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Properties of corresponding statistics of the technical analysis indexes on a SARV model

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Abstract: Bollinger bands, RSI and ROC for stochastic autoregressive volatility (SARV) model as real stock market were investigated. Under the given conditions, the stationarity and the law of large numbers of the corresponding statistics were proved .

Key words: SARV model; Bollinger bands; RSI; ROC; stationary process; strong mixing

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关于 SARV 模型技术分析指标的相应统计量的性质

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摘要: 研究了随机自回归波动率 (SARV) 模型作为真实股票市场技术分析指标的布林带、RSI和ROC相应统计量的概率性质，证明了这些统计量在给定条件下的平稳性和大数定律成立.

关键词: 随机自回归波动率模型; 布林带; RSI; ROC; 平稳过程; 强混合

0 Introduction

Since Charles H. Dow first introduced the Dow theory in the late 1800s, technical analysis has been extensively used in stock markets. Technical analysis tries to forecast the prices of financial securities by observing the pattern that the security has followed in the past. There are numerous methods within technical analysis, which are essentially independent from each other. Among the most important technical analysis tools in the stock market are Bollinger bands, RSI, ROC. The efficiency of these indexes is ‘proved’ by the observed relative frequency of the occurrence of the corresponding behaviors of stock prices. In other words, the traders use

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the daily (hourly, weekly, etc.) stock prices as samples of certain statistics and use the observed relative frequency to show the validity of these indexes. However, these samples are just the discrete observations of a realized path of a stochastic process, which are not independent, so the classical sample survey theory, especially the law of large numbers, does not apply to. But Liu *et al.*[1] and Zhu[2] found that some important technical analysis indexes are stationary process or the transformation of theirs. Liu *et al.*[1] discussed the Bollinger bands for Black-Scholes model as real stock market. They introduced the statistics U_t^n calculated according to the formulation of the Bollinger bands, which is stationary and $\{U_{t+kn}^{(n)}\}_{k=1,2,\cdots}$ are mutually independent for each fixed $t \geqslant 0$. Zhu[2] extended the above results to another index RSI for Black-Scholes model. Since we know that the frequency of the occurrence of stationary process can be computed, so those results have laid the theoretical foundation for statistical application of the technical analysis of stock prices.

Let S_t be observed stock price. Let us introduce the definitions of these technical analysis indexes as follows:

(1) Bollinger bands definition

$$\begin{aligned} \text{Middle Bollinger Band} &= n\text{-day weighted (or simple) moving average.} \\ \text{Upper Bollinger Band} &= \text{Middle Bollinger Band } +2 \times n\text{-day standard deviation.} \\ \text{Lower Bollinger Band} &= \text{Middle Bollinger Band } -2 \times n\text{-day standard deviation.} \end{aligned}$$

where n is the number of periods you select.Usally we take $n = 12$ or 20.

Denote by

$$\overline{S}_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} S_{t-i}, \qquad \hat{S}_t^{(n)} = \frac{1}{\sum_{i=1}^{n} i}\sum_{i=0}^{n-1}(n-i)S_{t-i},$$

and

$$\sigma_t^{(n)} = \sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}(S_{t-i} - \overline{S}_t^{(n)})^2}.$$

The curves $\gamma_t^- = \hat{S}_t^{12} - 2\sigma_t$ and $\gamma_t^+ = \hat{S}_t^{12} + 2\sigma_t$ are called the lower and the upper Bollinger bands, respectively. Bollinger[3] introduced this pair of bands to provide a relative definition of high and low for a stock price in the early 1980s. By definition the stock price is "high" at the upper band and "low" at the lower band. The closer the prices move to the upper band, the more overbought the market, and the closer the prices move to the lower band, the more oversold the market.

(2) RSI (Relative Strength Index) definition

If we denote

$$\Delta S_t = S_{t+1} - S_t, \qquad \Delta S_t^+ = (S_{t+1} - S_t) \vee 0,$$

then n-day RSI is defined as

$$V_t^{(n)} = 100 \times \frac{\sum_{i=1}^{n} \Delta S_{t-i}^+}{\sum_{i=1}^{n} |\Delta S_{t-i}|} \quad (\forall t > n),$$

where n is the number of periods you select.

