Multiquark states and the mixing of scalar meson *

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Abstract In order to describe Kaon-Nucleon scattering data, the mixing of scalar meson σ_0 and σ_8 must be introduced in the chiral $SU(3)$ quark model. Inspired by this, now the mixing of scalar meson is further considered to study some interesting dibaryons in the chiral $SU(3)$ quark model. The results show that the mixing of scalar meson has different effects on these dibaryons.

Key words chiral $SU(3)$ quark model, dibaryon, mixing of scalar meson

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1 Introduction

The kaon-nucleon (K-N) scattering has been studied in the constituent quark model using the resonating group method (RGM)[1]. In this model, the short-range interaction comes from one-gluon exchange (OGE), long-range interaction comes from confinement potential, also includes parameterized π and σ field exchanges between quarks. Their results show that they failed to describe the K-N interaction.

How to solve this problem? It seems that a reasonable and effective model is needed.

The quantum chromodynamics (QCD) is the basic theory of strong interaction. However, it is very difficult to solve nonperturbative QCD effect. Therefore, we still need QCD-inspired models to help. Among these models, the chiral $SU(3)$ quark model[2] has been quite successful in reproducing the energies of the baryon ground states, the binding energy of deuteron, the nucleon-nucleon (N-N) scattering phase shifts of different partial waves, and the hyperon-nucleon (Y-N) cross sections by the RGM calculation. In the chiral $SU(3)$ quark model, the nonet pseudoscalar meson exchanges and the nonet scalar meson exchanges are considered, and also includes the OGE and confinement potential as in Ref. [1].

Huang[3] et al. studied the K-N scattering by carrying on RGM calculation in the chiral $SU(3)$ quark model. They reproduced the experimental data quite well. Why does the chiral $SU(3)$ quark model can describe K-N interaction successfully? We made an analysis for these two models. Compared with Ref. [1], the mixing between σ_0 and σ_8 can be introduced in Ref. [3]. This is the main reason to successfully describe K-N interaction. When the mixing of scalar meson is considered, the attraction force of scalar meson between K and N can be reduced a lot, so the K-N scattering can be reasonably described.

Inspired by this, now the mixing of scalar meson is further considered to study some interesting dibaryons in the chiral $SU(3)$ quark model. In fact, in our previous works, many interesting dibaryons[4—8] have been predicted in this model. However, the mixing of scalar meson hasn't considered in all these works. The paper is organized as follows. In the next section, the framework of the chiral $SU(3)$ quark model are briefly introduced. The calculated results and some discussions are shown in Sec. 3 and the summary is given in Sec. 4.

2 Formulation

2.1 Model

The chiral $SU(3)$ quark model has been described in the literature[2] and we refer the reader to the work for details. Here we just give the salient feature of this

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model.

In the chiral $SU(3)$ quark model, the coupling between chiral field and quark is introduced to describe nonperturbative QCD effect. The interacting Lagrangian can be written as:

$$\mathcal{L}_I = -g_{\rm ch}\overline{\psi}\left(\sum_{a=0}^{8}\sigma_a\lambda_a + {\rm i}\sum_{a=0}^{8}\pi_a\lambda_a\gamma_5\right)\psi, \quad (1)$$

where λ_0 is a unitary matrix, σ_0,..., σ_8 are the scalar nonet fields, and π_0,...,π_8 the pseudoscalar nonet fields. We can prove that L_I is invariant under the infinitesimal chiral $SU(3)_{\rm L}\times SU(3)_{\rm R}$ transformation. We should mention here that only one coupling constant $g_{\rm ch}$ is needed by chiral symmetry requirement.

In this model, the total Hamiltonian of baryon-baryon systems can be written as

$$H = \sum_{i=1}^{6} T_i - T_G + \sum_{i<j=1}^{6} V_{ij} \ , \quad (2)$$

where $\sum_i T_i - T_G$ is the kinetic energy of the system, and V_{ij} represents the quark-quark interactions,

$$V_{ij} = V_{ij}^{\rm OGE} + V_{ij}^{\rm Conf} + V_{ij}^{\rm ch} \ , \quad (3)$$

where $V_{ij}^{\rm OGE}$ is the OGE interaction, and $V_{ij}^{\rm Conf}$ is the confinement potential. $V_{ij}^{\rm ch}$ represents the chiral fields induced effective quark-quark potential, which includes the scalar boson exchanges and the pseudoscalar boson exchange,

$$V_{ij}^{\rm ch} = \sum_{a=0}^{8} V_{ij}^{\sigma_a} + \sum_{a=0}^{8} V_{ij}^{\pi_a} \ . \quad (4)$$

More details can be found in Ref. [2].

