Nat. Hazards Earth Syst. Sci., 9, 211–216, 2009
www.nat-hazards-earth-syst-sci.net/9/211/2009/

Natural Hazards and Earth System Sciences

A non-extensive approach to risk assessment

F. Vallianatos

Technological Educational Institute of Crete, Department of Natural Resources and Environment, Geophysics and Seismology Laboratory, 3 Romanou str., Chania, 73133, Crete, Greece

Received: 28 August 2008 – Revised: 8 January 2009 – Accepted: 9 January 2009 – Published: 19 February 2009

Abstract. We analytically estimate the risk function of natural hazards (earthquakes, rockfalls, forestfires, landslides) by means of a non-extensive approach which is based on implementing the Tsallis entropy for the estimation of the probability density function (PDF) and introducing a phenomenological exponential expression for the damage function. The result leads to a power law expression as a special case and the b-value is given as a function of the non-extensive parameter q. A discussion of risk function dependence on the parameters of hazard PDF and damage function for various hazards is given.

1 Introduction

In recent years, there has been a sustained interest in the fractal nature and the possible power-law behaviour of a variety of natural hazards, e.g. earthquakes, landslides, rockfalls, forest fires etc. (Bak and Tang, 1987, 1989; Main, 1996; Bonnet et al., 2001; Malamud and Turcotte, 1999; Sornette and Sornette, 1989; Turcotte, 1997, 1999; Hergarten, 2002). To this effect, risk assessment is an extension of hazard assessment including terms for economic damage due to the natural disaster. The assessment of hazard is related to the probability of occurrence of a certain event, while the assessment of risk takes into account the effects of the corresponding disaster on life, urban environment and economy.

Following a well accepted definition of risk (Hergarten, 2004; McGuire, 2004), if N is the expected mean number of events of a certain type, in a certain region and time interval, and $<D>$ the expected damage caused by an event of the considered type, then the risk R is defined as

$$R = N < D > . \tag{1}$$

Correspondence to: F. Vallianatos
(fvallian@chania.teicrete.gr)

We note that Eq. (1) refers to the risk of a natural hazard of a certain size s (i.e. energy) and does not determine the risk of a natural hazard in a certain region and time. We clarify that s could be any physical quantity that measures the size of the hazardous event (e.g. released energy for earthquakes, displaced volume for landslides, destroyed area in forest fires etc.).

In order to include the dependence of (1) on event size, we must introduce an expression that includes: (a) all possible event sizes, (b) their frequency of occurrence and (c) the damage corresponding to a specific event size.

To this effect, if $F(s)$ is the probability that the size of an arbitrary hazard event is greater than, or equal to s, i.e. $F(s)$ is the cumulative distribution with probability density $p(s)=dF(s)/ds$, then the expected damage induced by an event of size s is given by the expression

$$< D(s) >= \int_{\Sigma} p(s)D(s)ds,$$

and the total risk R is (Hergarten, 2004; McGuire, 2004)

$$R = N \int_{\Sigma} p(s)D(s)ds \tag{2}$$

where Σ is the size range of natural hazards. Equation (2) indicates that in order to have an estimate of the expected risk, the probability density $p(s)$ and the dependence of the damage $D(s)$ on the event size s have to be evaluated. The expression for $p(s)$ involves a power-law size distribution $F(s)\approx s^{-b}$, indicating scale-invariant statistics (Aki, 1965; Kanamori and Anderson, 1975; Hergarten and Neugebauer, 1998; Hergarten, 2003; Guzzeti et al., 2002; Malamud et al., 2004; Turcotte, 1997, 1999).

It is straightforward to see that the estimation of $p(s)$ and $D(s)$ plays a crucial role in our attempt to derive an analytical expression for the risk function R. In the present work we apply for first time the concept of non-extensivity to the estimation of risk, starting from well known first principles

Published by Copernicus Publications on behalf of the European Geosciences Union.

by introducing a non-extensive formulation for $p(s)$ based on the maximum entropy principle and a phenomenological approach for $D(s)$. Furthermore and in order to support our theoretical results the special case of the expression of R which is based on power-law distributions $p(s)$ is given. We note that the power law expression for $p(s)$ results from the presented non-extensive formulation of the PDF under well defined conditions.

