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Extended proof of fibre-reinforced laminates with holes

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Methodology of research

ABSTRACT

Purpose: A calculation method for the strength prediction of tensioned laminated plates with circular notches is presented.

Design/methodology/approach: An extensive basis of experimental data from tensile fracture tests on carbon fibre reinforced specimens with circular notches was used to analyse factors determining the notched strength. The proposed extension of the known Whitney-Nuismer and Karlak Point Stress Criteria improves the strength prediction accuracy through introduction of a special coefficient function that takes into account the dependence between stress gradients in the notch near-field and the distance of the outer boundary.

Findings: It was found that the theoretical, notch size and outer boundary independent stress concentration factor is not sufficient for a reliable prediction of the notched strength. The presented results show that calculation procedure based on fundamental mechanical relations of the elasticity theory with few experimentally determined factors delivers results which good agree with experiments.

Research limitations/implications: The calculation procedure was verified on the representative population of tensile specimens made of several types of carbon fibre reinforced laminates used in the aerospace industry.

Practical implications: As openings represent relevant sites of failure in the construction, an accurate calculation of the notched strength is essential for the lightweight-oriented design of fibre-reinforced elements. The presented procedure is relevant for construction of practically important tensioned elements with circular holes.

Originality/value: The presented formulation of the phenomenological "characteristic distance" concept and its thorough experimental verification is original.

Keywords: Notch strength; Laminates with holes; Influence of geometry

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1. Introduction

The design of notched high-performance composites with fibre and textile reinforcement [2, 3, 14] requires a special proof of their strength, which includes the structural parameters of the component combination, material orientation and layer arrangement supported by appropriate experimental techniques [4, 13, 15]. For the determination of the strength of anisotropic plates with notches, semi-empirical criteria based on several limiting assumptions are used. These are e.g. the transferability of the stress concentrations from an infinite plate to a finite plate and rough approximation of the stress concentration decay [11]. Correction factors are usually derived from a stress analysis for isotropic materials. Semi-empirical failure models assume that a fibre/matrix composite plate with a notched region fails exactly when the stress concentration at a characteristic distance from the edge of the notch reaches the strength of a notch-free composite. Very often, the experimentally measured characteristic distances are considered to be independent of laminate design, fibre orientation and dimensions of notches; they are then transferred to the to a great variety of laminates made of the same type of fibre. The disadvantage of such analysis strategy is that physically inconsistent results are produced for some fibre orientation in highly anisotropic laminates. For this reason, such macroscopic failure models are applied for strength analysis of quasi-isotropic laminates only.

An improved stress analysis of notched anisotropic plates combined with a physically based strength analysis [1, 5, 9, 12] gives the possibility of developing more accurate failure models, which allow a detailed and realistic prediction of the reliability of notched fibre/matrix composite structures. In the first step, therefore, it is necessary to carry out an exact calculation of stress fields in notched multilayered plates with finite boundaries in dependence of all material and geometry-based influence factors. The main parameters are: fibre/matrix-combination, fibre orientation and distribution, laminate design, notch shape and dimensions as well as geometry of outer boundary. The notch stress field of infinitely contoured anisotropic composite plates with holes can be computed on basis of fundamental mechanical relations of the elasticity theory [8], whereby the notch stress factor K_t^{∞} (stress concentration factor) is only dependent on the material parameters and not on the cut-out size. Two facts give good reasons for a new approach:

- Enhanced semi-analytical approaches make it possible to compute the notch stresses of multilayered fibre-reinforced plates with a finite outer boundary [6, 7].
- The notch stress factor, which is determined for "large" plates, is not influenced by the hole size. Fracture tests on fibre-reinforced plates with cut-outs of various different sizes do indeed demonstrate that the strength of the notched composite plate (notch strength) is dependent on the notch size, such as it is the case for isotropic materials.

