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# Bivariate option pricing with GARCH-NIG model and dynamic copula

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**Abstract:** GARCH process was developed with the combination of dynamic copula for pricing bivariate contingent claims. In order to take into account the stylized factors in finance, such as skewness, leptokurtosis and fat tails, NIG distribution was fitted for residuals. Furthermore, the dynamic copula method was applied to describe the dependence structure between the underlying assets. The approach was illustrated with call-on-max option of Shanghai and Shenzhen Stock Composite Indices. The results showed the advantage of the suggested approach.

**Key words:** call-on-max option; GARCH process; NIG distribution; copula; dynamic copula

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# 基于 GARCH-NIG 模型和动态 Copula 的双标的型期权定价

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**摘要**: 结合动态 copula 和 GARCH 模型, 发展了双标的型未定权益的定价方法. 针对诸如非对称、尖峰态和厚尾现象等各种金融中的固有因素, 采用 NIG 分布拟合于残差量. 而标的资产之间的相关结构由动态 copula 来刻画. 以上海证券指数和深圳证券指数为双标的资产最大认购期权为例, 理论方法得到了有效的实证结果.

**关键词**: 最大认购期权; GARCH 过程; NIG 分布; copula; 动态 copula

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# 0 Introduction

Over the years, various of pricing models, following the great work of Black and Scholes[1], have been developed to price multivariate options. However, in most of these models, there are two main disadvantages. The first one is that correlation was used to measure the dependence between assets. It was pointed out by Embrechts et al.[2] that correlation may cause some confusion and misunderstanding. Indeed, it is a financial stylized fact that correlations observed under ordinary market conditions differ substantially from correlations observed in hectic periods. The second disadvantage concerns the heteroskedasticity of asset returns. Although GARCH option pricing models have experienced some empirical successes[3,4], the distribution of the residual term in GARCH process attracts a lot of attention. Engle[5] used Normal distribution as the conditional distribution. However, in order to fully capture the excess kurtosis and fat tails, alternative distributions such as $t$ distribution and GED distribution are considered. Unfortunately, these two distributions both have their own limitations: On the one hand, it is inappropriate to use $t$ distribution to model continuously compounded asset returns since its moment generating function with any finite degree of freedom does not exist; on the other hand, GED distribution is restricted to symmetry. Therefore, a more flexible and suitable distribution is called for.

In order to avoid the two main disadvantages in pricing models, we considered an approach by combining GARCH-NIG model with dynamic copula. The first reason for choosing such framework came from the properties of NIG (Normal Inverse Gaussian) distribution whose moment generating function exists. Although GARCH process introduced by Bollerslev[6] can capture excess kurtosis and fat tails of equity returns, NIG process is also proved to hold such advantages[7]. NIG process was introduced by Barndorrff-Nielsen[8], and then a lot of papers have been put forward in this field. With the aid of NIG distribution in the stationary NIG process, the residual term in the GARCH process can be handled more flexibly. Secondly, as copula has proven an ideal tool to measure the dependence structure, a dynamic copula approach was adopted for considering the change of dependence structure when the underlying assets cover a long time period. Therefore, in the present paper, a new dynamic approach to price bivariate options with GARCH-NIG process and time-varying copula was proposed.

Compared with the previous methods in this aspect, the proposed approach made the dynamic pricing more reasonable and tractable. In our empirical study, call-on-max option based on Shanghai and Shenzhen Stock Composite Indices was applied to illustrate the innovated approach.

The remainder of this paper is organized as follows. In Section 1, the requisite knowledge is reviewed. Section 2 explains in detail the new idea for pricing bivariate option with GARCH-NIG process and dynamic copula. In Section 3, empirical study is described and results are presented. Section 4 gives the conclusion.