2 Non-extensive modeling in natural hazards

The non-extensive statistical mechanics pioneered by the Tsallis group (Curado and Tsallis, 1991; Lyra and Tsallis, 1998; Tsallis, 1988; Tsallis et al., 1995, 1998; Tsallis and Bukman, 1996) offer a consistent theoretical framework, based on a generalization of entropy, to analyze the behavior of natural systems with fractal or multi-fractal distribution of their elements. Such natural systems where long – range interactions or intermittency are important, lead to power law behaviour. We note that this is consistent with a classical thermodynamic approach to natural systems that rapidly attain equilibrium, leading to exponential-law behavior.

In order to handle non-equilibrium states in systems with complex behavior an entropic functional was proposed (Tsallis, 1988):

$$S_q = k\frac{1}{q-1}\left[1 - \int_{\Sigma} p^q(s)ds\right],$$

which is, in some sense, a generalization of the classical entropic functional because in the limit $q \to 1$ for the Tsallis entropy, we obtain the well known Boltzmann-Gibbs (BG) entropy $S_{BG} = -k\int_{\Sigma} p \ln p ds$. The index q has been interpreted as the degree of non-extensivity, that accounts for the case of many non-independent or long-range interacting systems (Tsallis et al., 1998; Tsallis, 2001). The function $p(s)$ is the probability of finding an event of size s. The sum of all states in entropy is expressed through the integration in all size s of the natural hazard under investigation.

The maximum entropy formulation for Tsallis entropy involves the introduction of at least two constraints (Tsallis et al., 1998). The first one is the normalization of $p(s)$

$$\int_{\Sigma} p(s)ds = 1, \quad \text{where} \quad \Sigma = (0, \infty) \tag{3}$$

and the second is an ad hoc condition about the so-called q-mean value m_{sq}, which can be expressed as:

$$\int_{\Sigma} sp^q(s)ds = m_{sq} \tag{4}$$

In order to calculate $p(s)$ we apply the technique of Lagrange multipliers (see Kubo, 1981). Accordingly, we need to maximize the functional:

$$S_q^* = S_q - \lambda_0 \int_{\Sigma} p(s)ds - \lambda_1 \int_{\Sigma} sp^q(s)ds,$$

with result

$$p(s) = C\,[1+(q-1)\lambda_1 s]^{-\frac{1}{q-1}},$$

where

$$C = \left[\frac{\lambda_0(1-q)}{q}\right]^{\frac{1}{q-1}}.$$

Applying condition (3) we obtain:

$$\lim_{\xi\to\infty} \frac{C}{\lambda_1(q-2)}\left[\xi^{\frac{q-2}{q-1}} - 1\right] = \frac{C}{\lambda_1(2-q)} = 1,$$

which converges only if $1<q<2$. In a similar way condition (4) leads to:

$$m_{sq} = C\int_{\Sigma} \frac{\mathrm{sds}}{[1+(q-1)\lambda_1 s]^{\frac{q}{q-1}}} = \frac{1}{\lambda_1}.$$

Then the distribution and the escort probability are:

$$p(s) = \frac{\lambda_1(2-q)}{[1+\lambda_1(q-1)s]^{\frac{1}{q-1}}} \tag{5a}$$

$$P_{\mathrm{esc}}(s) = \frac{p^q(s)}{\int_{\Sigma} p^q(s)ds} = \frac{\lambda_1^q(2-q)^{q-1}}{[1+\lambda_1(q-1)s]^{\frac{q}{q-1}}} \tag{5b}$$

It is easy to verify that taking the limit $q \to 1$ in P_{esc} we get $\lambda_1 e^{-\lambda_1 s}$, which is the well known exponential distribution. From Eq. (5a) it is obvious that if

$$(q-1)s \gg 1/\lambda_1 = m_{sq}$$

then

$$p(s) \approx m_{sq}^{\frac{2-q}{q-1}} \frac{(2-q)}{(q-1)^{\frac{1}{q-1}}}\left(\frac{1}{s^{\frac{1}{q-1}}}\right)$$

indicating power-law behavior observed in a variety of natural hazards (Turcotte, 1997, 1999; Malamud et al., 1998, Hergarten, 2002).