RSI was proposed by Welles Wilder Jr. in 1978. The condition of market is reflected by calculating the correlative value of the strength in buys and sells in a period of time. In a normal market, the price can be stabilized only when the both sides of the business strength obtain the balance. RSI takes its values in [0, 100]. In general, RSI value maintains above 50 for a strong trend market, and is lower than 50 for a weak trend market. RSI may be used in judging the ultra-buy and ultra-sell in market. Take 9-day RSI as an example, RSI above 80 may be regarded as the ultra-buy area, and below 20 may regarded as the ultra-sell area. It is an early warning signal of the price possibly reverse when the market enters the ultra-buy area or the ultra-sell area. Investors always pay closely attention on the market when this signal appears.

(3) ROC (Rate of Change Index) definition

ROC = 100 × (closing price - closing price of n-day before) / closing price of n-day before. That is, n-day ROC is defined as

$$W_t^{(n)} = 100 \times \frac{S_t - S_{t-n}}{S_{t-n}} \qquad (\forall t > n),$$

where n is the number of periods you select. Usually we take $n = 12$. The ROC must establish the antenna and the grounding also. But unlike other ultra-buy or ultra-sell indexes, its antennas and grounds are indefinite. When ROC undulates in "normal scope", it is time to sell out while ROC rises to the first ultra-buy line (5) and it time to buy in while ROC drops to the first ultra-sell line (-5). After ROC breaking through the first ultra-buy line upward, the rising trend mostly ends when it reaches the second ultra-buy line (10). And the dropping trend mostly ends when ROC reaches the second ultra- sell line (-10) after it breaks through the first ultra-sell line downward.

It is well-known that discrete-time models was typically used in empirical financial literature. In this paper, we aim at discussing the technical analysis indexes Bollinger bands, RSI and ROC for discrete-time SARV model introduced by Andersen[4] as real stock market . We consider log return process $r_t = \log S_t - \log S_{t-1}$. Define the information set $\mathcal{F}_t$ of daily returns to be $\{r_t, r_{t-1}, \cdots, r_1\}$. In the following we assume that the log return r_t is generated as follows,

$$r_t = \mu(\sigma_t) + \varepsilon_t \tag{0.1}$$

Here, we consider a $\log AR(1)$ version of the stochastic autoregressive volatility model introduced by Andersen[4]:

$$\varepsilon_t = \sigma_t z_t, \quad \ln \sigma_t = \omega + \beta \ln \sigma_{t-1} + (\gamma + \alpha \ln \sigma_{t-1}) u_t, \tag{0.2}$$

where$\{z_t\}$ and $\{u_t\}$ are mutually independent i.i.d. random variables with zero means and unit variances, $\alpha + \beta > 0$, and $\alpha + \gamma > 0$.

In Section 1, the stationarity of the corresponding statistics of the technical analysis indexes Bollinger bands, RSI and ROC for SARV model as real stock market are proved under the given conditions. In Section 2, we show that the law of large numbers of these statistics hold under the above conditions and give the rate of convergence.

1 Stationarity

For model (0.2), the following result is due to Carrasco and Chen([5], Proposition 15).

Proposition 1.1 Assume that (i) $\{u_t\}$ is a sequence of i.i.d. real-valued random variables, independent of σ_0, with $E(u_t)=0$ and $E(u_t^2)=1$; the probability distribution of u_t has a continuous density (with respect to Lebesgue measure on real line), and its density $p(.)$ is positive on $(-\infty,+\infty)$. (ii) $|\beta|<1$ and there is an integer $s\geqslant 1$ such that

$$E|u_t|^s<\infty,\quad E|\beta+\alpha u_t|^s<1.$$

Then (1) $E[|\ln\sigma_t|^s]<\infty$. (2) If $\{\sigma_t\}$ is initialized from its stationary distribution, then the term $\{\sigma_t\}$ is strictly stationary and exponential β-mixing.

Let S_t be observed stock price by the model (0.1) and (0.2), then

$$S_t=S_0\exp\left\{\sum_{k=1}^{t}(\mu(\sigma_k)+\varepsilon_k)\right\}. \tag{1.1}$$

Denote

$$g(t,i,j)=\exp\left\{\sum_{k=t-i}^{t-j}(\mu(\sigma_k)+\varepsilon_k)\right\}. \tag{1.2}$$

Then we have the following:

Theorem 1.2 Suppose f be a measurable function: $\mathbf{R}^n\to\mathbf{R}$. Let $\Lambda_t^{(n)}=f(g(t,n-1,0),\cdots,g(t,n-1,n-1))$. Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then the process $\{\Lambda_t^{(n)}\}_{t\geqslant n}$ is stationary.

