Nonlin. Processes Geophys., 14, 103–108, 2007
www.nonlin-processes-geophys.net/14/103/2007/

Nonlinear Processes in Geophysics

Anomalies of critical state in fracturing geophysical objects

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Received: 28 December 2006 – Revised: 26 February 2007 – Accepted: 26 February 2007 – Published: 6 March 2007

Abstract. Non-linear time-sequences of fracture-related events were studied in drifting sea-ice and fracturing rock. A reversible drop of the b-value was detected prior to the large-scale sea-ice cover fragmentation, when the time sequence of impact interactions between ice-fields was fully decorrelated. A similar loss of the temporal invariance of the fracture process was revealed in the time sequence of microfracture events detected in a loaded rock sample. These temporal gaps in the continuous critical state of the considered self-organizing, open systems were attributed to the property of hierarchicity inherent in the geophysical objects. A combination of scaling and hierarchic features in the behavior of fracturing solids manifests itself in the heterogeneity of the temporal pattern of fracture process.

1 Introduction

The scaling properties of the fracture products, such as crack networks, crack surfaces and collections of broken pieces, are well established and described in relevant reviews (Bouchaud, 1997; Stanley, 1999; Weiss, 2003). The spatial invariance of this kind is usually interpreted as a sign of criticality of the fracture process (Petri et al., 1994; Caldarelli et al., 1999; Kapiris et al., 2004). However, the primary practical interest is connected with the temporal aspect of fracture since the temporal scaling contains a certain potentiality for forecasting the fracture events in both natural and engineering structures. The space and time invariance of fracturing takes place on various scale levels, as beginning from the nanoscopic crack nucleation (Chmel, 2003) and microfracturing (Weiss et al., 2001; Kuksenko et al., 2006; Mandelbrot, 2006), and up to earthquakes (Sornette and Sornette, 1989; Kapiris et al., 2003) and basin-wide sea-ice-cover fragmentations (Korsnes et al., 2004; Chmel et al., 2005). As a rule, the scaling properties of damaged solids are regarded as a result of nucleation and gradual, monotonic development of some self-similar structures, such as the ensemble of hypocenters in rocks or ice (Zang et al., 1998; Weiss, 2003; Amitrano, 2006), or fatigue microcracks in metals (Botvina, 2004). However, the suggested one-way self-organizing is ill-consistent with the hierarchic organization of fracturing (Gabrielov, 2000; Botvina, 2004; Tomilin and Kuksenko, 2004). The natural observations (Varotsos, 2002; Kapiris et al., 2003), laboratory experiments (Lockner et al., 1992; Kuksenko et al., 1996) and analytical consideration (Amitrano, 2006) evidence the presence of stages of clustering of damages including a multistage faulting (Tomilin and Kuksenko, 2004) when clusters of damages of lower size form damages of higher size, which, in turn, serve as precursors for subsequent fault nucleation. Two to three cycles of the hierarchic magnification were detected experimentally (Tomilin and Kuksenko, 2004) but up to now this phenomenon was not bound to virtual anomalies in the scaling properties of the process. Meanwhile, one should expect that the hierarchicity would affect the temporal invariance of fracturing due to the presence of short-term transitions between metastable stages of stationary accumulation of damages.

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In this work we consider the dynamics of the fracture processes on geophysical and laboratory scale levels. Respectively, the sea-ice cover fragmentation and compressional rock fracture were studied in order to reveal some transient statistical anomalies in the fracture process. In terms of critical dynamics, we present examples of avalanche periods occurring in fracturing systems sliding “along the border of chaos” (Obukhov, 1990). Our goal was to demonstrate some common statistically significant deviations from the scaling behavior of stressed heterogeneous solids with quite different physical and mechanical parameters, such as elastic properties, dimensions, heterogeneity, and, in addition, characterized by various rate of fracturing.

