**Nonlinear Processes
in Geophysics**

Statistical distribution of elapsed times and distances of seismic events: the case of the Southern Spain seismic catalogue

M. D. Martínez[1]**, X. Lana**[2]**, A. M. Posadas**[3, 4]**, and L. Pujades**[5]

[1]Departament de Física Aplicada, Universitat Politècnica de Catalunya, Av. Diagonal 649, E-08028 Barcelona, Spain
[2]Departament de Física i Enginyeria Nuclear, Univ. Politècnica de Catalunya, Av. Diagonal 647, E-08028 Barcelona, Spain
[3]Departamento de Física Aplicada, Universidad de Almería, E-04120, Almería, Spain
[4]Instituto Andaluz de Geofísica y de Prevención de Desastres Sísmicos, Universidad de Granada, E-18080, Granada, Spain
[5]Departament d'Enginyeria del Terreny, Cartogràfica i Geofísica, Universitat Politècnica de Catalunya, Gran Capità s/n, E-08034, Barcelona, Spain

Abstract. Several empiric cumulative distributions of elapsed times and distances between seismic events occurred in the Southern Iberian Peninsula from 1985 to 2000 (data extracted from the seismic catalogue of the Andalusian Institute of Geophysics) are investigated. Elapsed times and distances between consecutive seismic events of the whole catalogue, taking into account threshold magnitudes of 2.5, 3.0, 3.5 and 4.0, and of five seismic crises, without distinguishing magnitudes, are investigated. Additionally, the series of distances and elapsed times from the main event to every aftershock are also analysed for the five seismic crises. Even though a power law is sometimes a satisfactory model for the cumulative distribution of elapsed times and distances between seismic events, in some cases a fit with a Weibull distribution for elapsed times performs better. It is worth of mention that, in the case of the seismic crises, the fit achieved by the power law is sometimes improved when it is combined with a logarithmic law. The results derived might be a contribution to a better representation of the seismic activity by means of models that could be based on random-walk processes.

1 Introduction

The analysis of the space-time patterns of the seismic activity is a relatively complex problem. Several researches are devoted to the spatial and temporal clustering of the seismicity (Ouchi and Uekawa, 1986; Smalley et al., 1987 and Chen et al., 1998, among others). The chaotic and dissipative be-

haviour of the elapsed times and how can be characterised the predictability of the seismicity in terms of Lyapunov exponents have been also analysed for instance by Correig et al. (1997) by using the reconstruction theorem (Takens, 1981). More recently, nonlinear analyses of sequences of magnitude and interevent time intervals have been performed by Matcharashvili et al. (2000) and differences in the temporal distribution of small earthquakes before and after large events have been investigated by Matcharashvili et al. (2002). In the first case, significant evidence of low nonlinear dimension structure is detected. In the second case, it is verified that the low nonlinear dimension structure of the temporal distribution of earthquakes changes remarkably before and after the largest events. It is also worth of mention the book of Goltz (1998), where concepts of fractal and chaos are introduced and fractal and chaotic properties of earthquakes are broadly analysed.

From the point of view of a stochastic process, it has to be mentioned the model of the continuous time random walk, CTRW (Bouchaud and Georges; 1990) and Lévy distributions (Gnedenko and Kolmogorov, 1954; Feller, 1996), which try to establish relationships between interevent waiting times and hypocenter location. An illustrative application can be found in Sotolongo-Costa et al. (2000) and Posadas et al. (2002).

The main objective of the present study is to obtain statistical models to describe the elapsed times and distances between earthquakes occurred in Southern Spain. These results will be compared with those derived from several seismic crises occurred in the same area. Although the distribution of the elapsed times between consecutive events has been extensively studied (Smalley et al., 1987; Sornette and Knopoff, 1997; Wang and Kuo, 1998; Corral, 2003, 2004),

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Correspondence to: M. D. Martínez
(dolors.martinez@upc.edu)

Table 1. Range of magnitudes, Δm, recording period, Δt, number of events, latitude and longitude intervals, parameters b of the Gutenberg-Richter law and p of the Omori's law, and fractal dimension D_S of the spatial earthquake distribution for the five seismic crises of Adra, Agrón, Alborán, Antequera and Loja. Uncertainties on D_S, b and p are also specified.

	Adra	Agrón	Alborán
Δm	5.0–1.5	4.0–1.5	4.5–2.5
Δt	23 Dec. 1993	5 Dec. 1988	2 July 1997
	16 March 1994	15 Dec. 1998	24 Dec. 1997
events	346	101	56
latitude	36.4° N–37.0° N	36.99° N–37.05° N	36.3° N–36.7° N
longitude	2.5° W–3.3° W	3.8° W–3.9° W	2.8° W–3.4° W
b	0.87±0.02	0.97±0.02	0.86±0.03
p	0.983±0.001	0.89±0.01	0.86±0.01
D_S	2.10±0.03	2.01±0.04	1.79±0.01