# 1 Knowledge review

## 1.1 Option valuation

This paper concentrates on call-on-max option, but the technique is sufficiently general to apply for other multivariate options. The payoff of a unit amount call-on-max option is

$$\max\{\max(S_{1,T}, S_{2,T}) - K, 0\},$$

where $T$ is the maturity time, $S_{i,t}$ is the price of the $i$-th asset $(i=1,2)$ at time $t$ $(0 \leqslant t \leqslant T)$, and $K$ is the strike price. In the following, $r_{i,t}$ is used to denote the log-return on $i$-th index $(i=1,2)$ from time $t-1$ to time $t$, i.e., $r_{i,t} = \log(\frac{S_{i,t}}{S_{i,t-1}})$.

The fair value of the option is determined by taking the discounted expected value of the option's payoff under the risk-neutral measure. As the call-on-max is typically traded over the counter, price data are not available. Therefore, valuation models cannot be tested empirically. However, comparing models with different assumptions can be implemented.

### 1.2 NIG distribution

NIG (Normal Inverse Gaussian) distribution is a special case of generalized hyperbolic distribution. The density function of NIG distribution is

$$f_{\mathrm{NIG}}(x;\alpha,\beta,\delta,\mu) = \frac{\delta\alpha \exp(\delta\sqrt{\alpha^2-\beta^2})K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})\exp(\beta(x-\mu))}{\pi\sqrt{\delta^2+(x-\mu)^2}},$$

where $K_\lambda$ is the modified Bessel function of the third kind and $x \in \mathbb{R}$. If the random variable $X$ has a NIG distribution, we denote it as $X \sim \mathrm{NIG}(\alpha,\beta,\delta,\mu)$. The mean and variance of NIG distribution are given by

$$\mathrm{E}(X) = \mu + \frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}} \quad \text{and} \quad \mathrm{Var}(X) = \frac{\delta\alpha^2}{(\alpha^2-\beta^2)^{3/2}}.$$

The parameters $\alpha,\beta,\delta,\mu \in \mathbb{R}$ are interpreted as follows: $\mu$ is the location parameter and $\delta > 0$ is the scale parameter. The parameter $0 \leqslant |\beta| < \alpha$ describes the skewness and $\alpha > 0$ gives the kurtosis. Particularly, if $\beta = 0$, the distribution is symmetric, and if $\alpha \to \infty$, the Gaussian distribution is obtained in the limit.

### 1.3 Risk-neutralization for GARCH process

In order to derive the joint risk-neutral return process, the objective marginals should be specified. We propose to transform each marginal process separately: In the objective environment, the one-period log-return for every index is assumed to be conditionally distributed under probability measure $P$, together with a GARCH process, that is, for $i=1,2$:

$$\begin{aligned} r_{i,t} &= m_{i,t} + \sqrt{h_{i,t}}\varepsilon_{i,t}, \\ h_{i,t} &= \alpha_{i,0} + \textstyle\sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}h_{i,t-j}, \\ \varepsilon_{i,t}|\varphi_{i,t-1} &\sim D(0,1) \quad \text{under measure } P, \end{aligned} \tag{1.1}$$

where the conditional mean $m_{i,t}$ is any predictable process, the information set $\varphi_{i,t-1}$ contains all information up to and including time $t-1$. Under historical measure $P$, $\varepsilon_{i,t}$ follows some distribution $D$, whose distribution function is denoted as $F_D$, with zero mean and variance 1. Other restrictions are $p \geqslant 0, q \geqslant 0$; $\alpha_{i,0} > 0$; $\alpha_{i,j} \geqslant 0\,(j=1,\ldots,q)$; $\beta_j \geqslant 0\,(j=1,\ldots,p)$. To ensure the covariance stationarity of GARCH $(p,q)$ process, $\sum_{j=1}^{q}\alpha_{i,j}+\sum_{j=1}^{p}\beta_j$ is assumed to be less than 1.

In order to obtain the risk-neutral price, a generalized risk-neutral valuation relationship is proposed below:

**Assumption 1.1** The equilibrium pricing measure $Q$, defined over the interval $[t_l,t_u]$ is said to satisfy the generalized locally risk-neutral valuation relationship if, for $\forall\, t\in[t_l,t_u-1]$, the following conditions are all satisfied.