In the following chapters, this new approach to prediction of notch strengths and especially notch strength factors K_{tpr} depending on the geometry of the tensioned laminate plate with circular hole (OHT) will be presented, its procedure will be described and applied. The free parameters required by the approach will be exemplarily determined in fracture tests on notched laminate specimens of the CFRP family (0/±45/90).

2. Analytical background

Higher notch strengths are obtained in test when the holes are small and lower notch strengths when the holes are large. This influence depends on the decay behaviour of the peak stress, which means on the stress gradient. The decrease of the stresses in the near-field of the notch is particularly large in the case of a small notch, whereby just a small volume of the material is subjected to a very high stress. In case of ductile or quasi-ductile behaviour this highly stressed/strained domain can re-distribute load to the neighbouring, less stressed domain (so-called micro-support effect, based on [10]). The high stress gradient smoothes the situation. The behaviour of a plate with a larger hole is different as a larger material volume is involved and further, the failure probability – due to a higher number of micro-local "over-stressed" points – increases. This is valid for isotropic materials as well as for the laminates.

The local failure mechanisms are described by mechanical material strength models, which take account of the aforementioned influence of the stress gradients in the near-field of the notch by means of a characteristic quantity, characteristic distance d_c. These models are the so-called *notch strength criteria*.

For laminates, these criteria pursue an analogous procedure to that of Neuber. Well known are the point stress criterion (PSC) and the average stress criterion (ASC) [13]. The former will be taken here. In accordance with the PSC the notch stress at a characteristic distance d_c from the notch, $\sigma_y(x = r + d_c)$ is responsible for the failure of the multilayered composite plate Figure 1.

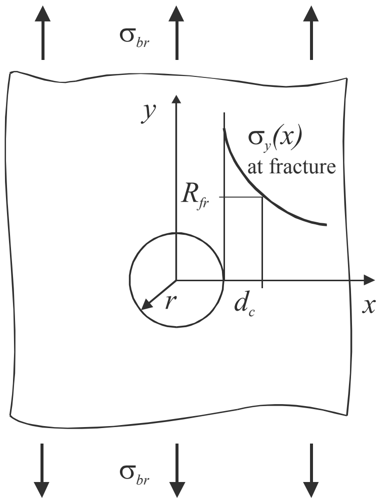

Fig. 1. Evaluation of notch stresses by means of the Point Stress Criterion (PSC)

When using such criteria, the realistic formulation of the characteristic distance by means of appropriate functional relations is a major problem. Here, the decay behaviour of the notch stress is approximated by means of a polynomial, which results from the transfer of analytical equations for notched isotropic plates to orthotropic ones.

In the event of failure (Figure 1), the failure condition of the unnotched laminate at the point $x = r + d_c$,

$$\sigma_y(x = r + d_c) = \sigma_{br} \quad (1)$$

can be taken as a failure condition for the notched laminate at the same point

$$\sigma_y(x = r + d_c) = \sigma_{NV}\left\{1 + \frac{1}{2}\xi^2 + \frac{3}{2}\xi^4 - (K_{tg}^{\infty} - 3)(5\xi^6 - 7\xi^8)\right\} \quad (2)$$

with

$$\xi = r/(r + d_c) \quad (3)$$

wherein

$$K_{tg}^{\infty} = 1 + \sqrt{2\left[\sqrt{\frac{E_y}{E_x}} - \nu_{yx}\right] + \frac{E_y}{G_{xy}}} \quad (4)$$

is the conventional stress concentration factor for orthotropic materials.

With the previously derived results one can define the predicted notch strength factor

$$K_{tgV} = \frac{\sigma_{br}}{\sigma_{NV}} \quad (5)$$

Its reciprocal value is used as the measure for the drop of strength caused by the hole.

The unknown characteristic distance d_c in Eq. (1) has to be determined for the application of the computation procedure. In the case of conventional formulations, simple relations are stated, which merely consider the influence of the micro-support effect based on the hole size and not based on the finite external edge (expressed by the ratio). The external edge is considered afterwards in this case by means of the introduction of general correction factors (FWC: Finite Width Correction), which establish a link between the gross stresses and the net stresses.