Proof From the expression of (1.2), we can see clearly that $g(t,i,j)_{0\leqslant j\leqslant i\leqslant n-1}$ is just a function of $(\sigma_k,z_k)_{k=t-n+1,\cdots,t}$. By Proposition 1.1, we know σ_t be independent of z_t and σ_t have stationary distribution for all $t\geqslant 0$. So $(\sigma_k,z_k)_{k=t-n+1,\cdots,t}$ is a two-dimensional stationary process. Then we have for any $m\geqslant 0$,

$$(g(t,n-1,0),\cdots,g(t,n-1,n-1))(=)(g(t+m,n-1,0),\cdots,g(t+m,n-1,n-1)).$$

where $X(=)Y$ denote X and Y have the same distribution. So

$$\{\Lambda_t\}_{t\geqslant n}(=)\{\Lambda_{t+m}\}_{t\geqslant n}$$

and the proof is complete.

Remark 1.3 Using the above conclusion, we can draw a series of stationary processes from SARV model so long as the desired conditions are satisfied. This fact can be applied to the technical analysis indexes explained before. The conclusions list in the following corollaries.

Corollary 1.4 Let S_t be the stock price generated by (0.1) and (0.2). Denote by

$$U_t^{(n)}=\frac{S_t-\hat{S}_t^{(n)}}{\sigma_t^{(n)}},\qquad(\forall t\geqslant n). \tag{1.3}$$

Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then the process $\{U_t^{(n)}\}_{t\geqslant n}$ is stationary.

Proof We can obtain immediately by using (1.2):

$$\begin{aligned} S_t &= S_{t-n}g(t,n-1,0), \\ \hat{S}_t^{(n)} &= S_{t-n}\frac{1}{\sum_{i=1}^n i}\sum_{i=0}^{n-1}(n-i)g(t,n-1,i), \\ \overline{S}_t^{(n)} &= S_{t-n}\frac{1}{n}\sum_{i=0}^{n-1}g(t,n-1,i), \end{aligned}$$

and

$$(n-1)[\frac{\sigma_t^{(n)}}{S_{t-n}}]^2 = \sum_{i=0}^{n-1}(g(t,n-1,i)-\frac{1}{n}\sum_{i=0}^{n-1}g(t,n-1,i))^2.$$

So $\{U_t^{(n)}\}_{t\geqslant n}$ is a function of $(g(t,n-1,0),\cdots,g(t,n-1,n-2))$.So by Theorem 1.2 we have $U_t^{(n)}$ is stationary.

Corollary 1.5 Let S_t be the stock price generated by the model(0.1) and (0.2). Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then the process

$$V_t^{(n)} = 100\times\frac{\sum_{i=1}^n \Delta S_{t-i}^+}{\sum_{i=1}^n |\Delta S_{t-i}|} \quad (\forall t > n)$$

is stationary.

Proof We can obtain immediately by using (1.2):

$$S_t = S_{t-n}g(t,n-1,0), \quad S_{t-i} = S_{t-n}g(t,n-1,i).$$

Then

$$V_t^{(n)} = 100\times\frac{\sum_{i=1}^n(g(t,n-1,i-1)-g(t,n-1,i))\vee 0}{\sum_{i=1}^n |g(t,n-1,i-1)-g(t,n-1,i)|},$$

here we decree $g(t,n-1,n)=1$. Then it is clearly that $V_t^{(n)}$ is a measurable function of $(g(t,n-1,0),\cdots,g(t,n-1,n-1))$. So by Theorem 1.2 we have $V_t^{(n)}$ is stationary.

Similarly, we have the following corollary.

Corollary 1.6 Let S_t be the stock price generated by the model(0.1) and (0.2). Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then the process

$$W_t^{(n)} = 100\times\frac{S_t - S_{t-n}}{S_{t-n}} \qquad (\forall t > n)$$

is stationary.