(1) $Q$ is mutually absolutely continuous with respect to the objective measure $P$;

(2) there exists a predictable process $\lambda_{i,t}$ such that $\Phi^{-1}[F_D(\varepsilon_{i,t})]+\lambda_{i,t}$, conditionally on $\varphi_{i,t-1}$, is a standard normal random variable with respect to measure $Q$;

(3) $E^Q(\frac{S_{i,t}}{S_{i,t-1}}|\varphi_{i,t-1})=\exp(r_t)$,

where $r_t$ denotes the one period risk free interest rate at time $t$, $\Phi(\cdot)$ denote the standard normal distribution function.

Under some sufficient conditions[9], the above valuation relationship holds, then the asset return can be simply characterized by a risk-neutral dynamic model:

**Theorem 1.1** Under the pricing measure $Q$, the model for one-period log-return $r_{i,t}$ is given by

$$\begin{aligned} r_{i,t} &= m_{i,t}+\sqrt{h_{i,t}}F_D^{-1}[\Phi(Z_{i,t}-\lambda_{i,t})], \\ h_{i,t} &= \alpha_{i,0}+\textstyle\sum_{j=1}^{q}\alpha_{i,j}\Big\{F_D^{-1}[\Phi(Z_{i,t-j}-\lambda_{i,t-j})]\Big\}^2+\sum_{j=1}^{p}\beta_{i,j}h_{i,t-j}, \\ Z_{i,t}|\varphi_{i,t-1} &\sim N(0,1) \quad \text{under measure } Q, \end{aligned} \tag{1.2}$$

where $\lambda_{i,s}$ is the solution to

$$E^Q[\exp(m_{i,s}+\sqrt{h_{i,s}}F_D^{-1}[\Phi(Z_{i,s}-\lambda_{i,s})])|\varphi_{i,s-1}]=\exp(r_s). \tag{1.3}$$

Theorem 1.1 provides a relatively easy transformation to local risk-neutral environment. According to this theorem, the terminal asset price is derived in the following corollary:

**Corollary 1.1** Under the pricing measure $Q$, the terminal price for the $i$-th $(i=1,2)$ asset can be expressed as

$$S_{i,T}=S_{i,t}\exp\{\sum_{s=t+1}^{T}[m_{i,s}+\sqrt{h_{i,s}}F_D^{-1}[\Phi(Z_{i,s}-\lambda_{i,s})]]\}. \tag{1.4}$$

where $h_{i,s}$, $Z_{i,s}$ and $\lambda_{i,s}$ are given in Equation (1.2) and (1.3).

It is necessary to note that the discount asset price process $\mathrm{e}^{-r_t t}S_{i,t}$ is a $Q$-martingale. Therefore, the call-on-max option, with exercise price $K$ maturing at time $T$, has the $t$-time value

$$V_t^{COM}=\exp(-\sum_{s=t+1}^{T}r_s)E^Q[\max(\max(S_{1,T},S_{2,T})-K,0)]. \tag{1.5}$$

### 1.4 Dependence measure: copula

From one point of view, copulas are functions that join or "couple" multivariate distribution functions to their one-dimensional marginal distribution functions[10]. Let $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ be an $n$-dimensional random variable with multivariate distribution function $F(x_1, x_2, \ldots, x_n)$ and continuous marginal distributions $F_1, F_2, \ldots, F_n$. Sklar's theorem[11] implies that there exists a unique copula $C$ such that

$$F(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \cdots, F_n(x_n))$$

for all $x_1, x_2, \ldots, x_n \in \mathbb{R}$. Conversely, for any marginal distributions $F_1, F_2, \ldots, F_n$ and any copula function $C$, the function $C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n))$ is a multivariate distribution function with given marginal distributions $F_1, F_2, \ldots, F_n$. This theorem provides the theoretical foundation for the widespread use of the copula approach in generating multivariate distributions from univariate distributions.