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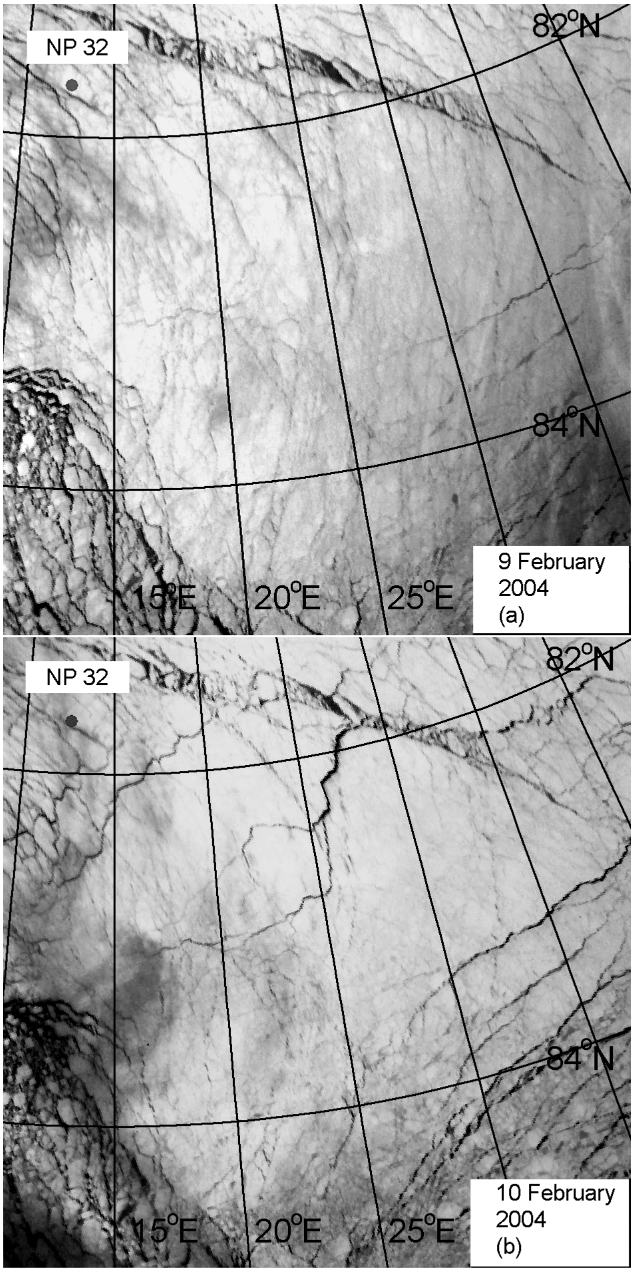

Fig. 1. Fragments of AVHRR images of the region of NP 32 drift obtained on 9 February **(a)** and 10 February **(b)** 2004. A few newly-formed giant leads are seen in (b).

2 Sea-ice cover fragmentation

The fracture phenomena in cryosphere could cover the frozen areas of the Arctic Ocean with dimensions comparable with dimensions of tectonic plates (Weiss, 2003); the energy release in impact interactions of mid-size ice-sheets reaches $\sim 10^7$ J (Smirnov, 1996). Herein we shall consider an event of large-scale fragmentation of the Arctic sea-ice cover (ASIC), which was detected and analyzed using a combination of satellite imaging and field observations carried out in the ice-research station North Pole 32 (NP 32) established on drifting ice pack. The ASIC is the open, dynamic system with well-pronounced scaling properties that manifest themselves in the temporal invariance of drift dynamics (Chmel et al. 2005a), fractal distribution of ice-field deformations (Marsan and al., 2004), self-similar geometry (Chmel et al., 2005) and size distribution of ice pieces (Rothrock and Thorndike, 1984; Korsnes et al, 2004). The fracture events (ice cracking, ridge formation etc.) that take place at the scale levels from 10^2 to 10^5 m cause substantial changes in the crack-and-ridge pattern, redistribution of areas of open water and lead propagation. These processes flow simultaneously and they are interrelated dynamically. From time to time, unfavorable combinations of meteorological factors, such as the atmospheric pressure and wind velocity, lead to ice cover fragmentation over the area up to 10^5 km^2.

Figure 1 shows two satellite images of the region north of Franz Josef Land and Spitsbergen where on 10 February 2004 a few large-scale leads (up to 400 km in length) formed in one day. The ice cover fragmentation of local scale was also detected in the research station NP 32 whose actual position in that period of time is denoted in the figure.

