**Nonlinear Processes
in Geophysics**

Meteorological uncertainty and rainfall downscaling

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Abstract. We explore the sources of forecast uncertainty in a mixed dynamical-stochastic ensemble prediction chain for small-scale precipitation, suitable for hydrological applications. To this end, we apply the stochastic downscaling method RainFARM to each member of ensemble limited-area forecasts provided by the COSMO-LEPS system. Aim of the work is to quantitatively compare the relative weights of the meteorological uncertainty associated with large-scale synoptic conditions (represented by the ensemble of dynamical forecasts) and of the uncertainty due to small-scale processes (represented by the set of fields generated by stochastic downscaling). We show that, in current operational configurations, small- and large-scale uncertainties have roughly the same weight. These results can be used to pinpoint the specific components of the prediction chain where a better estimate of forecast uncertainty is needed.

1 Introduction

Operational assessment of flood risk in small basins, with areas up to a few hundred square kilometers (such as those encountered in mountainous regions and along the Mediterranean coast), requires detailed precipitation forecasts on small spatial and temporal scales (Ferraris et al., 2002), as well as estimates of the associated prediction uncertainty. However, the smallest spatial scales reliably resolved by regional meteorological models, used for real-time ensemble predictions, rarely go below the size of these basins. In addition, computational limitations allow for regional meteorological ensembles with a relatively small number of members (not larger than about twenty).

An alternative approach to cope with small-scale ensemble predictions resorts to the use of stochastic models for rainfall downscaling (see e.g. Ferraris et al., 2003, for a comparison of different approaches). In this framework, an ensemble of small-scale stochastic rainfall fields with prescribed statistical properties, usually defined from larger-scale meteorological forecasts, is used to estimate the probability of occurrence of intense rainfall events. When the stochastic fields are fed into a rainfall-runoff model of selected basins, this method allows for estimating the probability of flood occurrence (Ferraris et al., 2002). Operational assessment of the probability of extreme rainfall events and of their uncertainty should thus be based on a mixed dynamical-stochastic rainfall forecasting chain, based on two main ingredients (Siccardi et al., 2005): (1) an ensemble of limited-area meteorological forecasts, used to capture the dynamical variability and the sensitivity to initial conditions of the atmosphere; (2) from each of these forecasts, the generation of an ensemble of rainfall fields produced by a stochastic downscaling model.

In this type of approach, the stochastic downscaling procedure plays two roles. On one hand, it allows for estimating the probability of occurence of intense precipitation on small spatial and temporal scales. On the other hand, it also introduces additional variability which "dresses" individual dynamical ensemble forecasts (Roulston and Smith, 2003), compensating for the usually very small number of dynamical ensemble members in meteorological forecasts.

The present article focusses on the second issue. In this work, we analyse for the first time the practical construction of a mixed dynamical-stochastic ensemble prediction chain and compare the large-scale uncertainty represented by the dynamical meteorological ensemble with that modelled by the use of stochastic downscaling at small scales. Ensemble meteorological predictions are provided by the COSMO-LEPS limited area ensemble prediction system (Molteni et al., 2001; Montani et al., 2001, 2003; Marsigli et al., 2001, 2004, 2005), and the downscaling procedure is based on the approach discussed by Rebora et al. (2006a,b). Both

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procedures have been shown to provide well calibrated, probabilistic forecasts of precipitation at large and small scales respectively. The results of the analysis will be used to provide some general guidelines on how to determine, in an empirical way, the specific components of the prediction chain where a better assessment of the forecast uncertainty leads to significant improvement of the overall uncertainty estimates.