## 2 Bivariate option pricing with GARCH-NIG process and dynamic copula

In our proposed pricing model, the objective marginals are characterized by GARCH-NIG process introduced in Equation (1.1), we specify the distribution $D$ as NIG distribution. When the pricing model is transformed to risk-neutral environment, the requirement that $\lambda_{i,s}$ is the solution to Equation (1.3) may be extremely difficult to deal with. Note that there exists a one to one correspondence between $\lambda_{i,s}$ and $m_{i,s}$, if $m_{i,s}$ is assumed to be measurable with respect to the information set $\varphi_{i,s-1}$, Equation (1.3) may be rewritten as

$$m_{i,s} = r_s - \ln E^Q[\exp(\sqrt{h_{i,s}} F_{\mathrm{NIG}}^{-1}[\Phi(Z_{i,s} - \lambda_{i,s})])|\varphi_{i,s-1}]. \tag{2.1}$$

Therefore, Equation (1.4) is displayed as

$$\begin{aligned} S_{i,T} = \ & S_{i,t} \exp\{\textstyle\sum_{s=t+1}^{T}[r_s - \ln E^Q[\exp(\sqrt{h_{i,s}} F_{\mathrm{NIG}}^{-1}[\Phi(Z_{i,s} - \lambda_{i,s})])|\varphi_{i,s-1}] \\ & + \sqrt{h_{i,s}} F_{\mathrm{NIG}}^{-1}[\Phi(Z_{i,s} - \lambda_{i,s})]]\}. \end{aligned} \tag{2.2}$$

In the empirical work, we assume that $\lambda_{i,s} = \lambda_i$ for all $s$, and the generalization will be discussed in further research.

Furthermore, the objective copula and risk-neutral copula are assumed to be the same.

**Proposition 1.1** The objective copula describing the dependence between the two assets' log-returns in Equation (1.1) and the risk-neutral copula for the dependence of the log-returns in Equation (1.2) are the same.

**Proof** We define an operator $\Psi$ by $\Psi(\varepsilon_{i,t}) \equiv \Phi^{-1}[F_{\mathrm{NIG}}(\varepsilon_{i,t})]$, where $\varepsilon_{i,t}$ is a $P$-measure NIG random variable with $\mathrm{E}(\varepsilon_{i,t}|\varphi_{i,t-1}) = 0$ and $\mathrm{Var}(\varepsilon_{i,t}|\varphi_{i,t-1}) = 1$. As $\Psi(\varepsilon_{i,t})|\varphi_{i,t-1}$ is a $P$-measure standard normal random variable, so there exists an one to one transformation with the same mean and variance, conditional on the past information:

$$\text{NIG r.v. (zero mean and unit variance)} \ \underset{\Psi^{-1}}{\overset{\Psi}{\rightleftarrows}} \ \text{standard Normal r.v}.$$

From the second conditon in Assumption 1.1, it can be seen that the risk neutralization merely casuses the transformed standard residuals to undergo a shift in mean, with magnitude of $\lambda_{i,t}$, that is, conditional on $\varphi_{i,t-1}$:

$$\begin{array}{ccc} \text{measure P} & \to & \text{measure Q}, \\ \text{std. Normal r.v.}\,\Psi(\varepsilon_{i,t}) & \to & \text{std. Normal r.v}\,\Psi(\varepsilon_{i,t})+\lambda_{i,t}\,(=Z_{i,t}), \\ \text{NIG r.v.}\,\varepsilon_{i,t} & \to & \text{NIG r.v.}\,\Psi^{-1}(Z_{i,t}-\lambda_{i,t})\,(=\varepsilon_{i,t}). \\ (\text{mean 0 and variance 1}) & & (\text{mean}\,\lambda_{i,t}\,\text{and variance 1}) \end{array}$$

In addition, after filtering the log-returns of underlying assets from GARCH-NIG process, the copula measuring the dependence actually acts on the standard residuals. So the standard residuals under the objective and risk-neutral measures are necessary to be presented as

$$\begin{array}{ccc} \text{measure P} & \to & \text{measure Q}, \\ \varepsilon_{i,t} & \to & \xi_{i,t}=\Psi^{-1}(Z_{i,t}-\lambda_{i,t})+\lambda_{i,t}=\varepsilon_{i,t}+\lambda_{i,t}. \end{array}$$