From Eq. (5a) we obtain

$$F(s) = \mathrm{Pr}ob(x>s) = \int_{s}^{\infty} p(s)ds = \frac{1}{[1+\lambda_1(q-1)s]^{\frac{2-q}{q-1}}}$$

which for $(q-1)s \gg m_{sq}$ exhibits a power-law behavior as well, of the form

$$F(s) \propto \frac{1}{s^{\frac{2-q}{q-1}}} \approx s^{-b}$$

where $b(q) = \frac{2-q}{q-1}$.

In recently published reviews (Dussauge et al., 2003; Guzzeti at al., 2002, 2003, 2006; Hergarten, 2003, 2004; Malamud et al., 2004; Stark and Hovius, 2001), the cumulative distribution function $F(s)\propto s^{-b}$ and the value of the exponent b has been reported for a variety of natural hazards. We note that when $q\rightarrow 1$ the quantity $b(q)$ increases rapidly, while when $q\rightarrow 2$, $b(q)$ approaches zero. For q=1.5 , b=1. For earthquakes, and taking into account earthquake energy as a size, the b-value falls into the range 0.5–0.8 (Main, 1996 and references therein) For landslides a rather strong variation exists in the exponent b, attributed to the triggering mechanism; most studies resulted to values between 1.0 and 1.6, if landslides size is measured in terms of affected area (for details see the review by Hergarten, 2003 and the references therein). For rockfalls, size distribution exhibits a power law if the event size is measured in terms of the volume of displaced rock; most power law exponents fall into the range between 0.4 and 0.7. For rockfalls a detailed analysis is reviewed by Dussauge et al. (2002, 2003). Regarding forest fires, the reported b values lie in the range 0.3 to 0.5, if burnt area is the measure of event size (Malamud et al., 1998). Taking into account the aforementioned range-value reported for b, we estimate the range of the non-extensivity parameter q (see Table 1). We observe that the q value estimated for earthquakes is within the range 1.55–1.67. The latter is in agreement with the values of q estimated using earthquake catalogues from New Madrid fault zone (q=1.63), San Andreas fault (q=1.6 to 1.7) (Viral et al., 2006; Vallianatos and Triantis, 2008).

3 A damage model

It is well accepted that the construction of the damage function $D(s)$ would take into consideration not only of the natural process (hazard) but also the danger to life and property. The proposed expressions for $D(s)$ vary from simple ones, (linear functions of the size s for forest fires provided they are not too large), to very complex ones for earthquakes. Nevertheless, any damage function has to be built on the basis of some principles, which are summarized as follows (Hergarten, 2004):

1. For any event below a certain size $s_{\min}$ the natural hazard does not cause any damage and thus $D(s)$=0.
2. An event with size $s_{\max}$ exists, for which any hazard with size greater than this results in total destruction, i.e. $D(s)=D_{\max}$ for $s\geq s_{\max}$.
3. For natural hazard sizes between $s_{\min}$ and $s_{\max}$ we assume a simple power-law dependence for $D(s)$, i.e. $D(s)\propto s^{\beta}$.
4. The damage function $D(s)$ could be discontinuous, depending on the particular facility hinted by the hazard with size s. For simplicity, we will not introduce discontinuities in our model.

Table 1. Observed power law exponents and the estimated non extensivity parameter q for various natural hazards (see text).

Natural hazard	Measured size	Observed b value	Estimated q parameter
Earthquakes	Energy	0.5–0.8	1.67–1.55
Landslides (area)	Affected area	1.0–1.6	1.5–1.38
Rockfalls (volume)	Displaced volume	0.4–0.7	1.71–1.59
Forest fires	Burnt area	0.3–0.5	1.77–1.67

Using conditions (a), (b) and (c) above, we introduce

$$D(s)=\frac{D_{\max}}{s_{\max}^{\beta}-s_{\min}^{\beta}}(s^{\beta}-s_{\min}^{\beta}) \quad \text{if} \quad s_{\min}<s<s_{\max}\,, \tag{6}$$

while $D(s)$=0 if $s<s_{\min}$ and $D(s)=D_{\max}$ when $s>s_{\max}$. We note that β is a parameter strongly controlled by the type of hazard and the particular environment connected. A review presentation of β for different types of natural hazards is given in Hergarten (2004).