The sea-ice drift excites the mechanical interaction of ice sheets between each other and with offshore structures. The ice drift is the self-organized process that maintains permanent temporal correlation of accelerations during quiet periods (Chmel et al., 2005). In this work we analyzed the local motion of an ice field in the time interval covering the mentioned above large-scale ice fragmentation occurred on 10 February 2004. The drift was monitored with the help of a GPS transducer established on the ice field; the accuracy of positioning was 10 m or better. On the base of these data, the ice-field speed (V) and absolute amplitudes of accelerations $A=|\Delta V|/\Delta t$ were determined (here ΔV is the variation of drift rate measured in regular intervals $\Delta t=10$ min).

2.1 Temporal correlation

A time sequence of values A determined during January-February, 2004 was used to construct a function of distribution of waiting times for the accelerations whose amplitudes exceeded a cut-off value $A_{\text{cut-off}}$, that is $N_{A>A\,\text{cut-off}}$ $(>\tau)$. A range of variation of times τ covered the range from 6×10^2 s (time resolution of field observations) to $\sim 1.3\times10^3$ s (waiting time for the most rare events). In this analysis we considered the contribution of accelerations with amplitudes $A>A_{\text{cut-off}}=8\times10^{-6}$ m/s^2 (the instrumental limit was equal to 1.2×10^{-6} m/s). These accelerations constituted $\sim$20 per cent of the capacity of actual database.

The functions $N_{A>A\,\text{cut-off}}$ $(>\tau)$ were found for three periods of observations: (a) "remote" period from 1 January to 31 January; (b) "precedent" period (prior to "catastrophic" sea-ice perturbation) from 1 February to 10 February; (c) "posterior" period from 11 February to 28 February. In each

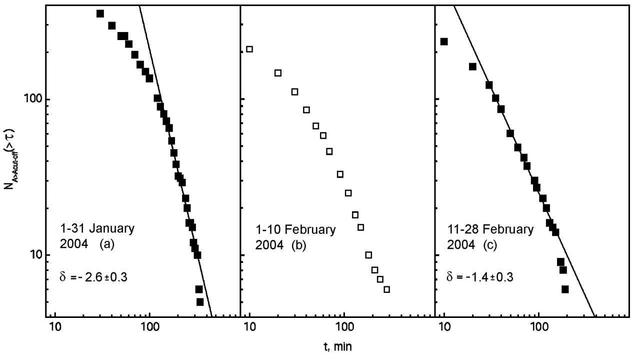

Fig. 2. Number of waiting times with τ higher than given by the corresponding abscissa. The straight lines fit the power law $N_{A>A\,\mathrm{cut-off}}\ (>\tau)\propto\tau^{\gamma}$. The accuracy of determination of the slopes of log-linear portions was estimated from the deviation of the slope at the end points of log-log plots with respect to the best-fit data used to draw the straight lines.

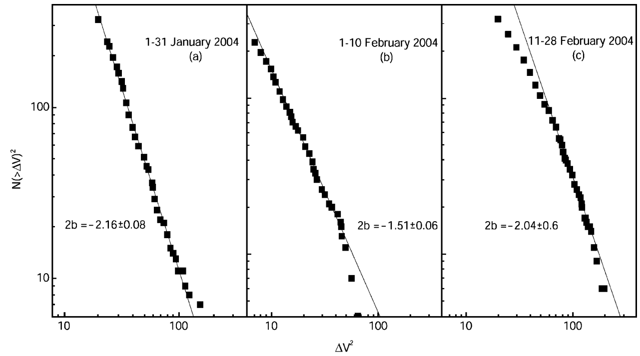

Fig. 3. Distributions $N>\Delta V^2$ versus ΔV^2 in three periods of field observations.

period, the number of subsequent events $N_{A>A\,\mathrm{cut-off}}$ separated by the time interval that exceeds τ was plotted against τ in doubly logarithmic coordinates (Fig. 2). One can see that three constructed distributions of waiting times are quite different. In Figs. 2a and c corresponded to "remote" and "posterior" periods, respectively, the $N_{A>A\,\mathrm{cut-off}}$ versus τ dependences exhibit a power law:

$$N_{\mathrm{A>Acut-off}}(>\tau)\propto\tau^{\gamma} \tag{1}$$

shown by straight lines in these figures. At the same time, the time sequence of ice-field accelerations does not follow the power law in the immediate interval prior to "catastrophic" perturbation (Fig. 2b). While the power law distribution of waiting times is an indication of the correlation between successive events, a transient disturbance of the scaling law (1) in attendance of the large-scale event signalizes the deviation from the critical time sequence of force interactions as approaching the crucial stage. We suppose that the physical mechanism of this disturbance is related with a limited mobility of individual sea-ice sheets in the consolidated ice in period of freezing cycles in the region.