2   Uncertainty and ensemble predictions

Every meteorological forecast is invariably affected by uncertainty: Measurement error and finite resolution in the initial conditions, incomplete representation of the physics of the problem and finite numerical resolution, corresponding to an incomplete representation of subgrid processes, all lead to an uncertainty in the forecast which grows with forecast time and rapidly propagates to all scales (see e.g. Palmer, 2000). Dynamical ensemble forecasting (Epstein, 1969; Leith, 1974; Toth and Kalnay, 1993; Palmer, 1993; Molteni et al., 1996; Toth and Kalnay, 1997) represents the current method of choice for taking into account some of these sources of uncertainty, by estimating the probability of occurrence of different meteorological scenarios from the relative frequency of different forecast ensemble members. Current computational constraints limit the resolution of operational global ensemble forecasts to spatial scales of at best 50 km and to few tens of ensemble members (the operational ECMWF ensemble with its 51 members is one of the largest ones). Stochastic perturbations of the model physics and a multi-model approach can be used to take into account model error (Palmer, 2001).

At a regional level, global ensemble forecasts can be extended to fine scales (currently around 10 km), either by independent integration on limited areas or by nesting high resolution, often non-hydrostatic, limited area models (LAM) in the general circulation scenarios. Examples of such systems are COSMO-LEPS, HIRLAM-LAMEPS (Frogner and Iversen, 2002; Sattler and Feddersen, 2005) and the NCEP SREF system (Hamill and Colucci, 1997, 1998; Du et al., 1997). However, due to the incomplete representation of small scale processess, to the limited resolution of the observational network used for initialization and data assimilation, and to numerical details (Adlerman and Droegemeier, 2002), currently these models have low prediction skill already at spatial scales of a few tens of kilometers, even if their nominal resolution is higher. Also for LAMs, computational constraints severely limit the number of ensemble members, and ensemble size has been shown to have a significant impact on performance statistics in these systems (Marsigli et al., 2005).

Both at large and small scales, the limited size of the ensembles is associated with a significant residual error in the forecast, due to undersampling of the forecast probability distribution function (Buizza and Palmer, 1998). This error can become severe when small regional areas are considered, leading to significant mistakes in the estimates of the probability of extreme precipitation events. Roulston and Smith (2003) suggested to take into account the residual errors in the forecasts by creating stochastic "daughter" ensembles which dress each ensemble member with a statistical error ensemble. With such methods, large stochastic ensembles can be easily generated and a smoother representation of future probability distributions can be obtained, but the resulting spatio-temporal fields do not necessarily preserve the statistical properties of the observed fields. As such, this approach is not suitable for hydrological applications, where spatial and temporal correlations are important.

An alternative approach is based on the construction of stochastic rainfall prediction ensembles, using downscaling methods (Ferraris et al., 2002). From a single deterministic forecast, these methods allow for the generation of large ensembles of higher resolution, stochastic, spatio-temporal fields, which share the desired (larger-scale) statistical properties with the original meteorological forecast. The small scale properties of precipitation in these models are purely stochastic and simply obtained by propagating the large-scale information in the forecast. In past years, several downscaling approaches have been proposed, including point-process models (see e.g. Waymire et al., 1984), autoregressive models (e.g. Bell, 1987), and fractal cascades (e.g. Lovejoy and Mandelbrot, 1985; Perica and Foufoula-Georgiu, 1996; Menabde et al., 1997), all of which have similar skills in reproducing the statistical properties of observed precipitation (Ferraris et al., 2003).

3   Sources of uncertainty in a precipitation prediction chain

In this work, we apply a stochastic downscaling procedure to each member of an ensemble of dynamical forecasts generated by the COSMO-LEPS limited-area ensemble prediction system with spatial resolution 10 km. The study area is a square region of 320 km on each side, centered over northwestern Italy, see Fig. 1a. For the purpose of this work, we chose four study cases of intense precipitation events (forecasts issued on 13 November 2002, 23 November 2002, 26 December 2002 and 20 January 2003). For each case, the COSMO-LEPS system operational in 2002/2003 produced an ensemble of $N_L=5$ forecast fields, issued at noon every day, with a total forecast time of 120 h. The precipitation forecasts are cumulated over a period of 6 h; Fig. 1a also shows an example LEPS forecast member.

