

Do negative numbers have a place on the mental number line?

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Abstract

The cognitive representation of negative numbers has recently been studied with magnitude classification and variable standard (Fischer, 2003b). Results suggested that negative numbers are cognitively represented on a “mental number line”. Conflicting with this observation we report here that response biases for parity classification of negative numbers are related to absolute digit magnitude only. A control experiment with magnitude classification relative to a fixed standard shows that our result does not reflect mere inattention to the minus sign. Together, the available evidence suggests that we process negative numbers less automatically compared to positive numbers.

Key words: mental number line, negative numbers, SNARC, MARC

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Our ability to understand numbers is under intense investigation. Systematic performance patterns across various number processing tasks have led to the concept of a *mental number line*, conceived as an analogue representation of numerosities (Moyer & Landauer, 1967; Restle, 1970). A recent hypothesis about the mental number line is that it might be spatially oriented. This hypothesis was inspired by the *SNARC effect* (for Spatial-Numerical Association of Response Codes): Digits representing relatively small magnitudes (e.g., 1, 2) facilitate left side responses, while digits representing relatively larger magnitudes (e.g., 8, 9) facilitate right side responses (Dehaene, Bossini & Giraux, 1993; Fias, Brysbaert, Geypens & d'Ydewalle, 1996; Fias 2001; Fischer, 2001, 2003a; Fischer, Warlop, Hill & Fias, 2004; Fischer, Castel, Dodd, & Pratt, 2003).

In a recent study, Fischer (2003) used this spatial congruency effect to investigate whether we have separate cognitive representations for positive and negative numbers. He presented digit pairs from the range -9 to 9 while keeping the numerical pair distances constant at 5. Classification decisions were significantly faster when the displays were spatially congruent with a negatively extended mental number line. This result suggested that we do indeed acquire an association of negative numbers with left space.

On the other hand, Nuerk, Iversen and Willmes (2004) asked participants to make parity judgments on negative numbers and found no reliable spatial association (although there was a trend). They did find a SNARC effect for negative numbers in their error data, and there was also a systematic association between parity status and space such that odd numbers were associated with left and even numbers with right responses. This latter observation has been termed the MARC effect, to suggest a linguistic markedness effect on number processing.

The present study reexamined the cognitive representation of negative numbers by replicating Nuerk et al.'s (2004) approach. In contrast to their study we manipulated participants' level of experience with numbers to see if this could account for the conflicting results. It is conceivable that participants with mathematical training develop stronger spatial associations for negative numbers than untrained participants. This factor had not been manipulated in the two previous studies.

Experiment 1

Method

Participants

20 students (18-31 years, average 21, 1 left-handed) consented to participate for pay. Ten studied mathematical topics (mathematics, physics, engineering), ten studied psychology.

Apparatus

The experiment was run on an Apple Power Macintosh 4400/200 with 39 x 29 cm screen, using black stimuli on white background. The response keys on the top row of the standard Apple keyboard were 12 x 12 mm wide and 380 mm apart. The keyboard was centered just below the display and viewing distance was about 50 cm.

Material and design

Digits from the ranges [0, 9], [-9, 0] and [-4, 5] were selected for the three experimental conditions. The order of ranges was counterbalanced as much as possible across participants.

Procedure

The task was to make a speeded parity judgment on each digit, using left and right response buttons according to an agreed rule. Participants were informed about the range of digits that would appear on the screen. After 90 trials the assignment of responses to response keys was reversed for the remaining 90 trials for this range. To reduce participants' confusion arising from such reversals, response assignments only changed within a range and were kept constant as participants moved to the next range condition. Data collection for each block was preceded by 12 practice trials.

In each trial, an empty frame appeared centered on the screen for 300 ms. It was a 22 x 32 mm rectangle drawn with a 1 mm wide line. Then the target digit appeared in it, and the participant's response was recorded over a period of 1,500 ms. Digits were 10 mm wide x 17 mm high and were shown either with a minus sign directly in front or without any sign. The frame and the digit were then blanked for 1,500 ms before the next trial began. Feedback was given if a response was incorrect or its latency outside the interval between 100 and 1,500 ms. These trials were repeated at the end of each block.