After the catastrophe, a new configuration of structural links forms, and the correlated motion of the sea-ice restores. A decrease of the exponent γ from 2.5±0.5 to 1.4±0.3 after the event of 10 February evidences a reduction of the number of larger waiting times owing to the increased mobility of more fragmented sea-ice.

In order to characterize the internal energy exchange in the sea-ice cover, the energy distribution of local accelerations in drifting ice was studied in connection with its response to the burst of fragmentation.

2.2 Energy distribution

The energy release due to impact interactions between adjacent ice fields, E, is directly proportional to the change in speed squared, $E\propto\Delta V^2$. Therefore, the distribution of energy release in events of acceleration is related directly with the distribution function $N(>\Delta V^2)$ (here N is the number of velocity variations exceeding ΔV^2). The distributions $N(>\Delta V^2)$ were calculated for the same three time intervals in January and February, and plotted in log-log coordinates (Fig. 3). All distributions exhibit log-linear portions satisfying the power law:

$$N(>\Delta^2)\propto\Delta V^{-2b} \tag{2}$$

Hence, for the energy release we have a scaling relation:

$$N(>E)\propto E^{-b} \tag{3}$$

In contrast to the temporal statistics, no periods of fully decorrelated energy exchange were revealed. Portions of the linear dependence $\log(N>\Delta V^2)$ versus $\log(\Delta V^2)$ manifest themselves in all the intervals selected. At the same time, the b-values calculated from the slopes of straight portions were found to be equal to 1.06±0.03 in the "remote" and "posterior" periods, and 0.76±0.06 in the "precedent" period.

A decrease of the b-value before the "global" fracture event is well established in seismology (Kapiris et al., 2004) and rock fracture (Zang et al., 1998). However the trajectory of the b-value variation in the case of fracturing sea-ice has a particular feature that distinguishes it from the trend established in other fracture processes. We observed not only the decrease of the b-value prior to a critical point but also its returning to the initial ("pre-catastrophic") value. The fluctuations of scaling parameters in the critical system are typical for self-organizing ensembles, and our results evidences that the ASIC is the case.

3 Rock fracture

3.1 Experimental

The behavior of the b-value and other attributes of fracture scaling on the laboratory level were verified in experiments

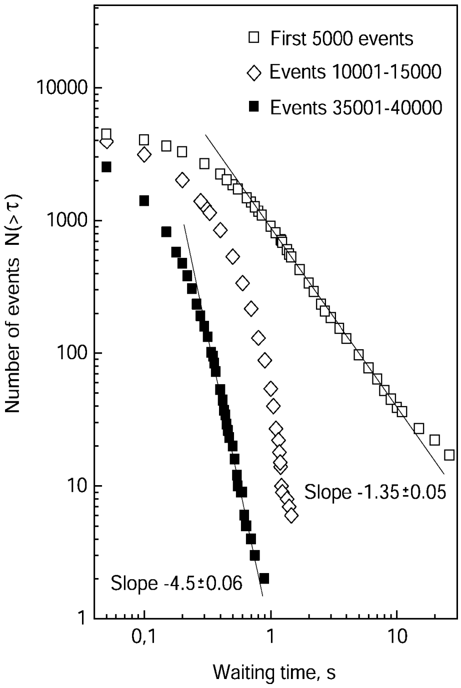

Fig. 4. Waiting time distributions in different time windows. The straight lines show the power law $N(>\tau)\propto\tau^{-\gamma}$.