4 Construction of the risk function

Introducing the probability density function $p(s)$ expressed by Eq. (5) and the damage function $D(s)$ of Eq. (6) into Eq. (2), we obtain the total risk R as:

$$R=N\int\limits_{\Sigma} p(s)D(s)ds=N\left[\int\limits_{s_{\min}}^{s_{\max}} p(s)D(s)ds+D_{\max}\int\limits_{s_{\max}}^{\infty} p(s)D(s)\right]$$

After some algebra we obtain

$$R=ND_{\max}\left[\frac{1}{s_{\max}^{\beta}-s_{\min}^{\beta}}\int\limits_{\Sigma} p(s)s^{\beta}ds-\frac{s_{\min}^{\beta}}{s_{\max}^{\beta}-s_{\min}^{\beta}}\right.$$
$$\left.\int\limits_{s_{\max}}^{\infty} p(s)D(s)+\int\limits_{s_{\max}}^{\infty} p(s)ds\right] \tag{7}$$

where

$$P_{12}=\int\limits_{s_{\min}}^{s_{\max}} |p(s)ds=[1+\lambda_1(q-1)s_{\min}]^{\frac{q-2}{q-1}}-[1+\lambda_1(q-1)s_{\max}]^{\frac{q-2}{q-1}}$$

and

$$P_{2\infty}=\int\limits_{s_{\max}}^{\infty} p(s)ds=[1+\lambda_1(q-1)s_{\max}]^{\frac{q-2}{q-1}}\ .$$

We note that since $1<q<2$ the exponent $\frac{q-2}{q-1}$ is negative. The first integral in Eq. (7) can be written as

$$\int_{s_{\min}}^{s_{\max}} p(s)s^{\beta}ds = (\frac{2-q}{q-1})(\frac{m_{sq}}{q-1})^{\beta}\int_{x_1}^{x_2}\frac{x^{\beta}dx}{(1+x)^{\frac{1}{q-1}}}, \qquad (8)$$

where $x_1=(q-1)(s_{\min}/m_{sq})$ and $x_2=(q-1)(s_{\max}/m_{sq})$.

In most cases $s_{\min}\ll s_{\max}$ leading to a good approximation for the integral of Eq. (8), of the form

$$I_R(\beta,q)=\int_0^{\infty}\frac{x^{\beta}}{(1+x)^{\frac{1}{q-1}}}dx=B(z,w),$$

where

$$z=\beta+1,\ w=\frac{1}{q-1}-(\beta+1)$$

and $B(z,w)$ is the Beta function (Abramowitz and Stegun, 1965).

In this case the risk function R could be written as:

$$R\approx ND_{\max}\left[\frac{2-q}{(q-1)^{\beta+1}}(\frac{m_{sq}}{s_{\max}})^{\beta}I_R(\beta,q)-(\frac{s_{\min}}{s_{\max}})^{\beta}P_{12}+P_{2\infty}\right]$$
$$\approx ND_{\max}\left[\frac{2-q}{(q-1)^{\beta+1}}(\frac{m_{sq}}{s_{\max}})^{\beta}I_R(\beta,q)+R_{2\infty}\right]$$

5 Discussion and concluding remarks

In the present work we apply the Tsallis entropy generalization that extends the traditional Boltzmann- Gibbs thermostatistics to natural hazard systems, where non-linearity, long-range interactions, long memory effects and scaling (fractal and multifractal) are important. The advantage of considering the Tsallis distribution is that based on an entropy principle, it can be related to statistical mechanics and reduces to the traditional BG statistical mechanics as a special case.