In the proposed approach, both the influence of the absolute hole size and also of the finite width (expressed by the ratio D/W) are included within the formulation of the characteristic distance. Based on theoretical and experimental investigations, the following approach

$$d_c = a(D/W)\left(\frac{r}{r_0}\right)^{\alpha} \quad (6)$$

can be used, where r_0 is a reference unit radius, so that the exponent α becomes non dimensional (recommended: $r_0 = 1$ mm if the unit of r is mm). The function $a(D/W)$ as well as the exponent α can be iteratively derived by an evaluation of fracture tests on notched multilayered composite specimens. For fibre-reinforced laminates the experiment outlines a linear approximation

$$a(D/W) = a_A \cdot D/W + b_A \quad (7)$$

which can be assumed a good regression function.

3. Calculation procedure

3.1. Input

- Laminate properties
  - Layer stacking sequence
  - Single layer properties (E_1, E_2, ν_{12}, G_{12})
  - Unnotched laminate strength σ_{br}
  - Laminate properties calculated according to the classical laminate theory
- Geometry of the notched specimens
  - Hole diameter D
  - Specimen width W
- Results from the tension tests
  - Notched strengths σ_{NV} (gross)

3.2. Iterative determination of α and a(D/W)

- Specification of α_k and determination of the values $a_1, a_2, \ldots a_n$ from (2) ($1 \ldots n$: individual tests) on the basis of the test values $(\sigma_{NV})_1, (\sigma_{NV})_2, \ldots (\sigma_{NV})_n$
- Approximation of $a = a(D/W)$ with a linear coefficient function $a(D/W) = a_A D/W + b_A$
- Calculation of the average deviation Δ_m between the predicted notched strengths $(\sigma_{NVB})_1, (\sigma_{NVB})_2 \ldots (\sigma_{NVB})_n$ and the notched strengths determined in experiments $(\sigma_{NV})_1, (\sigma_{NV})_2 \ldots (\sigma_{NV})_n$:

$$\Delta_m = \frac{1}{n}\sum_{i=1}^{n}\left|(\sigma_{NVB})_i - (\sigma_{NV})_i\right|$$

with

$$(\sigma_{NVB})_i = 2\sigma_{br}\Big/\left\{2 + \xi_i^2 + 3\xi_i^4 - \left(K_{tg}^{\infty} - 3\right)\left(5\xi_i^6 - 7\xi_i^8\right)\right\}$$

$$(d_c)_i = a((D/W)_i)r_i^{\alpha_k}$$

- Repetition of 1 to 3 with an appropriate iteration algorithm to determine the optimal exponent r whereby the deviation Δ_m is minimal.

4. Experimental basis

Tests were carried out on laminate tensile specimens made of prepreg plies (with HTA fibres and the 6376 resin system) of thickness of 0.25 mm. The stacking sequences (ply lay-ups) and the resulting laminate thicknesses and elastic properties of six selected laminates are listed in Table 1. The notched strengths (Table 2) have been determined for the total failure of the laminates (LPF: Last Ply Failure) for various D/W ratios at room temperature (RT).

The investigated D/W ratios varied within a range of 0.05 to 0.36 (lower ratios result inevitably in a lower D). The length of the specimens amounted to $L = 200$ mm, whereby $L/D \geq 10$ in order to avoid the effect from the load introduction on the notch stress field. The notch strength has been determined for the total failure of the laminates (failure according to inter-fibre-failure, fibre-failure or both) at Room Temperature (RT).