If we define $T(\varepsilon_{i,t})=\varepsilon_{i,t}+\lambda_{i,t}$, then $T$ is strictly increasing and $\xi_{i,t}=T(\varepsilon_{i,t})$. According to the invariant property of copulas[12], the copula of $(\varepsilon_{1,t},\varepsilon_{2,t})$ is exact the copula of $(\xi_{1,t},\xi_{2,t})$ for $\forall\, t$, that is, the objective copula is the same as the risk-neutral one, thus completes the proof.

Therefore, the transformation from Equation (1.1) to Equation (1.2), in conjunction with the same dynamic conditional copula for the objective and local risk-neutral environments, allows a particularly convenient pricing method for bivariate options.

As for the dynamic copula, our interest lies on the change of parameters in copulas with static copula family. We firstly use the moving window to observe the change trend of copula. On different subsamples divided by moving window, the best copulas are chosen according to AIC criterion[13]. If the results show that the copula family remains changeless while the copula parameters change, we then define a time-varying parameter function expressed in the following. With the standardized residuals $(\varepsilon_{1,t},\varepsilon_{2,t})$, the dynamic copula $C$ is assumed to have the time dependent parameter vector $\theta_t=(\theta_{1,t},\theta_{1,t},\ldots,\theta_{m,t})$, such that

$$\theta_{l,t}=\theta_0+\sum_{i=1}^{g}\eta_i\prod_{j=1}^{2}\varepsilon_{j,t-1}+\sum_{k=1}^{s}\zeta_k\theta_{l,t-k} \tag{2.3}$$

for $l=1,2,\ldots,m$ and $\eta_i\,(i=1,2,\ldots,g)$, $\zeta_k\,(k=1,2,\ldots,s)$ are scalar parameters that can be estimated by the maximum likelihood method. Equation (2.3) defines a dynamic structure, motivated by GARCH process, for the dependence parameters.

Benefiting from the identification result for the objective and local risk-neutral conditional copulas, pairs of standard normal random variables $Z_{i,t}$ $(i=1,2)$ in Equation (1.2) can be drawn from the dynamic copula. This procedure is accomplished with the aid of Monte Carlo simulations. These generated random variables are then applied to obtain the transformed residuals as shown in Equation (1.2). Eventually, the payoffs implied by these residuals are averaged and discounted at the risk-free rate, and the fair value of the call-on-max option expressed in Equation (1.5) can be obtained.

# 3 Empirical results

The empirical work deals with the call-on-max option on Shanghai and Shenzhen Stock Composite Indices. The sample contains 1 857 daily observations from January 4, 2000 to May 29, 2007. The log-returns of Shanghai and Shenzhen Stock Composite Indices are shown in Fig. 1. We restrict the parameters $\beta = 0$ and $\mu = 0$ for the NIG distribution fitting, then the results are shown in Tab. 1. In Fig. 2 the Q-Q plot for NIG distribution and Gaussian distribution are presented, from which it can be seen that the assigned NIG distributions fit well to the data, especially in the tails.