	Antequera	Loja
Δm	3.5–2.5	3.5–2.0
Δt	7 June 1989	3 Feb. 1985
	9 June 1989	13 June 1985
events	79	146
latitude	36.9° N–37.3° N	37.1° N–37.2° N
longitude	4.4° W–4.8° W	4.1° W–4.3° W
b	0.98±0.01	1.06±0.03
p	0.84±0.01	0.89±0.01
D_S	2.07±0.03	2.08±0.03

less attention has been paid to the distribution of distances between consecutive events. A power law, which derives from a truncated gamma distribution for small arguments, is assumed to model cumulative probabilities of both the elapsed times and distances. Additionally, the Weibull distribution and a logarithmic law have been also successful in some cases.
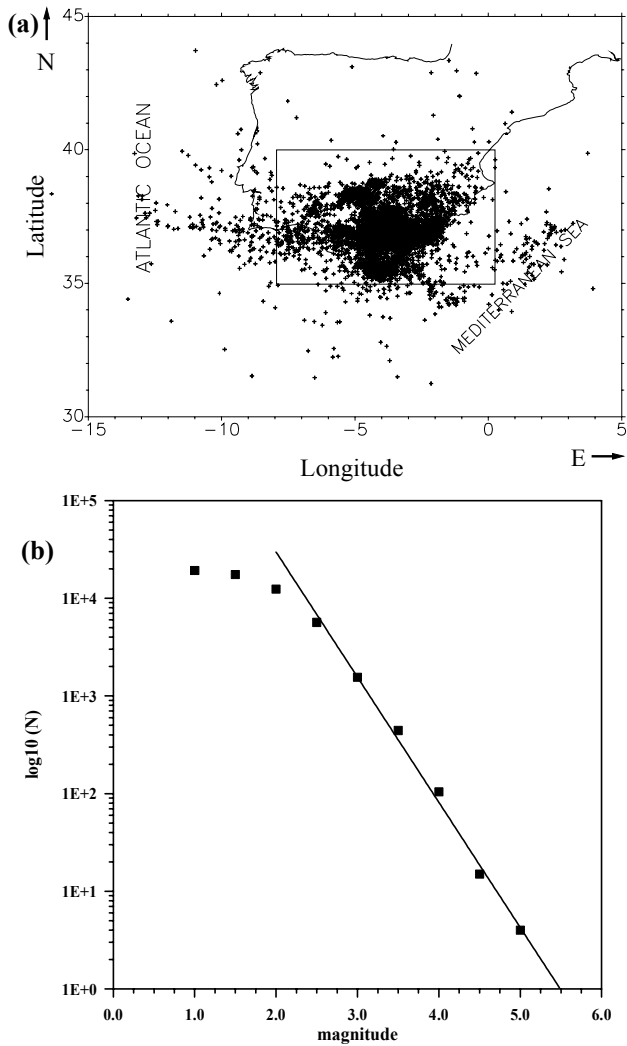
2　Seismological and tectonic settings

The southern part of the Iberian Peninsula is the most vulnerable to seismic risk in Spain. The area is situated in the central part of the Betic Cordillera (Fig. 1a). It includes the Granada Basin and several mountain ranges around it. The crust is characterised by thickness variations (Galindo-Zaldívar et al., 1997, 1999). The Granada Basin fault system created a set of blocks structured at different levels, thus allowing independent movements of them. These features fit into a general compressive framework, which produces contemporary extensional and compressive deformations on strike-slip faults. The seismogenetic areas are concentrated in three fracture systems having N10–30E, N30–60W and N70–100E directions. All the fracture systems are embedded in the Betic Area (Vidal, 1986; Peña et al., 1993; Posadas et al., 1993a, b) and the Granada Basin has a high activity of microearthquakes with hypocenters shallower than 20 km.

The Andalusian Seismic Network installed by the Andalusian Institute of Geophysics provided a wide seismic catalogue constituted by 20000 events with extreme body-

wave magnitudes reaching 5.5. According to the Gutenberg-Richter law (Fig. 1b), only events with magnitudes $M_L \geq 2.5$ have been considered, being 2.5 the minimum magnitude of completeness. This cut-off magnitude implies to finally dispose of a maximum number of 5648 events to be analysed. The study also includes the investigation of five seismic series, which locations are schematised in Fig. 2. These five seismic crises represent a varied sample of seismic activity and it should be interesting to observe if their interevent elapsed times and distances distribute with similar statistical parameters and functions. Some of the crises are characterised by a short duration (only three days) and others by a relatively long duration (more than 6 months). From the point of view of the magnitude range, some of the crises have a relatively wide range of available magnitudes (from 1.5 to 5.0), while others have narrower ranges, from 4.0 to 2.5.

1. The Adra-Berja area is located in the south-eastern Iberian Peninsula and it is characterised by a high level of seismicity; between December 1993 and March 1994, several hundred microearthquakes and earthquakes were recorded in this zone. The seismic series began with an earthquake of magnitude M_L=5.0 on 23 December and several aftershocks occurred, two of which reached M_L=4.0; a few days later, there was another earthquake of M_L=4.9.

2. In the Agrón seismogenetic region, several seismic crises have occurred in the 1985–2000 recording period. The main crisis began on 5 December with

Fig. 1. (a) Area (inner rectangle) covered by the seismic activity in Southern Spain. **(b)** Gutenberg-Richter law for the seismic events within the inner rectangle. Magnitudes less than 2.5 depart remarkably from the cumulative law.

Table 2. Parameters and least square regression coefficients for the power law (elapsed time and distances) and the Weibull distribution (elapsed times) when the whole seismic catalogue is considered. d_{max} and t_{max} represent the maximum distances an elapsed times predicted by the power law. Bold characters for u and K correspond to estimations achieved by maximum likelihood.