The dynamically forecasted fields are downscaled to finer resolution using the stochastic model RainFARM (Rebora et al., 2006b). This model is based on a nonlinear transformation of a linearly-correlated stochastic field, obtained by extrapolation to small scales of the large-scale power spectrum of the forecast. A power law functional form is assumed
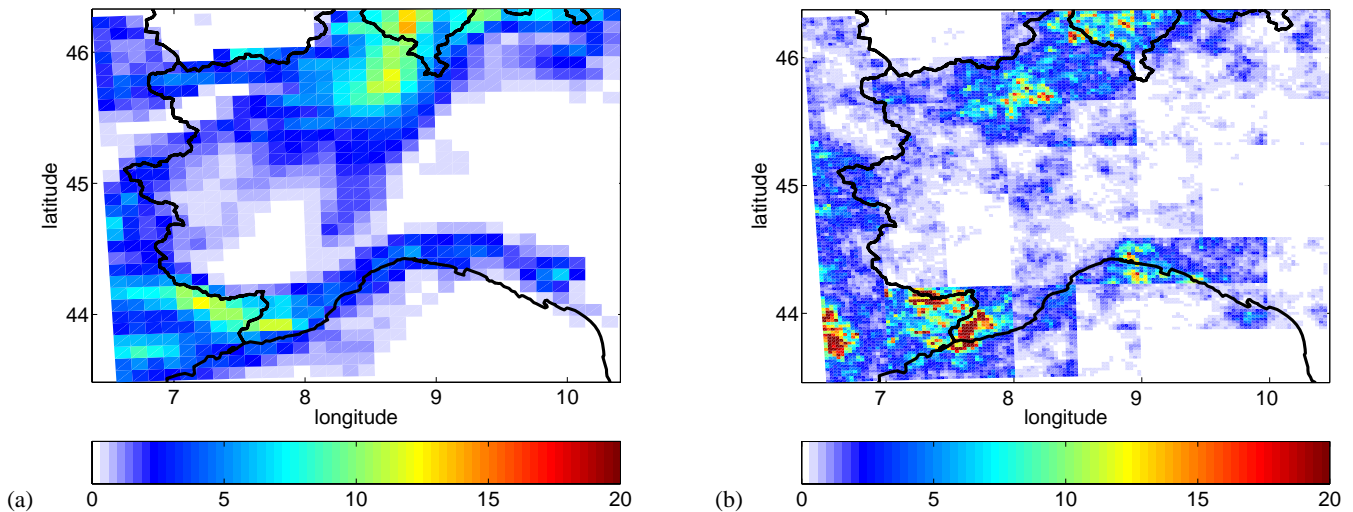
Fig. 1. The computational domain considered in this work, centered on north-western Italy, together with **(a)** a snapshot of a LEPS precipitation forecast with spatial resolution 10 km; and **(b)** the same field downscaled with RainFARM, using L_o=40 km and T_o=6 h. Both panels refer to the forecast issued on 13 November 2002 at forecast time τ=12 h. Precipitation is expressed in units of [mm/h] and averaged over 6 h in panel (a) and over 24 min in panel (b).

for the extrapolation and random Fourier phases are used at the unresolved scales. The downscaled fields produced by this model are easily nested in regional-scale meteorological forecasts. The rain fields generated by the downscaling procedure are characterized by stochastic variability only at scales smaller than the "reliability scales", L_o and T_o, and preserve the larger-scale properties of the member of the dynamical ensemble from which they are derived. These reliability scales represent the (empirically determined) scales below which the dynamical forecasts lose their predictive power; usually, L_o is about 4 to 6 times the nominal resolution of the model (Patterson and Orszag, 1971). The model produces fields which, when aggregated at scales larger than L_o and T_o, preserve total precipitation volumes, the location of large-scale structures, the spatial and temporal correlations of the original fields and the multifractal properties of the precipitation fields. A detailed verification of the properties of the fields generated by RainFARM is discussed in Rebora et al. (2006a,b).