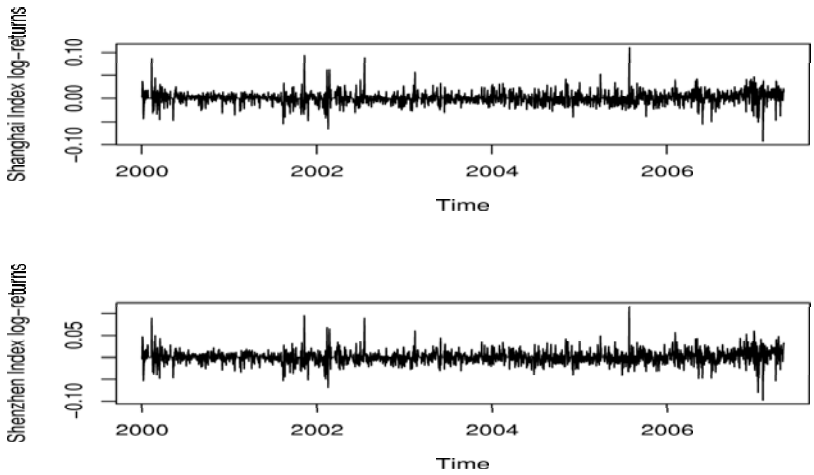


Fig. 1　Log-returns for Shanghai and Shenzhen Stock Composite Indices

Tab. 1　Estimates of NIG fitting parameters for marginal log-returns

| | Shanghai Index | | Shenzhen Index | |
|---|---|---|---|---|
| $\bar{\alpha}$ | 4.536e-01 | (4.539e-03) | 5.275e-01 | (6.117e-03) |
| $\bar{\beta}$ | 0.000 | | 0.000 | |
| $\mu$ | 0.000 | | 0.000 | |
| $\sigma$ | 1.409e-02 | (2.044e-07) | 1.516e-02 | (2.146e-07) |
| AIC | -10 971.40 | | -10 649.37 | |
| BIC | -10 960.34 | | -10 638.32 | |

Note: Figures in brackets are standard errors.

The parameter estimates of GARCH(1,1)-NIG models (see Equation (1.1)) are listed in Tab. 2, and in order to compare, the estimate results for GARCH-Gaussian models are also presented. From AIC values, GARCH-NIG models prove better for both Shanghai and Shenzhen Stock Composite Indices.

Several kinds of copulas are considered, including Gaussian, Frank, Gumbel, Clayton, Student $t$ copulas. The whole sample is divided into subsamples by the moving window. We let