	Power law			
δD (km)	$m= 2.5$	$m= 3.0$	$m= 3.5$	$m= 4.0$
a	0.028	0.041	0.042	0.015
c	0.598	0.535	0.512	0.660
ρ^2	0.988	0.970	0.990	0.979
d_{max}	395.1	391.7	488.61	580.1

	Power law			
δt (days)	$m= 2.5$	$m= 3.0$	$m= 3.5$	$m= 4.0$
a	0.648	0.383	0.203	0.103
c	0.504	0.433	0.456	0.453
ρ^2	0.963	0.945	0.975	0.976
t_{max}	2.36	9.17	33.01	151.11

	Weibull distribution			
δt (days)	$m= 2.5$	$m= 3.0$	$m= 3.5$	$m= 4.0$
u	0.769	2.597	9.835	44.625
	0.769	**2.495**	**9.532**	**42.280**
K	0.680	0.536	0.587	0.591
	0.697	**0.603**	**0.661**	**0.709**
ρ^2	0.987	0.981	0.986	0.963

300 microearthquakes and earthquakes were located in a very small area. The main event (3 February 1985) had M_L=3.7.

Table 1 summarizes the parameter b of the Gutenberg-Richter law, the parameter p of the Omori's law, the fractal dimension D_S of the spatial earthquake distribution, the magnitude range Δm for which completeness is assured by the Gutenberg-Richter law, the number of events, latitude and longitude range and recording period Δt of every seismic crisis. All these details are compiled from Posadas et al. (2002), who tested the reliability of the parameters b, p and D_S. The spatial clustering was estimated by a standard box-counting and the computation of the Morishita's index (Ouchi and Uekawa, 1986). It has to be pointed out that due to the close monitoring of these seismic crises by the IAG, the completeness of the seismic series is sometimes reasonably assured for magnitudes below 2.5. In this way, a low magnitude as 1.5 in the cases of Adra and Agrón has been accepted after the verification of the Gutenberg-Richter law for these crises.
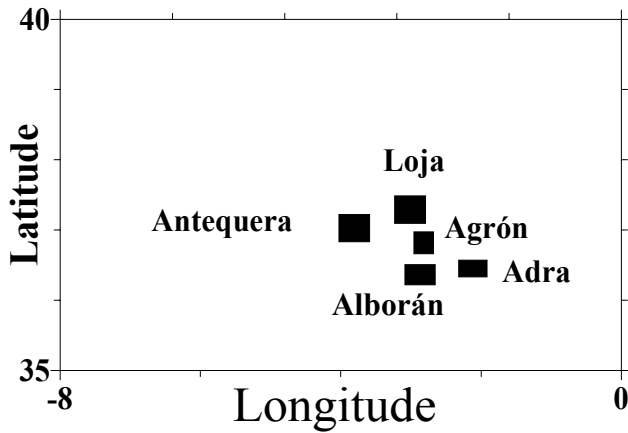
an earthquake of M_L=4.0 followed by another of M_L=3.9. Between 5 and 15 December 1988, 292 microearthquakes were recorded, with epicentres near the villages of Agrón and Cacín (Granada, Spain).

3. The Alborán Sea zone on the Mediterranean is close to that of Adra-Berja. The seismic series occurred from June 1997 until January 1998. After the beginning of the crisis with an event of M_L=4.5, more than 300 microearthquakes were located in this area.

4. In the Antequera area only one seismic crisis occurred with a very short duration, between 7 and 9 June 1989. At least 158 earthquakes of M_L <3.4 occurred.

5. The Loja region is an active zone where several seismic series have occurred, one of them in 1985, in a group of faults with orientation N40-55W. More than

Fig. 2. Emplacement of the five seismic crises in Southern Spain.

3 Results

3.1 Consecutive events for the whole catalogue

The set of elapsed times, δT, and distances, δD, between consecutive seismic events have been analysed for four different threshold magnitudes of 2.5, 3.0, 3.5 and 4.0. These threshold values reduce the number of available events to 5648, 1951, 441 and 103, respectively. The first threshold value is the lowest magnitude for which the Gutenberg-Richter law depicts completeness (Fig. 1b), while the last one is the maximum magnitude for which the series of δT and δD are long enough to obtain reliable empiric cumulative probabilities.

For the series of δT both a power law

$$P(x_0 \leq x) = ax^c \qquad (1)$$

and the Weibull distribution function

$$P(x_0 \leq x) = 1 - \exp\left\{-\left(\frac{x-\varepsilon}{u-\varepsilon}\right)^K\right\} \quad x \geq \varepsilon; u, K > 0 \quad (2)$$

are assumed. Besides its first applications in the analysis of fatigue of materials, the Weibull distribution has been often used in stochastic processes where the analysis of the elapsed time between certain events is investigated. As an example, two applications in different fields of the Earth's sciences such as seismology (Correig et al., 1997) and climatology (Burgueño et al., 2004) can be cited. Other distributions easier to manage, as the exponential function, could be also considered. Nevertheless, after some tests, the best fit was always performed by the power law and the Weibull distribution. It has to be remembered that the Weibull distribution includes as a particular case ($K=1$) the exponential function.