The reliability scales L_o and T_o represent the only free parameters of the model, since the other important parameters, such as the spectral slopes used to extrapolate the power spectrum to small scales, are derived from the large-scale properties of the precipitation field to be downscaled. With this procedure, for each member of the LEPS ensemble we generate a stochastic ensemble of N_R=100 downscaled fields with spatial resolution Δx=2.5 km and temporal resolution of Δt=24 min. By construction, each of these fields is identical to the original meteorological forecast when aggregated on the reliability scales. In the following, we explore different values for the reliability scales, and consider

L_o=[20, 40, 80, 160] km and T_o=[6, 12] h. As an example, Fig. 1b shows a snapshot of one field obtained by downscaling the LEPS forecast of Fig. 1a.

In the following, we indicate with $\mathbf{p}_{ik}(x, y, t)$ the k-th member (k=1 . . . N_R) of the ensemble of stochastic precipitation fields generated by downscaling, with i=1 . . . N_L indicating the member of the LEPS ensemble to which the downscaling procedure was applied. The position in space is (x, y) and t is time. Each precipitation field can be thought of as a vector in a high-dimensional space whose axes are defined by all spatial positions and temporal instants spanned by the field. From each stochastic ensemble, we derive the ensemble mean, $\bar{\mathbf{p}}_i(x, y, t) = \frac{1}{N_R} \sum_{k=1}^{N_R} \mathbf{p}_{ik}(x, y, t)$. We characterize the spread of the i−th stochastic ensemble using its r.m.s. deviation around the mean: $\sigma_i^2 = \frac{1}{N_R} \sum_{k=1}^{N_R} |\mathbf{p}_{ik} - \bar{\mathbf{p}}_i|^2$, where the operator $||^2$ indicates the mean quadratic distance between two spatio-temporal fields.

To quantify the distance between two stochastic ensembles generated from two different LEPS fields, we determine the distances between their centers: $D_{ij}^2 = |\bar{\mathbf{p}}_i - \bar{\mathbf{p}}_j|^2$, from which we derive the average distance of each stochastic ensemble from the others as $S_i^2 = \frac{1}{(N_L-1)} \sum_{j \neq i} D_{ij}^2$. We further define a measure of the average spread of stochastic ensembles, $\bar{\sigma}^2 = \frac{1}{N_L} \sum_{i=1}^{N_L} \sigma_i^2$, and an average spread of each dynamical ensemble, $\bar{S}^2 = \frac{1}{N_L} \sum_{i=1}^{N_L} S_i^2$. The different objects and some of the measures defined above are illustrated schematically in Fig. 2, which shows how each LEPS member, represented by crosses, is associated with a corresponding stochastic ensemble.
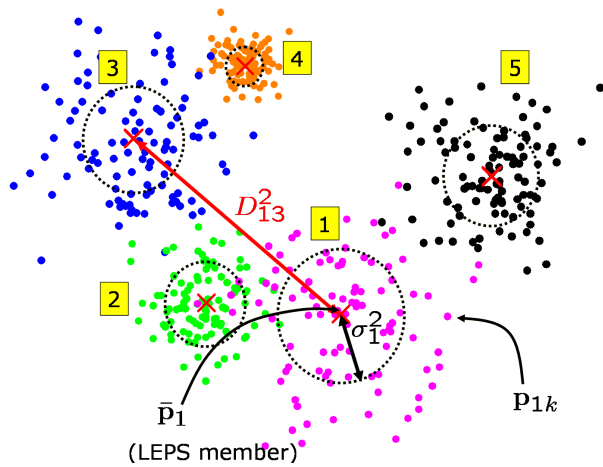
Fig. 2. An idealized illustration of the statistical quantities considered in the analysis, obtained by projecting onto a two-dimensional plane the high-dimensional space spanned by the precipitation fields. For each of the five LEPS ensemble members, a set of 100 stochastic rain fields were generated using RainFARM (the "stochastic ensembles"). The width of each stochastic ensemble is measured by its variance and is indicated by the dashed circles in the figures. The distances between ensemble centers, indicated by crosses, measure the spread of the dynamical LEPS ensemble. The figure indicates, as an example, the distance between the centers of the stochastic ensembles obtained by downscaling the first and third members of the LEPS ensemble.