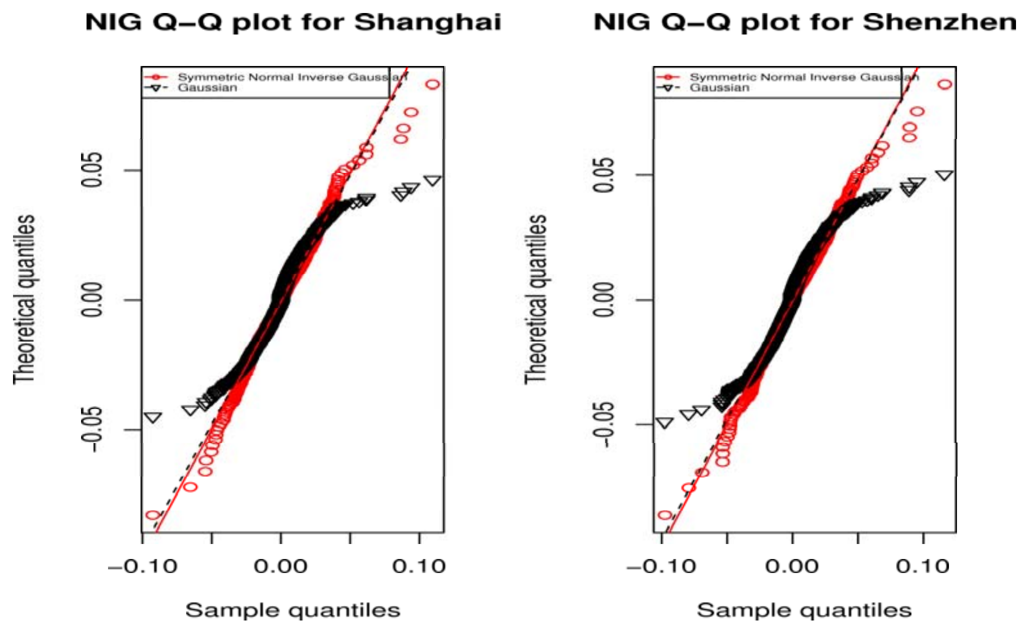


Fig. 2 Comparison of Q-Q plots for NIG fitting and Gaussian fitting

Tab. 2 Estimates of GARCH-NIG and GARCH-Gaussian parameters

| GARCH-NIG | Shanghai Index | Shenzhen Index |
|---|---|---|
| $m$ | 6.065e-04 (1.952e-04) | 7.260e-04 (2.300e-04) |
| $\bar{\alpha}$ | 8.103e-01 (2.807e-03) | 7.959e-01 (5.837e-03) |
| $\alpha_0$ | 3.597e-05 (2.032e-01) | 3.532e-05 (1.989e-01) |
| $\alpha_1$ | 2.758e-01 (3.865e-01) | 3.015e-01 (5.477e-01) |
| $\beta_1$ | 5.651e-01 (2.988e-01) | 5.558e-01 (7.747e-01) |
| AIC | -11 037.14 | -10 708.29 |

| GARCH-Gaussian | Shanghai Index | Shenzhen Index |
|---|---|---|
| $m$ | 3.833e-04 (2.419e-04) | 3.761e-04 (2.882e-04) |
| $\alpha_0$ | 5.136e-06 (7.682e-07) | 5.529e-06 (9.011e-07) |
| $\alpha_1$ | 8.115e-02 (4.726e-03) | 8.721e-02 (5.496e-03) |
| $\beta_1$ | 8.966e-01 (5.034e-03) | 8.950e-01 (5.249e-03) |
| AIC | -10 793.11 | -10 518.3 |

Note: Figures in brackets are standard errors.

the window consist of 300 observations, and we move it by 100 observations, thus 16 windows cover all the sample. The considered copulas are fitted to the standardized residual pairs from GARCH-NIG models on different subsamples. We then decide series of best fitting copulas by AIC criterion. The results for the best fitting copulas on all subsamples are shown in Tab. 3.

Results listed in Tab. 3 show that on almost all subsamples, Student $t$ copula turns out to be the best fitting copula for GARCH-NIG model. So we assume that the copula family remains static as Student $t$, while the parameter changes. In addition, it can be observed that the correlation changes little while the degree of freedom varies obviously. Therefore, it is

Tab. 3 Dynamic Copula Analysis for GARCH-NIG model

| Window | Copula | Parameter |
|---|---|---|
| 1 | Student $t$ | 9.109e-1(1.032e-1); 2.444(1.302) |
| 2 | Student $t$ | 9.064e-1(9.990e-2); 3.084(9.590e-1) |
| 3 | Student $t$ | 9.308e-1(9.505e-2); 5.594(9.384e-1) |
| 4 | Student $t$ | 9.451e-1(1.157e-1); 3.903(1.017) |
| 5 | Student $t$ | 9.602e-1(2.804e-1); 7.919(4.215) |
| 6 | Student $t$ | 9.697e-1(1.142e-1); 6.826(3.352) |
| 7 | Student $t$ | 9.654e-1(1.442e-1); 8.098(3.385) |
| 8 | Student $t$ | 9.598e-1(9.931e-2); 6.005(2.104) |
| 9 | Student $t$ | 9.444e-1(2.117e-1); 7.087(3.630) |
| 10 | Student $t$ | 9.385e-1(1.594e-1); 7.675(2.000) |
| 11 | Student $t$ | 9.419e-1(1.612e-1); 9.947(1.352) |
| 12 | Gussian | 4.450(2.167e-1) |
| 13 | Student $t$ | 9.228e-1(1.533e-1); 5.682(2.388) |
| 14 | Student $t$ | 8.831e-1(2.594e-1); 3.574(10.030) |
| 15 | Student $t$ | 8.727e-1(7.247e-2); 3.300(8.206) |
| 16 | Student $t$ | 8.493e-1(1.030e-1); 4.937(2.129) |

**Note**: Figures in brackets are standard errors. For the Student $t$ copula, the first parameter is the correlation, the second parameter is the degree of freedom.

reasonable to assume that the degree of freedom varies along time while the correlation remains static. The time-varying function for the degree of freedom of Student $t$ copula is

$$\nu_t = l^{-1}(s_0 + s_1\varepsilon_{1,t-1}\varepsilon_{2,t-1} + s_1 l(\nu_{t-1})), \tag{3.1}$$

where $s_0, s_1, s_2$ are parameters and $l(\cdot)$ is a function defined by

$$l(\nu) = \log(\frac{1}{\nu - 2})$$

to ensure that the degree of freedom is not smaller than 2.