The power law is very simple and characterise the fractal behaviour of seismicity. The parameters a and c of this law can be easily determined by means of a least square regression in log-log scales. The parameter ε of the Weibull function is set equal to zero, which is equivalent to assume that
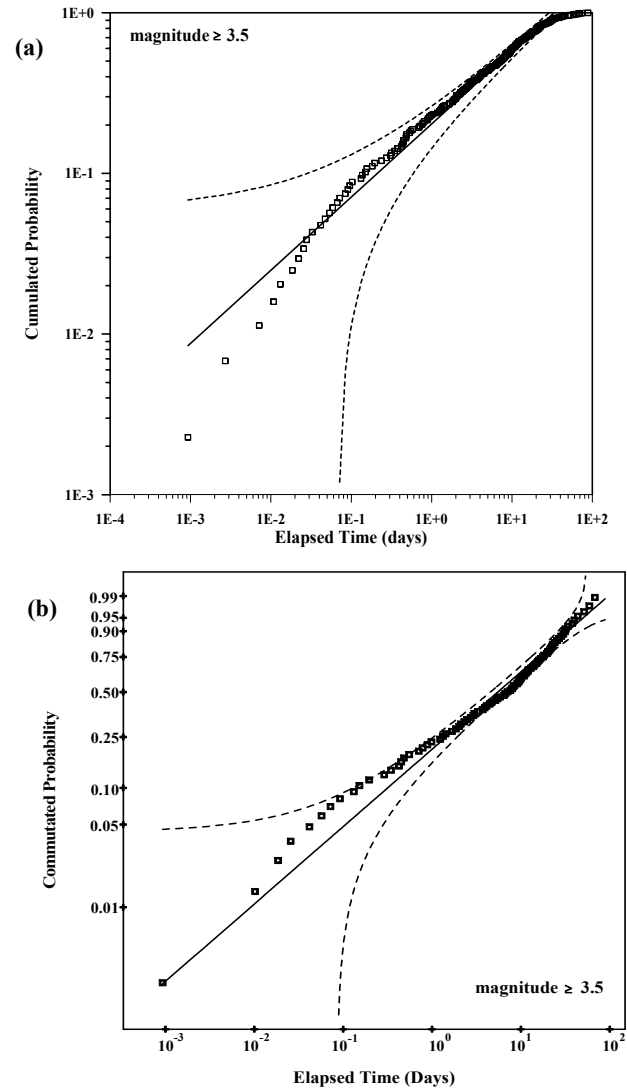

Fig. 3. Power law **(a)** and Weibull distribution **(b)** for the elapsed time between consecutive events for a threshold magnitude of 3.5.

the minimum elapsed time is almost null in comparison with the total length of the seismic episode, and parameters u and K can be easily estimated in a first approach by a least square regression in a log-log scale. Empirical and theoretical models are represented in terms of $\log\{-\log(1-p)\}$ versus $\log(x)$ for a first visual inspection of the possible linearity, since, in these scales, data accomplishing the Weibull model are evidenced by a linear behaviour. Better estimations of u and K are achieved by maximum likelihood and the Kolmogorov-Smirnov test (Lemeshko and Postovalov, 2001) in the case of composite hypothesis can be considered in order to decide if the Weibull distribution performs well empiric data. The cumulative probabilities for the series δD of distances are also modelled by the power law (Eq. 1).

Table 2 shows the values of the parameters and the corresponding least square regression coefficients ρ^2 for series δT and δD and the maximum elapsed times and distances

Table 3. Parameters and least square regression coefficients for the power and logarithmic laws and the Weibull distribution in the case of elapsed times computed from the starting (main) event. t_{max} (days) represents the maximum time predicted by the logarithmic laws. δT is the time interval for which every model fits well empiric data. Bold characters for u and K correspond to estimations achieved by maximum likelihood.

Crisis	Power law				Logarithmic law				
	a	c	ρ^2	δT	α	β	ρ^2	t_{max}	δT
Adra	11.49	1.012	0.985	0–0.02	0.992	0.196	0.958	1.1	0.02–7.0
Agrón	2.380	0.502	0.913	0–0.02	1.062	0.135	0.959	0.6	0.02–0.9
Alborán	0.581	0.287	0.946	0–0.02	0.638	0.103	0.991	33.6	0.02–20.0
Antequera	4.019	0.504	0.922	0–0.02	1.148	0.117	0.957	0.3	0.02–0.3
Loja	1.008	0.342	0.867	0–0.02	0.852	0.107	0.896	4.0	0.02–20.0