Table 1.
Layouts, elastic properties and strengths of the laminates

No.	Laminate	Layer orientation	Thickness[mm]	E_y [GPa]	E_x [GPa]	G_{yx} [GPa]	ν_{yx}	K_{tg}^{∞}	σ_{br} [MPa]
I	(9.1/36.4/54.5)	[90/+45/90$_2$/-45/0/-45/90$_2$/+45/90]	2.75	30.8	83.0	16.1	0.15	2.69	264
II	(11.2/44.4/44.4)	[+45/90$_2$/-45/0/-45/90$_2$/+45]	2.25	34.9	72.4	18.3	0.20	2.70	303
III	(14.3/57.1/28.6)	[+45/90/-45/0-45/90/+45]	1.75	40.3	55.6	21.9	0.30	2.72	397
IV	(16.7/66.6/16.7)	[+45/0/-45/90/+45/-45]$_S$	3.0	43.1	43.1	24.6	0.42	2.71	415
V	(20/40/40)	[90/+45/0/-45/90]$_S$	2.5	44.5	67.5	17.1	0.20	2.96	450
VI	(25/50/25)	[+45/0/-45/90]$_S$	2.0	51.7	51.7	19.9	0.30	3.00	472

Table 2.
Geometry and strengths of the test specimens

No.	D [mm]	W [mm]	σ_{NV} [MPa] Average value from 3 tests
1	3	15	270
2	3	30	326
3	3	45	339
4	3	60	337
5	5	15	218
6	5	30	271
7	5	45	288
8	5	60	299
9	10	30	201
10	10	45	237
11	10	60	252
12	16	45	187
13	16	60	211
14	20	60	193

5. Application examples

The strength for desired values of W and D can be estimated by means of the notched strength criterion (2) whilst including the value of the exponent α. Values of this exponent, found on the basis of experimental data for the considered laminates, are presented in the Table 3.

Table 3.
Optimized values of the exponent α

Laminate	I	II	III	IV	V	VI
α	0.6	0.7	0.6	0.4	0.4	0.4

For the laminate III (see Table 1) the procedure presented in the section 3 is applied with the following input data:

- Stacking sequence of laminate [45/90/-45/0-45/90/+45], 0.25 mm each ply
- Laminate properties
  $E_y = 40.3$ GPa, $E_x = 55.6$ GPa, $G_{yx} = 21.9$ Gpa
  $\nu_{yx} = 0.30$, $K_{tg}^{\infty} = 2.72$,
- Un-notched laminate strength $\sigma_{br} = 397.3$ MPa.

According to section 3.2, the value of $\alpha = 0.6$ was approximated. In this way, the coefficient function (see Fig. 2) takes the form:

$$a(D/W) = -2.94 \cdot \frac{D}{W} + 1.42 \text{ [mm]}.$$

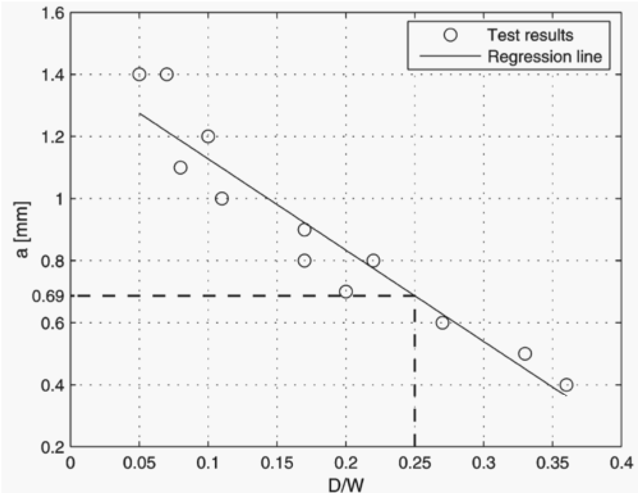

Fig. 2. Linear Approximation for a (example: laminate III)