Armed with these definitions, we now compare the spread of each stochastic ensemble, σ_i^2, with its average distance from other stochastic ensembles, S_i^2, for different values of the spatial and temporal reliability scales and for the four events considered. The statistics represented in the following figures are based on the spatio-temporal RMS distance between the downscaled fields. Spatial and temporal dependence appear in the choice of the reliability scales. Figure 3 shows that, for most of the events considered, variations in the reliability scales do not significantly affect the spread of individual stochastic ensembles. Note that increasing the reliability scales leads to a reduction in S_i^2. Indeed, by construction of the RainFARM procedure, the center of each stochastic ensemble roughly corresponds to the field obtained by aggregating the original meteorological forecast at the reliability scales. Increasing L_o and T_o thus leads to a reduction of the average distance between the stochastic ensemble centers.

When L_o and T_o are small, the spread of each stochastic ensemble typically stays well below the average distance between the ensemble centers. By contrast, for large values of L_o and T_o, the spread of the individual stochastic ensembles can become larger than the the average distance between the ensemble centers. One of the four cases (the forecast issued on 13 November 2002) represents an exception: the spread

of the stochastic ensembles exceeds the average distance between the ensemble centers for all choices of the reliability scales. What distinguishes this case from the others is a particularly large variance of the LEPS member fields themselves, which in RainFARM leads to a large spread of the stochastic ensembles.

The ratio of the average spread of the stochastic ensembles to that of the dynamical ensembles, $\bar{\sigma}^2/\bar{S}^2$, gauges the relative importance of these two components of total variability and is reported in Fig. 4 as a function of the reliability scales. There is a significant dependence of this ratio on the reliability scales. For most events, stochastic variability becomes larger than the dynamic one as reliability scales cross from 40 to 80 km (if T_o=6 h) or from 20 to 40 km (if T_o=12 h), with the only exception of the forecast issued on 13 November 2002.

4   Discussion and conclusions

This work can be considered a first step towards the practical construction of an operational mixed dynamical-stochastic forecasting chain for the management of hydrogeological risk. In particular, we compared the uncertainty associated with stochastic downscaling with the uncertainity present in a meteorological ensemble, in order to obtain a better quantification of the prediction uncertainty in small-scale rainfall prediction.

In order for stochastic downscaling to be a useful element of the prediction chain, the additional variability introduced by the downscaling procedure should be of the same order of the spread of the dynamical ensemble. When the spread of the dynamical ensemble is significantly larger than small-scale stochastic variability, the downscaling procedure can lead only to a marginal advantage in bracketing observations; viceversa a very large stochastic variability with respect to dynamical ensemble spread could completely swamp out the information provided by the dynamical ensemble.