Crisis	Weibull		
	u	K	ρ^2
Adra	0.157	0.773	0.988
	0.167	**0.698**	
Agrón	0.056	0.698	0.972
	0.061	**0.605**	
Alborán	0.933	0.392	0.982
	1.046	**0.396**	
Antequera	0.020	0.699	0.976
	0.023	**0.598**	
Loja	0.177	0.486	0.944
	0.207	**0.389**	
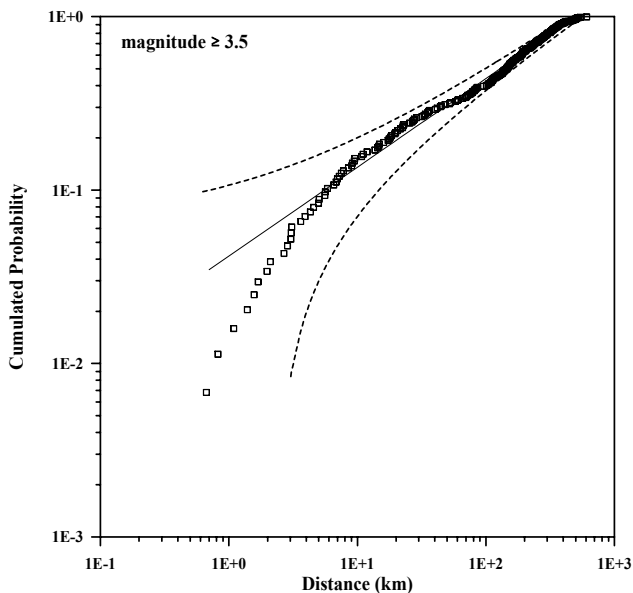
Fig. 4. Power law for the distance between consecutive events for a threshold magnitude of 3.5. Dashed lines correspond to 95% Kolmogorov-Smirnov uncertainty bands derived with the assumption of simple hypothesis.
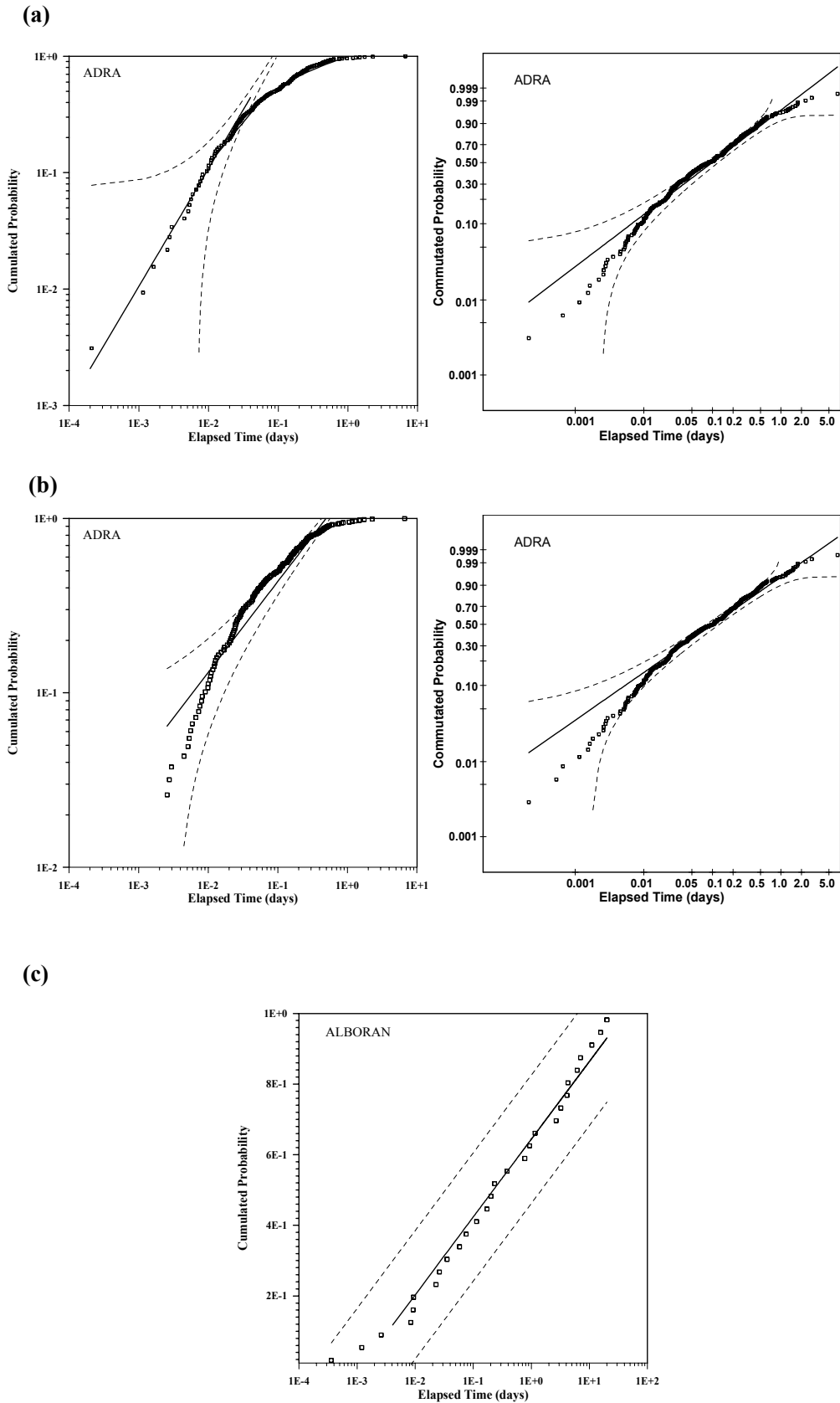
and a threshold magnitude of 3.5. In the case of the Weibull distribution (Fig. 3b), the straight line depicts the theoretical distribution computed from parameters u and K derived by means of maximum likelihood and dashed lines the 95% Kolmogorov-Smirnov uncertainty bands for composite hypothesis, which can be roughly estimated by $0.91/n^{1/2}$, n being the number of elements of the series. Dashed lines in Fig. 3a corresponds to the 95% Kolmogorov-Smirnov uncertainty bands for simple hypothesis (Benjamin and Cornell, 1970), roughly estimated by $1.36/n^{1/2}$. Figure 4 shows the fit performed by the power law (Eq. 1) for the series of distances and for the same threshold magnitude. Dashed lines of this figure are similar to those corresponding to Fig. 3a.