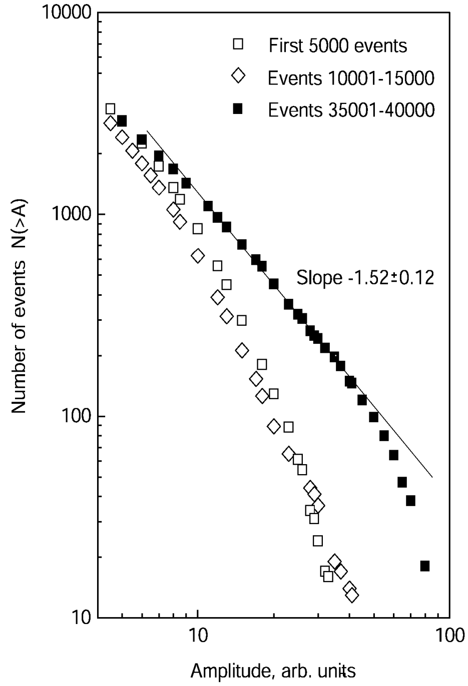

Fig. 5. Amplitude distributions in different time windows. The solid straight lines shows the power law $N_I(>A)\propto A^{-\delta}$ in time window T_3.

on the rock fracture. The axial hydrostatic compression was applied on the ends of cylinders cut from Westerly granite with the diameter and height equal to 76 mm and 190 mm, respectively. The load varied during the experiment in dependence of the acoustical emission (AE) activity in order to keep it at the approximately constant level. The drive control of this kind allowed one to extend the cycles of loading up to a few hours. AE signals resulted from events of microfracturing in the sample were detected by a set of resonance (0.6 MHz) piezoelectric transducers fixed on the sample's surface. The output of transducers was 1.0 to 18 mV.

3.2 Temporal correlation

The waiting-time (τ) distributions, $N(>\tau)$ versus τ, at various stages of loading were constructed for three time windows (here N is the number of intervals between successive signals whose value exceeds τ). Each time window included 5000 successive events and, consequently, the window widths, T_i, (i is the conventional number of window) were not the same. We calculated the time distributions for first 5000 events (T_1); events 10 001–15 000 (T_2); and events 35 001–40 000 (T_3). The experiment covered 4.4×10^4 detected events; its total duration was 11.4×10^3 s. The result is depicted in Fig. 4. Once can see that the waiting times in windows T_1 and T_3 are distributed in accordance with a scaling law

$$N(>\tau)\propto\tau^{-\gamma} \quad (4)$$

but the difference between initial and final periods of loading manifests itself in the graphs' slope, and correspondingly, the values of exponent γ are not the same: $\gamma(T_1)=-1.35\pm0.05$ and $\gamma(T_3)=-4.50\pm0.06$. This difference demonstrates a trend to reduction of the intervals between AE pulses at the terminal stage of loading, that is the damage acceleration before failure.

In the intermediate time window (T_2) the waiting-time statistics does not follow Eq. (4).

3.3 Energy release

A similar procedure was also performed in respect to the amplitude distribution of the AE signals. The calculated distributions $N(>A)$ versus A in three time intervals are shown in Fig. 5 (here N is the number of signals whose amplitude exceeds A). A portion that can be approximated with a straight line is well pronounced only at the final stage of loading. The exponent of the power law

$$N(>A)\propto A^{-2b} \quad (5)$$

determined from the graph's slope is equal to -1.52 ± 0.12.

The distribution of the energy release in AE bursts is directly corresponded with the amplitude distribution, $E\propto A^2$, and, hence, this is described by the scaling law:

$$N(>E)\propto E^{-b} \quad (6)$$

(here N is the number of events with the energy greater than E). For the exponent we have b=0.76±0.06. Unlike the case of the sea-ice cover fragmentation where the b-value was defined over the entire time period of observation, the scaling behavior of the energy distribution in rock fracture events could be clearly seen only before the macroscopic failure, since the laboratory experiment cannot model the recovery process.

4 Discussion

The fracture is a cumulative, self-organizing process where the fractal damage structure results from a chain of correlated events. A term "self-organization" implies that a system starts in state with uncorrelated dynamics and evolves towards more and more organized behavior characterized by a sequence of events correlated both in space and time (Paczuski et al., 1996). The process of fracture could be a direct illustration of this paradigm if one considers a situation when in a certain moment the external load is applied to the solid without any forcing prehistory and, consequently, without previously induced damages. No events – no correlation. However, in real geophysical systems an observer deals with the objects subjected in the past to various stress effects whose action inevitably had to produce a variety of stable defect sites. Therefore, one should expect that fracturing of natural heterogeneous materials, such as rocks or sea-ice sheets studied in this communication, would exhibit the critical dynamics as beginning from the start of observation. Despite the great difference in the scale of the considered phenomena as well as in the physical and mechanical properties of the objects studied (including their capability to restore initial state), it should be stressed that the fracture process in both cases took place in highly heterogeneous systems. The heterogeneity of materials is of great importance since it is this property that determines the delocalized nature of fracturing in contrast to single crack initiation to be expected in homogeneous media.