2.2 Determination of the parameters

We briefly give the procedure for the parameter determination. The three initial input parameters, i.e, the harmonic-oscillator width parameter $b_{\rm u}$, the up (down) quark mass $m_{\rm u(d)}$ and the strange quark mass $m_{\rm s}$, are taken to be the usual values: $b_{\rm u}$ = 0.5 fm, $m_{\rm u(d)}$ = 313 MeV, and $m_{\rm s}$ = 470 MeV. the coupling constant for scalar and pseudoscalar chiral field coupling, $g_{\rm ch}$, is fixed by the relation

$$\frac{g_{\rm ch}^2}{4\pi} = \frac{9}{25}\frac{m_{\rm u}^2}{M_{\rm N}^2}\frac{g_{\rm NN\pi}^2}{4\pi} \ , \quad (5)$$

with the experimental value $g_{\rm NN\pi}^2/4\pi = 13.67$. The mass of the mesons are taken to be experimental values, except for the σ meson which is a adjustable parameter and can be decided by fitting the experimental binding energy of deuteron. The OGE coupling constants $g_{\rm u}$ and $g_{\rm s}$ and the strengths of the confinement potential are fitted by baryon masses and their stability conditions. For mixing of scalar meson, the definition is as follow:

$$\sigma = \sigma_8 \sin\theta^S + \sigma_0\cos\theta^S,$$
$$\epsilon = \sigma_8\cos\theta^S - \sigma_0\sin\theta^S, \quad (6)$$

here we consider two cases: no mixing and ideally mixing. For no mixing, the $\theta^S = 0°$. For ideally mixing, the $\theta^S = 35.3°$, which means that σ meson only acts on the u(d) quark, and ϵ meson on the s quark.

We list the model parameters and binding energy of deuteron in Table 1 and Table 2, respectively.

Table 1. Model parameters.

	θ^S	m_σ
Set 1	0°	595 MeV
Set 2	35.3°	560 MeV

Table 2. Binding energy of deuteron.

	θ^S	$B_{\rm NN}$
Set 1	0°	2.13 MeV
Set 2	35.3°	2.10 MeV

3 Results and discussions

3.1 Theoretical analysis

We solve the RGM equation of the Hamiltonian to study the structure of dibaryon[9—11]. In RGM calculation, the trial wave function is taken to be

$$\Psi_{ST} = \sum_i c_i \Psi_{ST}^i(\overrightarrow{s_i}) \ , \quad (7)$$

with

$$\Psi_{ST}^i(\overrightarrow{s_i}) \ = \ \mathcal{A}(\phi_{\mathcal{A}}(\overrightarrow{\xi_1},\overrightarrow{\xi_2})\phi_{\mathcal{B}}(\overrightarrow{\xi_3},\overrightarrow{\xi_4})\times \chi(\overrightarrow{R}_{AB}-\overrightarrow{s_i})Z(\overrightarrow{R}_{CM})), \quad (8)$$

where A and B shows two clusters, and ϕ, χ and Z represent internal, relative and center of mass motion wave function, respectively. $\overrightarrow{s_i}$ is the generator coordinates, and $\mathcal{A}$ is the antisymmetrizing operator between cluster A and cluster B which is defined as

$$\mathcal{A} \ = \ \left(1-\sum_{i\in A, j\in B} P_{ij}\right)(1-P_{AB}), \quad (9)$$