3.2 Events belonging to a seismic crisis

In relation to the seismic crises described in Sect. 2, two types of series are considered: 1) interevent elapsed times and distances; 2) elapsed times and distances from the first (main) shock. Tables 3 to 6 summarise the values of the parameters and least square regression coefficients for series δT and δD, both taking as reference the previous event of the crisis and the mainshock of the seismic sequence. In some cases, a combination of the power law (Eq. 1) and the logarithmic law

$$P(x_0 \leq x) = \alpha + \beta \log(x) \qquad (3)$$

improves the modelling of the empiric distribution of distances and elapsed times. The parameters α and β can be easily determined by a least square regression in a linear-log

derived from the power law (Eq. 1). Bold characters correspond to parameters u and K estimated by maximum likelihood. Figure 3 shows the fit performed by the power law (Eq. 1) and the Weibull distribution (Eq. 2) for the series δT

(a)

(b)

(c)

Fig. 5. Power law and Weibull distribution for elapsed times of aftershocks referred to the main event **(a)** and between consecutive aftershocks **(b)**, both in the case of Adra. The case of Alborán for consecutive aftershocks **(c)** is well performed by the logarithmic law.

Table 4. Parameters and least square regression coefficients for the power and logarithmic laws and the Weibull distribution of the elapsed times in the case of consecutive events of a seismic crisis. t_{max} (days) represents the maximum time predicted by the logarithmic law. δT is the time interval for which every model fits well empiric data. Bold characters for u and K correspond to estimations achieved by maximum likelihood.

Crisis	Power law				Logarithmic law				
	a	c	ρ^2	δT	α	β	ρ^2	t_{max}	δT
Adra	4.765	0.825	0.980	0–0.07	0.955	0.177	0.902	1.3	0.07–8.0
Agrón	6.294	0.678	0.950	0–0.05	1.030	0.111	0.952	0.8	0.05–1.0
Alborán					0.644	0.104	0.993	30.7	0–20.0
Antequera	6.891	0.569	0.981	0–0.03	1.080	0.083	0.937	0.4	0.03–0.4
Loja	3.764	0.627	0.973	0–0.04	0.824	0.098	0.904	6.1	0.04–20.0

Crisis	Weibull		
	u	K	ρ^2
Adra	0.170	0.751	0.990
	0.181	**0.639**	
Agrón	0.056	0.701	0.975
	0.061	**0.603**	
Alborán	0.938	0.389	0.985
	1.047	**0.395**	
Antequera	0.020	0.701	0.975
	0.023	**0.599**	
Loja	0.196	0.507	0.962
	0.226	**0.413**	

Table 5. Parameters and least square regression coefficients for power and logarithmic laws in the case of distances from the mainshock to every aftershock. d_{max} (days) represents the maximum distance predicted by the logarithmic law, except for the case of Agrón, for which a power law performs well all available data. δd is the distance interval for which every model fits well empiric data.

Crisis	Power					Logarithmic				
	a	b	ρ^2	d_{max}	δd	α	β	ρ^2	d_{max}	δd
Adra	0.001	2.311	0.985		2.0–20.0	0.389	0.156	0.890	50.2	20.0–50.0
Agrón	0.364	1.290	0.960	2.2	0.1-5.0					
Alborán	0.004	2.031	0.967		3.0–15.0	0.463	0.142	0.916	43.9	15.0–50.0
Antequera						−0.280	0.465	0.974	15.7	1.0–15.0
Loja	0.057	1.456	0.987		1.0–4.0	−0.090	0.411	0.988	14.2	4.0–22.0

scale. In a few cases, the best fit is given either by the power or by the logarithmic law and the combination of both models does not perform better. As in Table 2, maximum elapsed times and distances deduced either from Eq. (1) or Eq. (2) are included in Tables 3 to 6 and bold characters for u and K correspond to estimations derived by maximum likelihood.

As an example, Fig. 5 shows the elapsed times from the mainshock and the interevent elapsed times for the seismic crisis of Adra, with the power law and the Weibull distribution. The good fit of the logarithmic law for the Alborán crisis interevent elapsed times is also shown. 95% uncertainty bands have been estimated in terms of composite hypothesis for the Weibull distribution. For the other three cases, 95% uncertainty bands derived from the simple hypothesis have been represented. Figure 6 illustrates two examples of the series of distances, as interevent distances and distances

from the mainshock. 95% uncertainty bands correspond to the Kolmogorov-Smirnov test for simple hypothesis. If the power law is assumed for the Adra seismic crisis, the law itself manifests the necessity of a complementary logarithmic law to explain the distribution for the largest distances and elapsed times. For instance, empirical data depart significantly from the power law for distances exceeding approximately 10 km (distances to mainshock) and 5 km (interevent distances).