Using a non extensive approach we conclude that for $(q-1)s\gg m_{sq}$, a power law behaviour exists, with a probability density function given by Eq. (5a) and $b(q)=(2-q)/(q-1)$. We proceed now to evaluate the risk function for the special case of a power law. Assuming that $s_{\max}$ is much larger than $s_{\min}$ (i.e. $s_{\max}\gg s_{\min}\gg m_{sq}/(q-1)$) we obtain

$$R=\frac{ND_{\max}M_{sq}}{bs_{\max}^{\beta}}\left[1+\frac{b}{\beta-b}s_{\max}^{\beta-b}-\frac{\beta}{\beta-b}s_{\min}^{\beta-b}\right]$$

where $\beta\neq b$ and

$$M_{sq}=m_{sq}^{\frac{2-q}{q-1}}\frac{(2-q)}{(q-1)^{1/(q-1)}}\frac{1}{s^{1/(q-1)}}.$$

If $\beta>b>0$ the term $s_{\max}^{\beta-b}$ is much larger than $s_{\min}^{\beta-b}$ and thus

$$R\approx\frac{ND_{\max}M_{sq}}{bs_{\max}^{\beta}}\left[1+\frac{b}{\beta-b}s_{\max}^{\beta-b}\right].$$

In such a case, when the second term in the above Equation is much greater than unity,

$$R\approx\frac{ND_{\max}M_{sq}}{\beta-b}\frac{1}{s_{\max}^{b}}=\frac{ND_{\max}}{\beta-b}s_{\max}p(s_{\max}).$$

The latter expression indicates that the risk is mainly determined by the largest event.

In the case where $0<\beta<$b, the term $s_{\max}^{\beta-b}$ becomes much smaller than $s_{\min}^{\beta-b}$ and the risk is

$$R=\frac{ND_{\max}M_{sq}}{bs_{\max}^{\beta}}\left[1+\frac{\beta}{b-\beta}\frac{1}{s_{\min}^{b-\beta}}\right].$$

When the term $\left[\frac{\beta}{b-\beta}\frac{1}{s_{\min}^{b-\beta}}\gg 1\right]$,then

$$R=\frac{ND_{\max}}{b}\left[\frac{\beta}{b-\beta}(\frac{s_{\min}}{s_{\max}})^{\beta}\right]s_{\min}p(s_{\min}).$$

The latter expression indicates that the increase of damage with event size is insufficient to compensate for the decrease in the frequency of event occurrence and the risk arises from a significant number of small events.

The aforementioned equations are valid in the case where $\beta\neq b$. When $\beta=b$ we obtain

$$R=\frac{ND_{\max}M_{sq}}{s_{\max}^{\beta}}\ln\frac{s_{\max}}{s_{\min}}=ND_{\max}s_{\max}p(s_{\max})\ln\frac{s_{\max}}{s_{\min}}.$$

The above defined expressions suggest that the crucial parameter in the behavior of risk is the difference $\beta-b(q)=\beta-\frac{2-q}{q-1}$ which involves the thermodynamic parameter q expressing the non-extensivity of the system.

Although quantifying the damage caused by natural hazards (i.e. the estimation of parameter β) is difficult, it can be expected that for earthquakes, the damage increases nonlinearly (i.e. $\beta>1$) with released energy (see Hergarten, 2004; McGuire, 2004). Taking into account that for earthquakes the parameter q is between 1.55 and 1.67, the difference $\beta-\frac{2-q}{q-1}$ is always positive, leading to the expected conclusion, that the risk resulting from earthquakes is dominated by the largest event. For forest fires, the damage parameter β varies between 0.5 and 1, scaling with burnt area (Turcotte and Malamud, 2004; Hergarten, 2004); this leads to $\beta-b(q)$ values in the range from 0 to 0.7 and to the conclusion that the risk resulting from forest fires is dominated by the largest event. Only in the case where $b=\beta=0.5$ we conclude that large and small events contribute evenly to the total risk. For landslides, the selection of an appropriate damage model is crucial. In the case of a simple linear model, $\beta=1$ the damage function scales linearly with the affected area; then $\beta-b(q)\leq 0$, leading to the conclusion that landslide risk is coming mainly from small-sized events. However, we note that slightly increasing β (e.g. from 1 to 1.6), may change the quantity $\beta-b(q)$ from negative to positive, highlighting the importance of selecting appropriate damage models. For rockfalls, the damage function is scaled with the displaced volume (or mass). Even in the simple linear case when β=1, the quantity $\beta - b(q)$ is positive supporting the result that the total risk rockfalls arises from the largest events.