The results obtained here show that the ratio between the variability introduced by small-scale stochastic downscaling and that associated with the dynamical ensemble at larger scales depends on the value of the smallest scales above which the meteorological predictions are considered reliable. The best choice of these scales for the COSMO-LEPS system is currently uncertain. Nonetheless, numerical diffusion problems usually make numerical models unreliable at resolutions which are below 4/6 times their grid resolution (Patterson and Orszag, 1971). Using this criterion, we can approximately estimate the spatial reliability scale of COSMO-LEPS forecasts as L_o=40/60 km. Figures 3 and 4 show that for three of the four events considered, this choice of reliability scales leads to dynamical and stochastic ensembles which contribute almost equally to the total variability of the forecast. Thus, the proposed operational forecasting chain can be considered well equilibrated, since both components
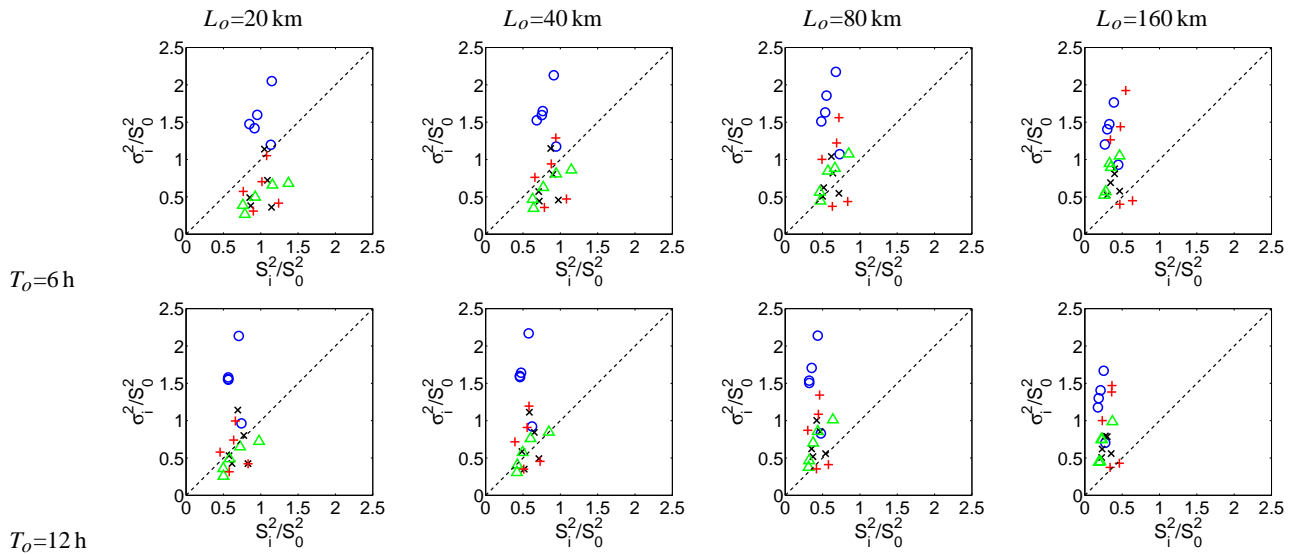
Fig. 3. Comparison of the width of each stochastic ensemble, σ_i^2, with its average distance from the other stochastic ensembles, S_i^2, for various choices of the reliability scales L_o and T_o. Both measures are normalized by $\bar{S}_o^2$, the average spread of the dynamic ensemble at L_o=20 km, T_o=6 h. Each event is characterized by a different value of $\bar{S}_o^2$. Different symbols indicate the four events (case studies) considered (circles: 13 November 2002; x-crosses: 23 November 2002; triangles: 26 December 2002; plus signs: 20 January 2003). The dotted line indicates the 45 degree line for reference.
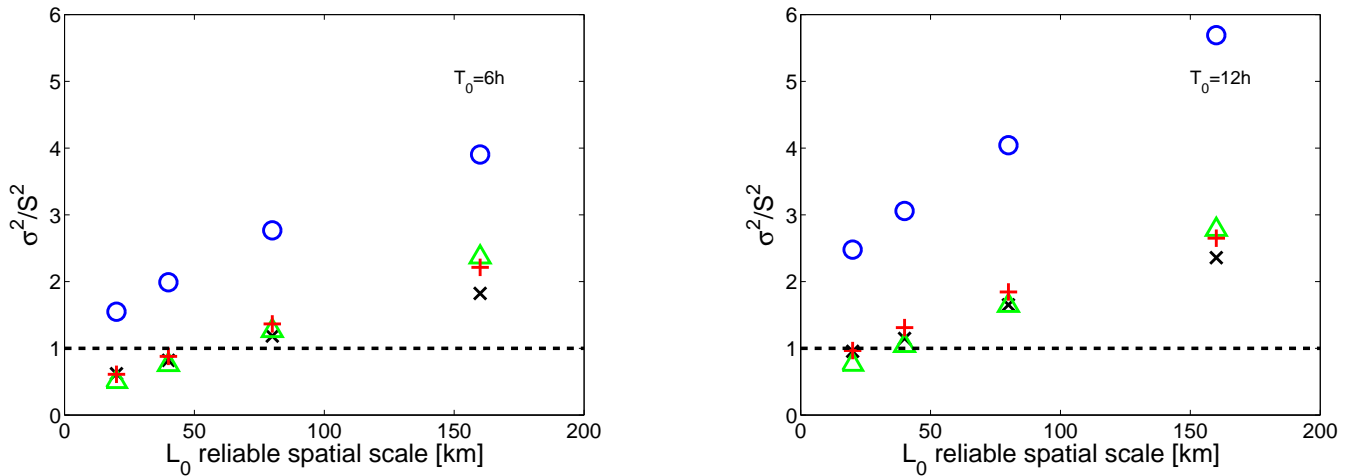

