0.0740	1.3889	1.5012	1.6226	1.7539
0.3	0.5769	0.4351	0.3282	0.2475	0.4500	0.3119	0.2161	0.1498	1.2821	1.3953	1.5185	1.6525
0.4	0.7143	0.5904	0.4880	0.4034	0.6000	0.4596	0.3520	0.2696	1.1905	1.2847	1.3864	1.4962
0.5	0.8333	0.7457	0.6673	0.5971	0.7500	0.6349	0.5374	0.4549	1.1111	1.1746	1.2417	1.3126
0.6	0.9375	0.8992	0.8625	0.8273	0.9000	0.8420	0.7877	0.7369	1.0417	1.0680	1.0951	1.1228

From the above table, we see that $\psi(u)>\psi_0(u)$ for all u and p, which coincides with the result in Chen et al.[2]. It is easy to see that the impact of the correlation coefficient p on the ruin probability is prominent. The last columns indicates that the ratios of $\psi(u)/\psi_0(u)$ get larger as u increases.

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References

[1] Boikov, A.V., The Cramer-Lundberg model with stochastic premium process, *Theory of Probability and its Application*, **47(3)**(2003), 489–493.

[2] Chen, S., Wang, G. and Wang, Z., The application of thinning process in risk problems, *Chinese Journal of Application of Statistics and Management*, **20(5)**(2001), 26–30.

[3] Yao, D. and Xu, L., Gerber-Shiu penalty function in the compound Poisson risk model with a random premium, *Chinese Journal of Applied Probability and Statistics*, To appear, 2007.

[4] Wu, R., Wang, G. and Zhang, C., On a joint distribution for the risk process with constant interest force, *Insurance Math. Econom.*, **36**(2005), 365–374.

[5] Yuen, K.C., Wang, G. and Wu, R., On the renewal risk process with stochastic interest, *Stochastic Processes and their Application*, **116**(2006), 1496–1510.

稀疏风险模型的期望折扣罚金函数

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本文考虑了一类风险模型, 其中保费到达过程是一个参数为$\lambda > 0$的Poisson过程, 而理赔过程是保费到达过程的稀疏过程. 在该模型下, 我们得到了期望折扣罚金函数所满足的积分方程, 积分–微分方程以及递推公式, 并且当保费和理赔额均为指数分布时, 我们使用积分–微分方程获得了破产时刻的Laplace变换和在破产时刻的赤字的闭式表达式.

关键词: 稀疏, Poisson过程, 期望折扣罚金函数.

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