2 Law of large numbers

Denote for $i\geqslant n$, $K_{\Gamma,i}^{(n)} = I_{[\Lambda_i^{(n)}\in\Gamma]}$, $\Gamma\in\mathcal{B}(\mathcal{R})$. Then from Theorem 1.2, $E[K_{\Gamma,i}^{(n)}] = P[\Lambda_i^{(n)}\in\Gamma] = P[\Lambda_n^{(n)}\in\Gamma]$. Let

$$V_{N,\Gamma}^{(n)} = \frac{1}{N+1}\sum_{i=0}^{N}K_{\Gamma,n+i}^{(n)}$$

which is the observed frequency of the events $[\Lambda_{n+i}^{(n)} \in \Gamma]$ $(i = 0, 1, ..., N)$. We have

Theorem 2.1 Under the conditions of Theorem 1.2, the law of large numbers holds:

$$E|V_{N,\Gamma}^{(n)} - P[\Lambda_n^{(n)} \in \Gamma]|^2 \leqslant \frac{C_0}{N+1},$$

where C_0 is a constant.

Proof First we will show that the process $Z_t = (\sigma_t, z_t)$ has β-mixing property. It is clearly that Z_t is a Markov chain on $\mathbf{R}^+ \times \mathbf{R}$ with its Borel σ-field. Since $\{\sigma_t\}$ is strictly stationary and β-mixing with exponential decay.Moreover σ_t and z_t is independent each other, so we can denote $P_Z = P_\sigma \bigotimes P_z$ as the stationary distribution of Z_t, where P_σ is that of σ_t and P_z is that of z_t. Denote by $P_Z^t(z, .)$ the transition probabilities of the Markov chain (Z_t). Then we have, for $z = (x, y) \in \mathbf{R}^+ \times \mathbf{R}, A_1 \in \mathcal{B}(\mathcal{R}^+), A_2 \in \mathcal{B}(\mathcal{R})$ and $t > 0$,

$$\begin{aligned} P_Z^n(z, A_1 \times A_2) &= P(\sigma_n \in A_1, z_n \in A_2 | \sigma_0 = x, z_0 = y) \\ &= P_z(z_n \in A_2) P(\sigma_n \in A_1 | \sigma_0 = x) \\ &= P_z(z_n \in A_2) P(\sigma_n \in A_1 | \sigma_0 = x) \\ &= P_z(z_n \in A_2) P_\sigma^n(x, A_1). \end{aligned}$$

Hence $\|P_Z^n(z, .) - P_Z(.)\| \leqslant \|P_\sigma^n(x, .) - P_\sigma(.)\|$.

Then we can deduce that Z_t is β-mixing with exponential decay by the β-mixing of $\{\sigma_t\}$. So by Lu and Lin[6] we know Z_t is also α-mixing with exponential decay. i.e., there exist $0 < \rho < 1$ and $c > 0$ such that $\alpha_Z(n) \leqslant c\rho^n, \forall n \in \mathbf{Z}_+$.

From the proof of Theorem 1.2 we know that $\{\Lambda_t^{(n)}\}_{t \geqslant n}$ is a function of $Z_k = (\sigma_k, z_k)_{k=t-n+1,\cdots,t}$, then $\Lambda_t^{(n)}$ is strong mixing and $\alpha_\Lambda(k) \leqslant \alpha_Z(k-n)$ for any $k \geqslant n$. We write $\xi_i = K_{\Gamma,N+i}^{(n)}$. Then $D\xi_i \leqslant \frac{1}{4}$ for any i , $\mathrm{cov}(\xi_i, \xi_j) = P(\Lambda_{n+i}^{(n)} \in \Gamma, \Lambda_{n+j}^{(n)}) \in \Gamma) - P(\Lambda_{n+i}^{(n)} \in \Gamma) P(\Lambda_{n+j}^{(n)}) \in \Gamma)$. Then $|\mathrm{cov}(\xi_i, \xi_j)| \leqslant \frac{1}{4}$ for any i, j and $|\mathrm{cov}(\xi_i, \xi_j)| \leqslant \alpha_\Lambda(|i-j|) \leqslant \alpha_Z(|i-j-n|)$ when $|i-j| > n$. Hence we have

$$\begin{aligned} E|V_{N,\Gamma}^{(n)} - P[\Lambda_n^{(n)} \in \Gamma]|^2 &= \frac{1}{(N+1)^2} \sum_{0 \leqslant i,j \leqslant N} \mathrm{cov}(\xi_i, \xi_j) \\ &\leqslant \frac{1}{(N+1)^2} \Big\{ \sum_{0 \leqslant i,j \leqslant N, |i-j| \leqslant n} |\mathrm{cov}(\xi_i, \xi_j)| + \sum_{0 \leqslant i,j \leqslant N, |i-j| > n} |\mathrm{cov}(\xi_i, \xi_j)| \Big\} \end{aligned}$$