For example, the notch strength of a plate made of the laminate III, having a width of $W = 60$ mm and a hole of $D = 15$ mm will be approximated. At first, the ratio $D/W = 15/60 = 0.25$ delivers (Fig. 2) – under the presumption of $\alpha = 0.6$ (Table 3) – the value of $a = 0.69$ mm. Then, for $r = D/2 = 7.5$ mm and applying $r_0 = 1$ mm with Eq. (6) is obtained:

$$d_c = 0.69 \cdot (7.5/1.0)^{0.6} = 2.30 \text{ mm}.$$

Finally, either the Eq. 2 may be used inserting

$K_{tg}^{\infty} = 2.72$, $\sigma_{br} = 397$ MPa und $\xi = 7.5/(7.5 + 2.3) = 0.765$

on the right side or utitlizing Figure 5:

$$\sigma_{NV} = 0.55 \cdot \sigma_{br} = 217 \text{ MPa}.$$

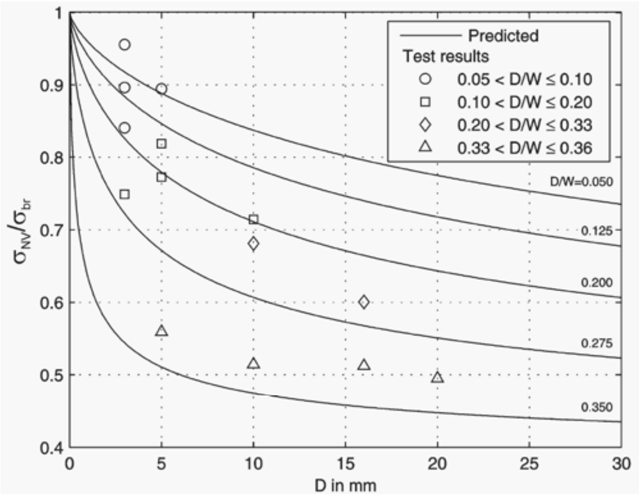

Fig. 3. Laminate I: (9.1/36.4/54.5) (%0° / %45° / %90°)

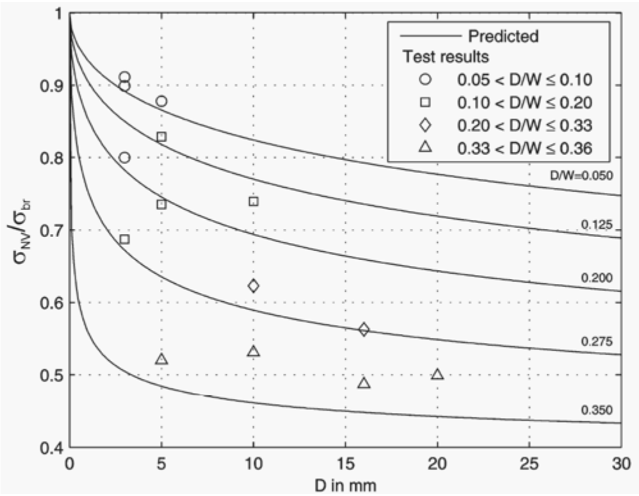

Fig. 4. Laminate II: (11.2/44.4/44.4)

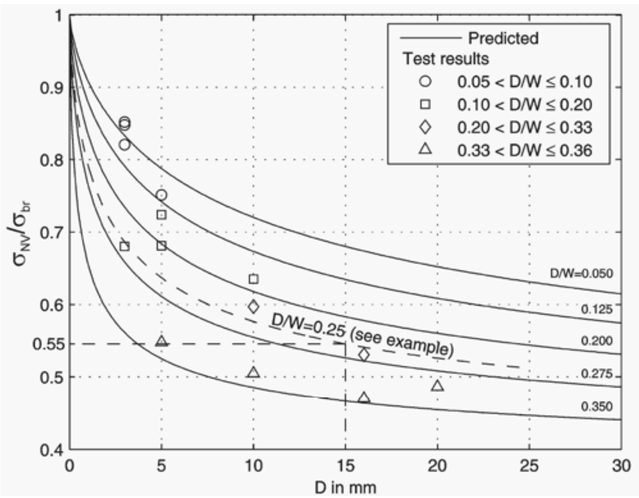

Fig. 5. Laminate III: (14.3/57.1/28.6)