Analyses

Mean reaction times (RT) for correct responses were computed for each participant, each digit and each side of response. An ANOVA evaluated effects of mathematical skill (high vs. low) and type of block (positive, negative, mixed). Next, for each digit range, mean RT was analyzed in a 2 (mathematical skill) x 2 (response side) x 5 (digit magnitude) x 2 (digit parity) ANOVA. A significance level of .05 was adopted throughout.

The presence of a SNARC effect is reflected in a significant interaction between side of response and digit magnitude. To quantify the size of this effect and to examine the orientation of the interaction, the difference between the mean RT for the right and the left side response (dRT) was computed for each participant and each digit. This new variable was regressed on stimulus digits for each participant to obtain a regression weight. This procedure was repeated for each number range. The resulting set of regression weights was then tested against zero to assess the direction and strength of the SNARC effect.

We noticed only during data analyses that one occurrence of the digit 1 in the negative range had inadvertently been displayed without the minus sign. Decision latencies for this unexpected positive digit within negative number range condition were separated from the remaining data and compared with those for the same digit in the mixed and positive range conditions to investigate list context effects on number processing and how such list context effects might depend on number processing skills. This was done with an ANOVA on mathematical skill level (high vs. low; between subjects) and type of block (positive, negative, mixed; within subjects).

Results

The average error rate was 4.2% and did not exceed 6.5% per participant. There was no speed-accuracy trade-off, as indicated by a small positive correlation between RT and number of errors, $r = 0.28$, $p > .23$. Mean RTs were 559 ms for positive digits, 549 ms for negative digits, and 546 ms for mixed digits. This effect of type of block was marginally significant, $F(2, 36) = 2.69$, $p = .08$, but there was no main effect of skill level and no interaction, both $F < 1$. The average decision times in the skilled and unskilled groups were 541 ms (SD 26) and 561 ms (SD 18), respectively, $t(18) = 0.63$, $p > .53$.

Positive digits

For positive digits, magnitude produced a significant main effect, $F(4, 72) = 9.03$, $p < .01$, and a significant interaction with response side, $F(4, 72) = 5.62$, $p < .01$. This indicates a SNARC effect. Digit parity had a reliable main effect, $F(1, 18) = 5.87$, $p < .05$, and also interacted with response side, $F(1, 18) = 5.56$, $p < .05$. This interaction signaled a MARC effect due to faster left than right responses for odd numbers (564 and 569 ms, respectively) and faster right than left responses for even numbers (537 and 566 ms, respectively). The only other significant result was the interaction digit magnitude X digit parity, $F(4, 72) = 4.79$, $p < .01$, due to faster even than odd responses for small and large but not intermediate magnitudes.

The mean non-standardized regression weight for the positive digits was $\eta = -6.54$. It deviated significantly from zero, $t(19) = -3.281$, $p < 0.01$, showing an association of smaller digits with the left side and larger digits with the right side of space. This association is depicted in the top left panel of Figure 1 (see also Table 1, left column). For the mathematically skilled group the mean regression weight was $\eta = -4.32$, for the less skilled group it was $\eta = -8.75$. Replicating Dehaene et al.'s (1993) finding, there was a weaker SNARC effect in the skilled group. However, this difference failed to reach significance here, $t(18) = 1.12$, $p = 0.28$.

Figure 1:

Decision time differences in Experiment 1 between right hand and left hand responses are plotted against the presented stimulus. Positive values indicate faster left-hand responses, and lines show the best-fitting linear regressions. Data are averaged across all 20 participants and shown separately for the positive range (top left panel, $r^2 = 0.33$), the negative range (top right panel, $r^2 = 0.27$), and the two segments of the mixed range (bottom left panel, negative sub-range: $r^2 = 0.26$; bottom right panel, positive sub-range: $r^2 = 0.96$).