here $P_{ij} = P_{ij}^r P_{ij}^{\sigma f c}$ is the permutation operator of i and j quark, and P_{AB} of baryon A and B. When two cluster is closed together and $L=0$, $\langle P_{ij}^r\rangle \approx 1$. Thus $\langle P_{ij}^{\sigma fc}\rangle$ is very important to measure the quark exchange effect for various spin-flavor states. There are three case: the first case is $(1-\sum_{i\in A,j\in B}\langle P_{ij}^{\sigma fc}\rangle)\approx 2$, the quark exchange effect makes two baryon cluster closer; the second case is $(1-\sum_{i\in A,j\in B}\langle P_{ij}^{\sigma fc}\rangle)\approx 1$, the quark exchange effect is not important; the third case is $(1-\sum_{i\in A,j\in B}\langle P_{ij}^{\sigma fc}\rangle)\approx 0$, the Pauli Block Effect is very serious. In all of dibaryon systems, only 6 of them belong to the first case, they are: $(\Omega\Omega)_{ST=00}$, $(\Xi^*\Omega)_{ST=0\frac{1}{2}}$, $(\Delta\Delta)_{ST=30}$, $(\Delta\Delta)_{ST=03}$, $(\Sigma^*\Delta)_{ST=0\frac{5}{2}}$, $(\Sigma^*\Delta)_{ST=3\frac{1}{2}}$. For the second case, $(\mathrm{N}\Omega)_{ST=2\frac{1}{2}}$ and $(\Delta\Omega)_{ST=3\frac{3}{2}}$ dibaryons are interesting.

3.2 Results

Now we come to discuss the results of binding energy for different dibaryons. The definition of the binding energy is:

$$B_{AB}=M_A+M_B-M_{AB}\ . \tag{10}$$

M_A, M_B and M_{AB} are the masses of baryon A and B, and dibaryon (AB), respectively.

3.2.1 $(\Omega\Omega)_{ST=00}$ dibaryon

The strangeness -6 $(\Omega\Omega)_{ST=00}$ dibaryon has been studied and found it deeply bound state[4]. Now we further consider the mixing of scalar meson. The results are given in Table 3. From Table 3, we can see that with no mixing, the binding energy is 171 MeV, it is a deeply bound state. In this case, σ meson and ϵ all contribute attractive forces. However, with ideally mixing, i.e. there is no σ meson exchange between s quarks, the binding energy is reduced to 61 MeV, but still quite large. It seems the quark exchange effect is very important, so this state can become a bound state.

Table 3. Binding energy of $(\Omega\Omega)_{ST=00}$ dibaryon.

	θ^S	$B_{\Omega\Omega}$	contribution
Set 1	0°	171 MeV	$\sigma+\epsilon$
Set 2	35.3°	61 MeV	ϵ

3.2.2 $(\Xi^*\Omega)_{ST=0\frac{1}{2}}$ dibaryon

In Table 4, we list the results for strangeness -5 $(\Xi^*\Omega)_{ST=0\frac{1}{2}}$ dibaryon. From Table 4, we can see that the binding energy is 117 MeV with no mixing and becomes 31 MeV with ideally mixing where only ϵ meson exchange contribute $\Xi^*-\Omega$ interaction.

Table 4. Binding energy of $\Xi^*\Omega$ dibaryon.

	θ^S	$B_{\Xi^*\Omega}$	contribution
Set 1	0°	117 MeV	$\sigma+\epsilon$
Set 2	35.3°	31 MeV	ϵ

3.2.3 $(\Sigma^*\Delta)_{ST=0\frac{5}{2}}$ dibaryon

In Table 5, we list the results for strangeness -1 $(\Sigma^*\Delta)_{ST=0\frac{5}{2}}$ dibaryon. From Table 5, we can see that the binding energy is 27.3 MeV with no mixing and becomes 15.7 MeV with ideally mixing. We make an analysis for this state and find no matter what kind of mixing, mainly σ meson exchange provides attractive force, ϵ meson exchange provides very weakly attractive force. Therefore, it is a stable bound state.

Table 5. Binding energy of $\Sigma^*\Delta$ dibaryon.

	θ^S	$B_{\Sigma^*\Delta}$	contribution
Set 1	0°	27.3 MeV	$\sigma+\epsilon$
Set 2	35.3°	15.7 MeV	σ

3.2.4 $(\Sigma^*\Delta)_{ST=3\frac{1}{2}}$ dibaryon

In Table 6, we list the results for strangeness -1 $(\Sigma^*\Delta)_{ST=3\frac{1}{2}}$ dibaryon. From Table 6, we can see that the binding energy is 37.8 MeV with no mixing and becomes 20.0 MeV with ideally mixing. Here we consider the coupling of S-wave and D-wave. The same as $(\Sigma^*\Delta)_{ST=0\frac{5}{2}}$ dibaryon, σ meson exchange dominantly provide attractive force, so no matter what kind of mixing, $(\Sigma^*\Delta)_{ST=3\frac{1}{2}}$ is a stable bound state.