The corresponding estimate result for the dynamic copula parameter in Equation (3.1) is listed in Tab. 4.

Tab. 4 Parameter estimates for dynamic parameter of Student $t$ copula

| $\rho$ | $\nu$ |
|---|---|
| | $s_0$: 4.384e-01 (1.497) |
| 9.176e-01 (2.361e-02) | $s_1$: -6.055e-02 (7.407e-01) |
| | $s_2$: -9.414e-01 (4.165e-01) |

**Note**: Figures in brackets are standard errors and $\rho$ denotes the correlation estimate.

Standard normal random variables can then be generated from this dynamic Student $t$ copula, and the transformed residuals can be sampled to compute the option price. It is assumed here that the initial asset prices are normalized to unity, and the claim maturity is 1 month. Moreover, the strike price is set at levels between 0.5 and 2.7, the risk-free rate is assumed to be 6% per annum, and $\lambda_i$ is considered as 5%. The Monte Carlo study is based on 100 000 replications.

Using the proposed GARCH-NIG model with dynamic copula, the option prices are represented in Fig. 3, compared with the option prices implied by GARCH-Gaussian dynamic model. It can be observed that the GARCH-Gaussian model generally underestimates the price.

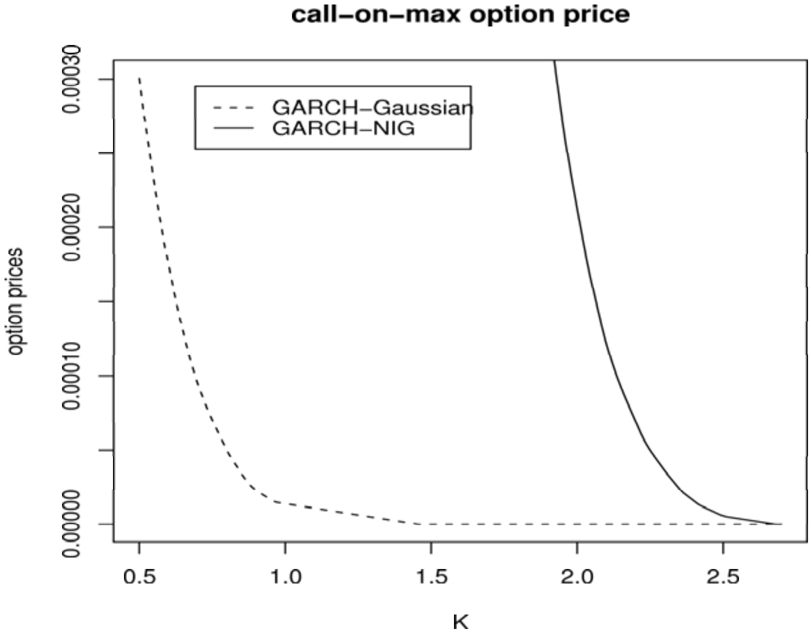


Fig. 3 Option prices from dynamic GARCH-NIG and GARCH-Gaussian models

# 4 Conclusion

In this paper, a systematic new approach for bivariate option pricing with GARCH-NIG model and dynamic copula has been introduced. The contributions of the proposed approach can be generalized as follows: (1) GARCH-NIG model seems more suitable to the real data than GARCH-Gaussian model; (2) Analyzing the change trend of copula allows to create the dynamic model more effectively; (3) The dynamic copula with time-varying parameter enables the changes of dependence to be more tractable; (4) The dynamic method is not restricted only to one-parameter copulas, multi-parameter copulas can also be considered.

## [ References ]

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