4 Discussion and conclusions

Some previous considerations have to be introduced for a right interpretation of the results. The first point concerns with the fact that the most robust estimation of the Weibull

Table 6. Parameters and least square regression coefficients for power and logarithmic laws in the case of distances between consecutive aftershocks of a seismic crisis. d_{max} (days) represents the maximum distance predicted by the power law, excepts for Adra and Loja, which need for a combination with a logarithmic law. δd is the distance interval for which every model fits well empiric data.

Crisis	Power					Logarithmic				
	a	b	ρ^2	d_{max}	δd	α	β	ρ^2	d_{max}	δd
Adra	0.038	1.345	0.979		0.2–2.0	−0.105	0.308	0.989	36.1	2.0–50.5
Agrón	0.230	1.228	0.924	3.4	0.1–9.0					
Alborán	0.063	0.887	0.954	22.6	0.4–5.0					
Antequera	0.049	1.174	0.967	13.1	0.6–16.0					
Loja	0.028	1.579	0.951		0.5–3.0	−0.408	0.502	0.981	16.5	3.0–20.0

distribution parameters is that based on the maximum likelihood. In spite of quite good values of the least square regression coefficient, the minimization process in a $\log(1-\log(P))$ scale leads to small deviations for small elapsed times and strong deviations for intermediate elapsed times. Second, parameters for the power and logarithmic laws can be assumed as derived from graphical analysis (probability paper representations), which lead to linear graphs in log-log and linear-log scales. In these cases, the least square regression coefficient has to be carefully considered because it is not again the best factor to decide the vicinity of empiric and theoretical distributions. One more time differences between theoretical and empiric distribution are not directly minimised. A third point concerns the uncertainty bands derived from the simple and composite Kolmogorov-Smirnov test hypothesis. For the case of the Weibull distribution, Kolmogotrov-Smirnov test with composite hypothesis have to be used, given that the same empiric data is used for the test and for the parameter estimation. Nevertheless, this kind of test seems to be unknown for the power and logarithmic laws. Consequently, simple hypothesis bands have to be assumed as a criterion to decide if power and logarithmic laws are rejeced. Given that these bands can be assumed wider than those derived from composite hypothesis, empirical distributions within simple hypothesis bands are not a guarantee that a power or logarithmic law is the best representation of the distribution of elapsed times or distances. Graphical analyses, least square regression and simple hypothesis bands only suggest that these laws could be a right model.

The analysis of the whole catalogue shows that the power law reasonably explains the empirical distribution of distances and elapsed times when the threshold magnitude varies. In spite of the fit is not perfect, it is much better than those observed for the seismic crises. It has to be remembered that, when fitting power laws, it is quite usual to detect discrepancies between empiric data and the theoretical power function for values of the cumulative probability very close to 1.0. For example, it can be observed in Fig. 3a empirical data out of the Kolmogorov-Smirnov simple band only for cumulative probabilities exceeding 0.9. Therefore, it could be assumed a universal law for both quantities and some kind of fractal structure in reason to the mathematical character of

this law. In fact, by following classic definitions of fractal dimension, such as those associated with rescaled analysis, clustering dimension or correlation dimension (Grassberger and Procaccia, 1983a, b; Korvin, 1992; Dicks, 1999), the parameter c, ranging from 0.4 to 0.7, should be considered the fractal dimension of the series analysed. The Weibull distribution for elapsed times also fits quite well the empiric data. Nevertheless, empiric data are sometimes out of the 95 uncertainty bands for some inter-event time interval, as Fig. 3b manifests.

The analysis of the five seismic crises shows a more complex behaviour and suggests the breakdown of the fractal behaviour for all cases where a power law is not sufficient to perform empiric data. When the elapsed times are computed from the first (main) shock, the best fit is furnished by the combination of power and logarithmic laws and the transition from one law to another is observed close to 0.02 days in all cases. It is worth of mention that, opposite to the case of the whole catalogue, elapsed times of the order of 0.02 days are not associated with cumulative probabilities close to 1.0. As an example, the case of Adra crisis (Fig. 5a) is characterised for the necessity of changing to a logarithmic behaviour for cumulative probabilities exceeding 0.3. This elapsed time of transition would be the value from which the approach of the cumulative gamma distribution to a power law (Arfken, 1985; Abramowitz and Stegun, 1970) becomes inaccurate. Once more, the Weibull distribution offers a similar satisfactory fit for empirical series, when comparing with the other two laws. Nevertheless, the Loja crisis seems to be much better reproduced by the Weibull distribution than by the combined power and logarithmic laws.

For the interevent times, in some cases the fit with one law performs better (see for example the case of the Alborán crisis, which is very well-fitted by the logarithmic law). When the combined laws are necessary, the elapsed time of transition (0.03–0.7 days) is quite similar to that obtained for times referred to mainshock. Once more, the Weibull distribution offers quite similar fits. Moreover, assuming that, as in the case of the whole catalogue, parameter c could represent the fractal dimension of the series, estimated now by only considering the range for which the power law performs well, it is observed that this dimension does not change remarkably

from one crisis to another. Nevertheless, notable changes are detected when comparing cases corresponding to times from mainshock and interevent times. Additionally, it is worth of mention that only fractal dimensions for Adra, according to Tables 3 and 4, are notably different to those obtained for the whole catalogue and the four threshold magnitudes.