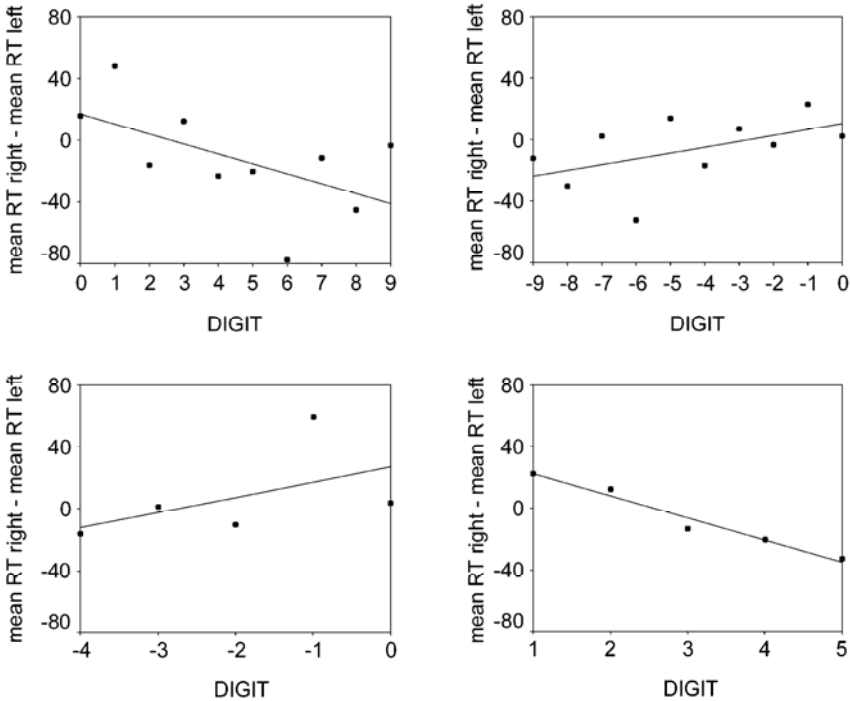


Table 1:

Results for the regression of RT (right hand) – RT (left hand) on number magnitude in Experiment 1.

		Positive Numbers	Negative Numbers	Mixed Numbers	
		[0 to 9]	[-9 to 0]	[-4 to 0]	[1 to 5]
skilled group	Mean	-4.32	3.34	19.45	-20.44
	SD	8.86	9.70	51.38	21.21
unskilled group	Mean	-8.75	4.26	23.53	-7.91
	SD	8.84	5.93	37.11	14.43
Total	Mean	-6.54	3.80	21.49	-14.18
	SD	8.91	7.84	43.67	18.79

Note: The Table contains unstandardized beta weights.

Negative digits

For negative digits the main effect for magnitude was also significant, $F(4, 72) = 4.59$, $p < .01$, and interacted significantly with skill, $F(4, 72) = 2.83$, $p < .05$. In addition, magnitude again interacted with response side, $F(4, 72) = 3.08$, $p < .01$, indicating a SNARC effect. This interaction was further modulated by skill, $F(4, 72) = 3.13$, $p < .05$, and will be explored below. A marginally reliable interaction of parity with response side signaled the MARC effect, $F(1, 18) = 4.13$, $p < .06$. There were faster left than right responses for odd numbers (545 and 552 ms, respectively) and faster right than left responses for even numbers (537 and 557 ms, respectively). The interaction between magnitude and parity, $F(4, 72) = 6.04$, $p < .001$, was also modulated by skill, $F(4, 72) = 3.39$, $p < .05$. There were again faster even than odd responses for small magnitudes in both groups, but no consistent pattern for other magnitudes.

The mean non-standardized regression weight for the negative digits was $\eta = 3.80$, which differed significantly from zero, $t(19) = 2.17$, $p < .05$, indicating an association of large negative digits (e.g., -9) with the right side and small negative digits (e.g., -2) with the left side of space (see top right panel of Figure 1 and middle of Table 1). No reliable differences between the groups were observed in the mean regression weights. In sum, spatial-numerical associations were independent from the accompanying minus signs.