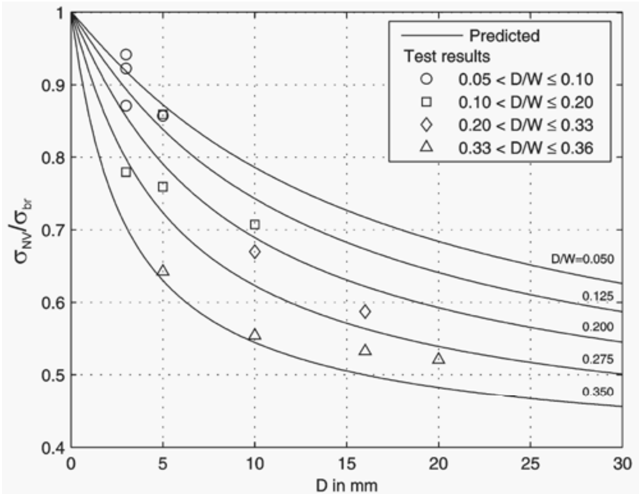

Fig. 6. Laminate IV: (16.7/66.6/16.7)

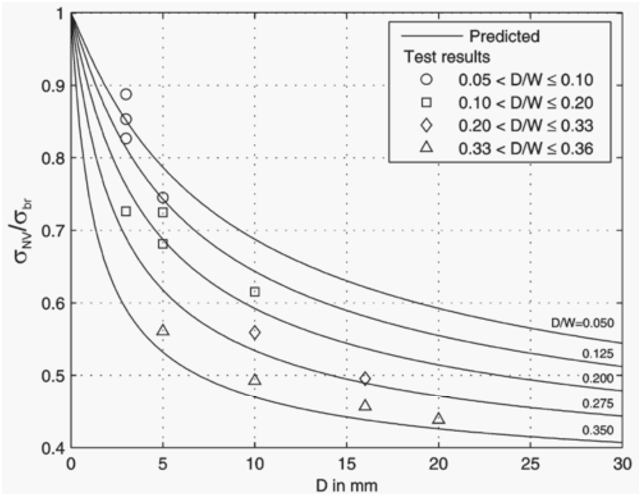

Fig. 7. Laminate V: (20/40/40)

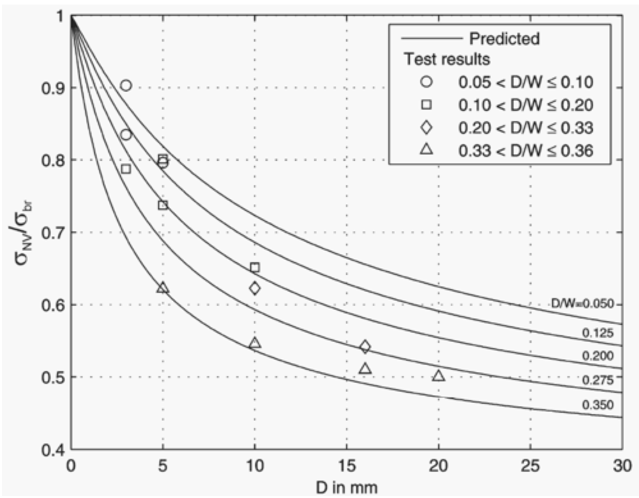

Fig. 8. Laminate VI: (25/50/25)

6. Conclusions

The given "Notch Strength Diagrams" (Figs. 3-8) refer to the experimentally examined laminates and the geometry of the notched specimens (see Tables 1 and 2). Families of curves for various D/W result from assumption of the geometry-dependent characteristic distance d_c. It is apparent that this concept improves the notch strength prediction compared with typical Point Stress Criteria, which would deliver a single curve for each laminate.

Since the presented notch strength criterion is based on fundamental mechanical relations of the elasticity theory, it is to be assumed, that similar dependences result in the case of comparable material properties, lay-ups and notch geometries. It could be expected, that for strongly differing material behaviour and dimensions (e.g. GFRP composites, UD-reinforcement, highly material-nonlinear composite behaviour or thick-walled composites), the notch strength approach (6) is to be adapted accordingly.

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