A scaling hypothesis implies the correlated development of the fracture process at all temporal stages, but due to specific properties of the heterogeneous matter it could be governed by different power-law exponents in different time intervals, with the uncorrelated behavior in transition periods (Contoyiannis et al., 2005). The variability of the self-organized process is closely related with that fact that the heterogeneous systems tend to exhibit some hierarchic properties. In fracturing system, the hierarchicity manifests itself in clustering of smaller defects with formation of larger damages. The clustering specifies a transient state when the "reservoir" of smaller damages is already depleted, and the ensemble of larger damages is yet few in number to exhibit the cooperative behavior. This transition period is similar to a period of hierarchic transition whose duration and spatial issue are determined by the structural peculiarities of the material. Temporal gaps between cycles of accumulation and clustering (or fault nucleation) should, time to time, break the permanent time invariance of the fracture process.

The results obtained in this work confirm the presence of time intervals when the correlation between events induced by the external forcing is temporarily disturbed.

In the case of the ASIC fragmentation, a sudden formation of giant leads followed the period of time characterized by the loss of temporal invariance of the sea-ice drift dynamics. In terms of the classification of the hierarchicity of the ASIC suggested by Overland et al. (1995), this was a transition of the fracture process from the regional scale level to the climate scale level.

It should be stressed that the concept of hierarchic structure of the ASIC was introduced by Overland et al. (1995) from considering the scales of the rate of the main process but not on the base of searching for characteristic areas or other dimensional parameters of the sea-ice cover. This approach is of particular importance, since the temporal parameters manifest themselves only in dynamics. The period precedent to the large-scale fragmentation is characterized by the failure of high-rate local processes to compensate the energy income, and the energy exchange was realized through more powerful but "slower" processes of upper temporal level. The temporal correlation of the internal forcing (characterized by the events of ice-field accelerations) became disturbed.

A deviation from the time-invariant development of the ensemble of hypocenters was also detected in our experiments on the rock fracture. At a certain stage of loading the initially invariant distribution of waiting times between fracture events became uncorrelated, and then returned to a power-law representation. A similar scenario as referred to the seismic activity was described by Contoyiannis et al. (2005) who explained the interplay between persistent and antipersistent series of stress-induced electromagnetic signals by the presence of weak and strong volumes in a heterogeneous medium. The heterogeneity of this kind is responsible for the multifocal development of fracturing with simultaneous variations of temporal and spacial scaling properties.

At the same time, the invariance of the energy release in the ASIC was not interrupted in the period of large-scale fragmentation. In contrast to the total loss of the temporal self-similarity prior to fragmentation, only a reversible variation of the b-value was detected. A decrease of the b-value that occurred prior to the "catastrophe" is in agreement with numerous laboratory experiments and field observations reported by many authors (Sobolev et al., 1991; Kapiris et al., 2004; Amitrano, 2006). A distinguishing feature of the ASIC dynamics is the reversibility of the b-value variation, while in conventional geophysical objects the decrease of the b-value is usually considered as a precursor of terminal state (Amitrano, 2006, and references therein). In the case of the AE experiment, the variation of b-value could not be determined reliably due to quite limited range of the power-law behavior at the initial stage of loading (less than an order of magnitude, Fig. 5).

To summarize, the heterogeneity of the temporal pattern of the fracture process in geophysical objects of various scale level and nature manifests itself in the presence of time gaps during which the distribution of waiting times between individual fracture events does not follow the power-law statistics thus indicating a transient disturbance of the critical state of externally driven system. These periods of temporal decorrelation are related with the presence of interlevel transitions in hierarchic systems where upper levels appear as aggregate properties of lower levels (Overland et al., 1995) with disturbing the long-term memory in the system: the time sequences of events belonging to a particular level cannot store the information on events that occurred on the adjacent level. The waiting time distributions are self-similar on all scale levels but characterized by different power-law exponents.

Acknowledgements. Authors are indebted to D. Lockner from the U.S. Geological Survey for presented AE data.

Edited by: A. Tarquis
Reviewed by: K. Eftaxias

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