Table 6. Binding energy of $\Sigma^*\Delta$ dibaryon.

	θ^S	$B_{\Sigma^*\Delta}$	contribution
Set 1	0°	37.8 MeV	$\sigma+\epsilon$
Set 2	35.3°	20.0 MeV	σ

3.2.5 $(\Delta\Delta)_{ST=03}$ dibaryon

In Table 7, we list the results for nonstrangeness $(\Delta\Delta)_{ST=03}$ dibaryon. From Table 7, we can see that the binding energy is 22.3 MeV with no mixing and becomes 21.7 MeV with ideally mixing. The same as above, the σ meson exchange dominantly provide attractive force to this state. Therefore, no matter what kind of mixing, it is a stable bound state.

Table 7. Binding energy of $\Delta\Delta$ dibaryon.

	θ^S	$B_{\Delta\Delta}$	contribution
Set 1	0°	22.3 MeV	$\sigma+\epsilon$
Set 2	35.3°	21.7 MeV	σ

3.2.6 $(\Delta\Delta)_{ST=30}$ dibaryon

In Table 8, we list the results for nonstrangeness $(\Delta\Delta)_{ST=30}$ dibaryon.

Table 8. Binding energy of $\Delta\Delta$ dibaryon.

	θ^S	$B_{\Delta\Delta}$	contribution
Set 1	0°	48.0 MeV	$\sigma+\epsilon$
Set 2	35.3°	45.0 MeV	σ

From Table 8, we can see that the binding energy is 48.0 MeV with no mixing and becomes 45.0 MeV with ideally mixing. We also consider the couplings of partial wave and hidden color channel. Again the σ meson exchange dominantly provide attractive force, so no matter what kind of mixing, it is a stable bound state.

3.2.7 $(N\Omega)_{ST=2\frac{1}{2}}$ dibaryon

In Table 9, we list the results for strangeness -3 $(N\Omega)_{ST=2\frac{1}{2}}$ dibaryon.

Table 9. Binding energy of NΩ dibaryon.

	θ^S	$B_{N\Omega}$	contribution
Set 1	0°	4.7 MeV	$\sigma+\epsilon$
Set 2	35.3°	unbound	no σ, no ϵ

From Table 9, we can see that with no mixing, the binding energy is 4.7 MeV, it is very weakly bound state. We made an analysis and found that the σ meson exchange provide attractive force, however, the ϵ meson exchange provide relatively larger repulsive force. Therefore, it is easy to understand why $(N\Omega)_{ST=2\frac{1}{2}}$ is a weakly bound state. When ideally mixing is taken, there is no σ and ϵ mesons exchange, so this state becomes unbound.

3.2.8 $(\Delta\Omega)_{ST=3\frac{3}{2}}$ dibaryon

In Table 10, we list the results for strangeness -3 $(\Delta\Omega)_{ST=3\frac{3}{2}}$ dibaryon.

Table 10. Binding energy of ΔΩ dibaryon.

	θ^S	$B_{\Delta\Omega}$	contribution
Set 1	0°	3.1 MeV	$\sigma+\epsilon$
Set 2	35.3°	unbound	no σ, no ϵ

From Table 10, we can see that with no mixing, the binding energy is 3.1 MeV, it is also a very weakly bound state. The same as $(N\Omega)_{ST=2\frac{1}{2}}$ dibaryon, here the σ meson exchange provide attractive force and the ϵ meson exchange provide relatively larger repulsive force, so this state is a weakly bound state. When ideally mixing is taken, there is no σ and ϵ mesons exchange, so this state becomes unbound state.

4 Summary

In this work, the binding energies of deuteron are firstly studied and the results show that we can reproduce the binding energy of deuteron with and without mixing of scalar meson. Then, using the same parameters, we studied some interesting dibaryons.

For the case $(1-\sum_{i\in A,j\in B}\langle P_{ij}^{\sigma fc}\rangle)\approx 2$, the quark exchange effect is very important, which makes two baryon cluster closer. No matter what kind of mixing is taken, they are still bound states.

However, for the case $(1-\sum_{i\in A,j\in B}\langle P_{ij}^{\sigma fc}\rangle)\approx 1$, the quark exchange effect is not important. With no mixing, they are weakly bound states, but with ideally mixing, they become unbound states.

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