Moreover,

$$\begin{aligned} \sum_{0 \leqslant i,j \leqslant N, |i-j| \leqslant n} |\mathrm{cov}(\xi_i, \xi_j)| &\leqslant 2(N+1)(n+1)\frac{1}{4} = \frac{1}{2}(N+1)(n+1), \\ \sum_{0 \leqslant i,j \leqslant N, |i-j| > n} |\mathrm{cov}(\xi_i, \xi_j)| &= 2 \sum_{k=n+1}^{N} (N+1-k) |\mathrm{cov}(\xi_0, \xi_k)| \\ &\leqslant 2 \sum_{k=n+1}^{N} (N+1-k) \alpha_Z(k-n) \\ &\leqslant 2 \sum_{k=1}^{N-n} (N+1-k-n) \alpha_Z(k) \end{aligned}$$

$$\begin{aligned}
&\leqslant 2c\sum_{k=1}^{N-n}(N+1-n-k)\rho^{k}\\
&= 2c\rho^{N+1-n}\sum_{k=1}^{N-n}(N+1-n-k)\rho^{-(N+1-n-k)}\\
&= 2c\rho^{N+1-n}\sum_{k=1}^{N-n}k\rho^{-k}\\
&= \frac{2c(N-n)}{\rho^{-1}-1}+\frac{2c(1-\rho^{N-n})}{(1-\rho^{-1})^{2}}.
\end{aligned}$$

We take $C_0=3\max\{\frac{1}{2}(n+1),\frac{2c}{\rho^{-1}-1},\frac{2c}{(1-\rho^{-1})^2}\}$, then we have

$$E|V_{N,\Gamma}^{(n)}-P[\Lambda_n^{(n)}\in\Gamma]|^2\leqslant\frac{C_0}{N+1}.$$

Thus we complete the proof of Theorem 2.1.

Remark 2.2 From the above theorem, it is reasonable to use the stationary distribution of $\Lambda_n^{(n)}$ to calculate the observed frequency $V_{N,\Gamma}^{(n)}$. Since $U_t^{(n)},V_t^{(n)}$ and $W_t^{(n)}$ change between a normal scope.The normal scope of $U_t^{(n)}$ is $[-2,2]$. $V_t^{(n)}$ always changes between 20 and 80. $W_t^{(n)}$ also has indefinite antennas and grounds. For simplicity, we unify the lower value of the normal scope be α and the upper value be β. As a application, we have the following corollaries.

Corollary 2.3 Denote for $i\geqslant n$, $H_i^{(n)}=I_{[|U_i^{(n)}|\geqslant 2]}$. Let

$$J_N^{(n)}=\frac{1}{N+1}\sum_{i=0}^{N}H_{n+i}^{(n)}.$$

Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then there exist a constant C_0 such that

$$E|J_N^{(n)}-P[U_n^{(n)}\in\Gamma]|^2\leqslant\frac{C_0}{N+1}.$$

Corollary 2.4 Denote for $H_i^{(n)}=I_{[V_i^{(n)}\in\Gamma]}$, where $\Gamma=[0,20]\bigcup[80,100]$. Let

$$J_N^{(n)}=\frac{1}{N+1}\sum_{i=0}^{N}H_{n+i}^{(n)}.$$

Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure,then there exist a constant C_0 such that

$$E|J_N^{(n)}-P[V_n^{(n)}\in\Gamma]|^2\leqslant\frac{C_0}{N+1}.$$

Corollary 2.5 Denote for $H_i^{(n)}=I_{[W_i^{(n)}\in\Lambda]}$, where $\Lambda=[-\infty,\alpha]\cup[\beta,\infty]$. and (α,β) are the indefinite antenna and ground of ROC. Let

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$$J_N^{(n)} = \frac{1}{N+1}\sum_{i=0}^{N} H_{n+i}^{(n)}.$$

Under the conditions of Proposition 1.1, if σ_0 is initialized from the invariant measure, then there exist a constant C_0 such that

$$E|J_N^{(n)} - P[W_n^{(n)} \in \Lambda]|^2 \leqslant \frac{C_0}{N+1}.$$

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