Concerning the analysis of distances, when they are computed from the location of the mainshock, the combination of power and logarithmic laws is not always necessary and the transition from one law to another varies remarkably from 4.0 to 20.0 km. The series of distances for Antequera crisis is well performed by only a logarithmic law and Agrón series by only a power law. When distances are computed between consecutive aftershocks, the logarithmic law is not necessary for three crises (Agrón, Alborán, Antequera) and fits offered by the power law are reasonably good. Due to only two examples can be revised, nothing can be safely stated with respect to distances for which the transition from power to logarithmic laws is observed. With respect to the fractal dimension, the parameter c for the whole catalogue reaches values (0.5 to 0.7) clearly lower than those derived for the seismic crises. Additionally, whereas c ranges from 0.9 to 1.6 for distances between consecutive aftershocks, the fractal dimension increases up to 1.3–2.3 for distances referred to the mainshock.

In short, different behaviours have been observed for series derived from the whole seismic catalogue and from subsets of seismic crises. Nevertheless, a single power law with changing parameters could explain more or less satisfactorily in some cases the distribution of elapsed times and distances. The most relevant difference is the necessity of a logarithmic law to complement elapsed time and distance distributions derived from seismic crises. The elapsed time for which the change from the power to the logarithmic law takes place is always within a narrow time interval from 0.02 to 0.07 days. In this way, a different behaviour in the seismic crises for relatively short and long waiting times could be assumed. This behaviour in the case of distances could be in some way justifiable if we imagine a rough division between aftershocks near or away from the mainshock. Unfortunately, nothing can be safely stated concerning the transition from power to logarithmic laws for the case of seismic crises and distance distributions. In the case of elapsed times, it seems to be sometimes a distinction between aftershocks close and remote in time to the mainshock.

In spite of the necessity of an upper bound for the argument of the power and logarithmic laws, this fact should not be a shortcoming. Firstly, there is a maximum empirical distance imposed by the geographical extension covered by the seismic crisis or the whole catalogue. Secondly, it is meaningless to expect elapsed times tending to infinite for the whole catalogue. It would imply the finish of the activity in the seismotectonic area covered by the catalogue. Only in the case of the seismic crises, it could be partially justified in terms of the time decay of aftershocks, quantified by the Omori's law. From this point of view, although the Weibull distribution for elapsed times would be sometimes a better
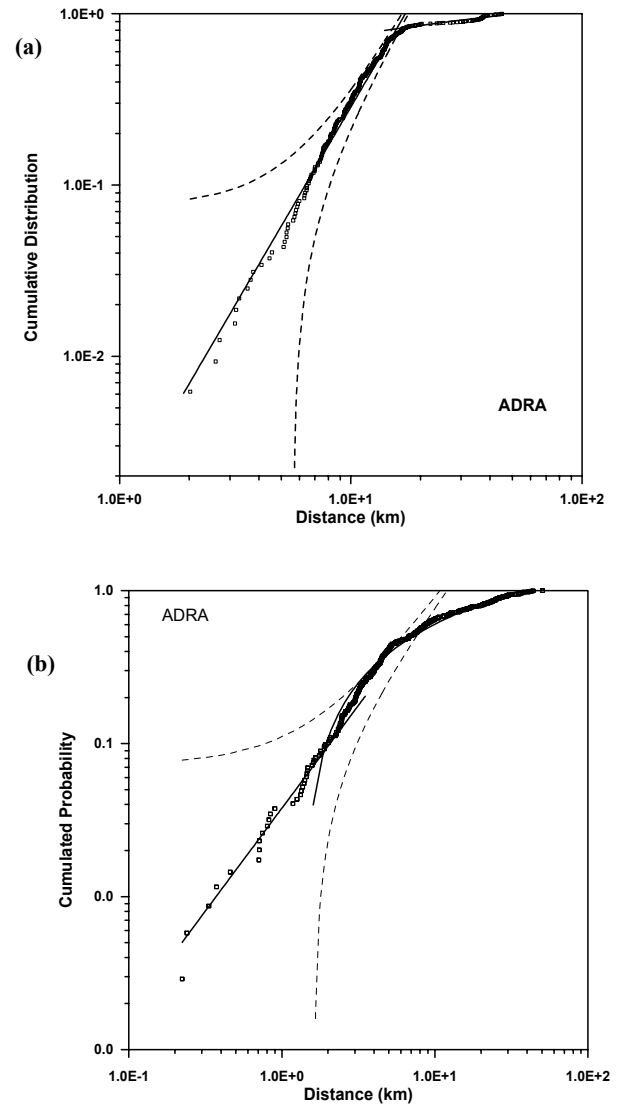
Fig. 6. Distribution of distances from the mainshock to the aftershocks **(a)** and distances between consecutive aftershocks **(b)**.

statistical option than the power law, a shortcoming should be that of an argument extended to infinite.

Finally, the set of results concerning functions used to modelling the elapsed time and distances between seismic events have to be considered as a contribution to the theory of continuous time random walks (CTRW), which is usually described in terms of Lévy distribution functions and tries to explain distribution of waiting times and localisation of hypocenters in a seismotectonic region. The results derived here for the Southern Iberian Peninsula should be considered for future CTRW models concerning space-time diffusion of the seismic activity. It could be especially useful for the analysis of the whole seismic activity of Southern Iberia, due to in this case simple power laws seem to be sufficient for a quite good description of waiting times and distances. The case of the seismic crises is a bit more complex, due to sometimes combined power and logarithmic laws are necessary.

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