In concluding, we point out that in the present work we indicate that the non extensivity viewpoint is applicable to natural hazard processes. In the frame of a non-extensive approach which is based on Tsallis entropy for the construction of the probability density function (PDF) and a phenomenological exponential expression for the damage function, we analytically calculate the risk function of natural hazards (earthquakes, rockfalls, forestfires, landslides). For the lowest size (i.e. energy level) of the natural hazard the PDF can be deduced on the basis of the maximum entropy principle using BG statistics. In the low energy regime the correlation between the different parts of elements involved in the evolution of natural hazards are short-ranged. As the size (i.e. energy) increases, long range correlation becomes much more important, implying the necessity of using Tsallis entropy as an appropriate generalization of BG entropy. The power law behaviour for the PDF is derived as a special case, leading to b-values being functions of the non-extensivity parameter q. The analysis of risk function dependence on the parameters of hazard PDF and damage function for various hazards indicates that earthquakes, rockfalls and forest fires exhibit similar behaviour, in which the total risk arises from the largest events, while for landslides, in a first linear approximation, risk is coming from the smaller events. The latter result is strongly governed by the selection of an appropriate damage model (i.e. the exponent β).

Acknowledgements. This work is supported by the project “Advanced techniques for SEismic RISK reduction in Mediterranean Archipelago Regions (SE RISK)”, A.1.034, Measure 3.3 “Management, prevention and reduction of natural risks: drought, desertification, fires, earthquakes etc.”, Priority Axis 3, ARCHIMED, INTERREG IIIB, co-financed by the European Regional Development Fund (ERDF) and national resources.

Edited by: M. Contadakis
Reviewed by: E. Papadimitriou and another anonymous referee

References

Abramowitz, M. and Stegun, I.: Handbook of mathematical functions, Dover, New York, 258–260, 1965.

Aki, K.: Maximum likelihood estimate of b in the formula $\log N = a - bM$ and its confidence limits, B. Earthq. Res. I. Tokyo, 43, 237–239, 1965.

Bak, P. and Tang, C.: Earthquakes as a self-organized critical phenomenon, J. Geophys. Res., 94, 15 635–15 637, 1989.

Bak, P., Tang, C., and Wiesenfeld, K.: Self-organized critically, Phys. Rev. A, 38, 364–374, 1987.

Bonnet, E., Bour, O., Olding, N. E., Davy, P., Main, I., Cowie, P., and Berkowitz, B.: Scaling of fracture systems in Geological media, Rev. Geophys., 39(3), 347–383, 2001.

Curado, E. M. F. and Tsallis, C.: Generalized statistical mechanics: Connection with thermodynamics, J. Phys. A-Math. Gen., 24, L69–72, 1991.

Dussauge-Peisser, C., Helmstetter, A., Grasso, J.-R., Hantz, D., Desvarreux, P., Jeannin, M., and Giraud, A.: Probabilistic approach to rock fall hazard assessment: potential of historical data analysis, Nat. Hazards Earth Syst. Sci., 2, 15–26, 2002, http://www.nat-hazards-earth-syst-sci.net/2/15/2002/.

Dussauge, C., Grasso, J. R., and Helmstetter, A.: Statistical analysis of rockfall volume distributions: Implications for rockfall dynamics, J. Geophys. Res.-Sol. Ea., 108(6), 2–11, 2003.

Guzzetti, F., Malamud, B. D., Turcotte, D. L., and Reichenbach, P.: Power-law correlations of landslide areas in central Italy, Earth Planet. Sc. Lett., 195, 169–183, 2002.