Fig. 4. Ratio between the average spread of stochastic ensembles, $\bar{\sigma}^2$, and the average spread of the dynamical LEPS ensemble, $\bar{S}^2$, as a function of the reliability scales L_o and T_o. The symbols refer to the same events as described in Fig. 3.

are important for representing the total uncertainty. In one of the examples considered, the variance of the individual meteorological fields is so high that it leads to a spread of the stochastic ensemble which is definitely larger than the spread of the dynamical ensemble for all choices of the reliability scales. This suggests that there may not exist just one single choice of the reliability scales for all cases and that the correct choice could be conditioned on the properties of synoptic circulations. For an operational system, however, this would add an extra dimension of complexity.

The results reported above can help determining the components of the prediction chain where an improvement in uncertainty estimates would be most effective. Given that a dynamical forecasting system with a very large number of ensemble members with very high spatial and temporal resolution is currently unfeasible, the finite computational resources impose a trade-off between increasing small-scale resolution and enlarging the number of members in the dynamical ensemble.

Numerical meteorological models and assimilation networks will presumably continue to improve in the future, possibly leading to smaller values of the reliability scales. In this case, the spread of the individual stochastic ensembles could become significantly smaller than the average distance between the different members of the dynamical ensemble. In such a scenario, ensemble stochastic downscaling and uncertainty below the resolved scales become less important; further effort should be put into the task of increasing the number of dynamical ensemble members and uncertainty estimates above the reliability scales. On the opposite, if a more detailed comparison between data and numerical predictions indicates that the reliability scales are significantly larger than the 40/60 km assumed here, most of the variability in the forecast would be associated with the stochastic downscaling procedure and with small-scale processes. In this case, increasing the number of dynamical ensemble members would be superfluous, and the effort should be put into achieving smaller, numerically reliable spatial and temporal scales.

The conclusions reported in this work rely on the assumption that the spread of the LEPS ensemble and the spread of the stochastic ensembles are correctly representing the uncertainity of the forecast respectively above and below the reliability scales. Indeed, COSMO-LEPS has been verified to provide satisfactory probabilistic forecasts of precipitation, particularly for intense events (Marsigli et al., 2005), and it can be considered appropriate for representing uncertainty above the reliability scales. Further, the RainFARM downscaling procedure has been shown to correctly reproduce the statistics of observed precipitation at small scales (Rebora et al., 2006b), and the spread of the stochastic ensembles can be considered to correctly represent uncertainty below the reliability scales.

Of course, stochastic downscaling does not capture the true physics of small-scale precipitation dynamics, and it simply propagates the information contained in large-scale forecasts. More insight could be gained from the analysis of a truly dynamical downscaling, i.e., by reaching small-scale resolution not by stochastic methods but by an ensemble of nested high-resolution numerical simulations. At the moment, however, this strategy seems difficult to follow in operational frameworks.

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