Guzzetti, F., Reichenbach, P., Cardinali, M., Ardizzone, F., and Galli, M.: The impact of landslides in the Umbria region, central Italy, Nat. Hazards Earth Syst. Sci., 3, 469–486, 2003, http://www.nat-hazards-earth-syst-sci.net/3/469/2003/.

Hergarten, S. and Neugebauer, H. J.: Self-organized criticality in a landslide model, Geophys. Res. Lett., 25(6), 801–804, 1998.

Hergarten, S.: Self-organized criticality in Earth Systems, Springer, Berlin, 269 pp., 2003.

Hergarten, S.: Landslides, sandpiles, and self-organized criticality, Nat. Hazards Earth Syst. Sci., 3, 505–514, 2003, http://www.nat-hazards-earth-syst-sci.net/3/505/2003/.

Hergarten, S.: Aspects of risk assessment in power-law distributed natural hazards, Nat. Hazards Earth Syst. Sci., 4, 309–313, 2004, http://www.nat-hazards-earth-syst-sci.net/4/309/2004/.

Kanamori, H. and Anderson, D. L.: Theoretical basis of some empirical relations in seismology, B. Seismol. Soc. Am., 65, 1072–1096, 1975.

Lyra, M. L. and Tsallis, C.: Nonextensivity and multifractality in low-dimensional dissipative systems, Phys. Rev. Lett., 80, 53–56, 1998.

Main, I.: Statistical physics, seismogenesis and seismic hazard, Rev. Geophys., 34, 433–462, 1996.

Malamud, B. D., Turcotte, D. L., Guzzetti, F., and Reichenbach, P.: Landslide inventories and their statistical properties, Earth Surf. Proc. Land., 29(6), 687–711, 2004.

Malamud, B. D., Turcotte, D. L., Guzzetti, F., and Reichenbach, P.: Landslides, earthquakes, and erosion, Earth Planet. Sc. Lett., 229, 45–59, 2004.

Malamud, B. D. and Turcotte, D. L.: Self-organized criticality applied to natural hazards, Nat. Hazards, 20, 93–116, 1999.

Malamud, B. D., Morein, G., and Turcotte, D. L.: Forest fires: an example of self-organized critical behaviour, Science, 281, 1840–1842, 1998.

McGuire, R.: Seismic hazard and risk analysis, EERI, Oaklans, CA, USA, 219 pp., 2004.

Sornette, A. and Sornette, D.: Self-organized criticality and earthquakes, Europhys. Lett., 9, 197–202, 1989.

Stark, C. P. and Hovius, N.: The characterization of landslide size distributions, Geophys. Res. Lett., 28(6), 1091–1094, 2001.

Turcotte, D. L.: Fractals and Chaos in Geology and Geophysics, Cambridge University Press, Cambridge, 2nd edn., 216 pp., 1997.

Turcotte, D. L.: Application of statistical mechanics to natural hazards and landforms, Physica A, 274, 294–299, 1999.

Turcotte, D. L. and Malamud, B.: Landslides, forest fires and earthquakes: example of self-organized critical behaviour Physica A, 340, 580–589, 2004.

Tsallis, C.: Possible generalization of Boltzmann-Gibbs statistics, J. Stat. Phys., 52, 479–487, 1988.

Tsallis, C., Levy, S. V. F., Souza, A. M. C., and Maynard, R.: Statistical mechanical foundation of the ubiquity of Lévy distributions in nature, Phys. Rev. Lett., 75, 3589–3593, 1995.

Tsallis, C. and Bukman, D. J.: Anomalous diffusion in the presence of external forces: Exact time-dependent solutions and entropy, Phys. Rev. E, 54, R2197–2200, 1996.

Tsallis, C., Mendes, R. S., and Plastino, A. R.: The role of constraints within generalized nonextensive statistics, Phys. Rev. A, 261, 534–554, 1998.

Vilar, C. S., Franca, G., Silva, R., and Alcaniz, J. S.: Nonextensivity in geolological faults?, Physica A, 377, 285–290, 2006.

Vallianatos, F. and Triantis, D.: Scaling in Fracture: From fracture to earthquake physics, Proc. Of the 2nd IASME Conference on Seismology, Cambridge, UK, 84–88, 2008.