Mixed digits

For the mixed list the main effect for magnitude was also significant, $F(4, 72) = 8.01$, $p < .001$, and again interacted with response side, $F(4, 72) = 3.37$, $p < .05$. This again demonstrates the SNARC effect. Parity had a significant main effect, $F(1, 18) = 4.74$, $p < .05$, and interacted with magnitude, $F(4, 72) = 4.02$, $p < .01$. Even digits were classified faster than odd digits in all magnitude ranges except for the 0/1 range. The triple interaction side \times magnitude \times parity, $F(4, 72) = 3.12$, $p < .05$, was due to another exception for the range -2/-1 when left responses were required.

In a linear regression of dRT on the full range of digits, the mean regression weight was $\eta = -2.64$, which failed to differ significantly from zero, $t(19) = -1.76$, $p = 0.10$. However, the relation between dRT and magnitude was positive for the interval [-4, 0] and negative for the interval [1, 5]. Indeed, both regression weights ($\eta = 21.49$ for the interval [-4, 0] and $\eta = -4.18$ for the interval [1, 5]) deviated significantly from zero, with $t(19) = 2.20$, $p < .05$, and $t(19) = -3.38$, $p < .001$, respectively (see Figure 1, bottom panels and Table 1, right side).

Expectancy violation

Finally, the positive digit 1 (without sign) had inadvertently appeared in all range conditions, including the negative range. This allowed us to evaluate effects of an expectancy violation on decision speed. Parity judgments were significantly slower when the digit 1 appeared in the negative range (629 ms) compared to either the positive (567 ms) or mixed ranges (555 ms), $F(2, 36) = 4.09$, $p < .05$. This was true for both skill levels, $F(1, 18) < 1$.

Discussion

We found faster left-hand decision for small digits and faster right-hand decisions for large digits - the SNARC effect. This was true regardless of sign, indicating that the mental number line does *not* extend to the left of zero, in conflict with the conclusion of Fischer (2003b). This result did not depend on the range of available numbers. We also replicated Nuerk et al.'s (2004) association of odd numbers with left space and even numbers with right space. Finally, mathematical skill affected only negative number processing and diluted the SNARC effect slightly.

One problem remains with the interpretation of the similar SNARC effects for positive and negative digits. Did participants focus their attention on the known location of the forthcoming number and completely ignore the task-irrelevant minus sign? The following experiment investigated this issue experimentally.

Experiment 2

A number categorization task was used where participants decided whether a given digit was larger or smaller than zero. Responses were made with two stimulus-response assignments: Participants responded either to positive digits with the right button and to negative digits with the left button, or to positive digits with the left button and to negative digits with the right button. According to the notion of a leftward extension of the mental number line for negative numbers, there should be an advantage for right-button responses to positive digits and an advantage for left-button responses to negative digits. The presence of a distance effect was used to assess whether participants selectively attend to the sign to solve the task. We also looked at whether the MARC effect would be obtained when number parity is not task-relevant.

Method

Participants

Fourteen new participants (23.5 years old, range 20 to 38, 9 female and 5 male, 1 left-handed) consented to participate for pay.

Apparatus and stimuli

The 18 digits from -9 to $+9$, excluding zero, were presented in 30-point boldface Times New Roman font. The sign was shown for both positive and negative digits. They appeared in black on a white background on a 15 inch Shinho AV2 monitor interfaced with a 486 PC compatible computer. Participants were seated in front of the center of the computer screen at a distance of approximately 40 cm. The keyboard was also centered in their mid-sagittal plane. Left and right hand responses were recorded on the "1" and "*" keys in the row below

the function keys of a standard QWERTY keyboard with integrated numerical keypad (38 cm center to center distance).

Design and procedure

The task was a rapidly classify each digit as larger or smaller than zero, using left and right response buttons according to an agreed rule. In each trial, a fixation star (40 point boldface Times New Roman) was presented in the center of the blank screen for 1,000 ms. It was then replaced by the stimulus for up to 2,000 ms or until a response had occurred. No feedback was given, and trials with erroneous responses were not repeated. Stimuli were presented in individually randomized order, and the order of response assignments was counterbalanced between participants in blocks of 180 trials. Participants had 36 practice trials prior to data collection in each response condition.

Analyses

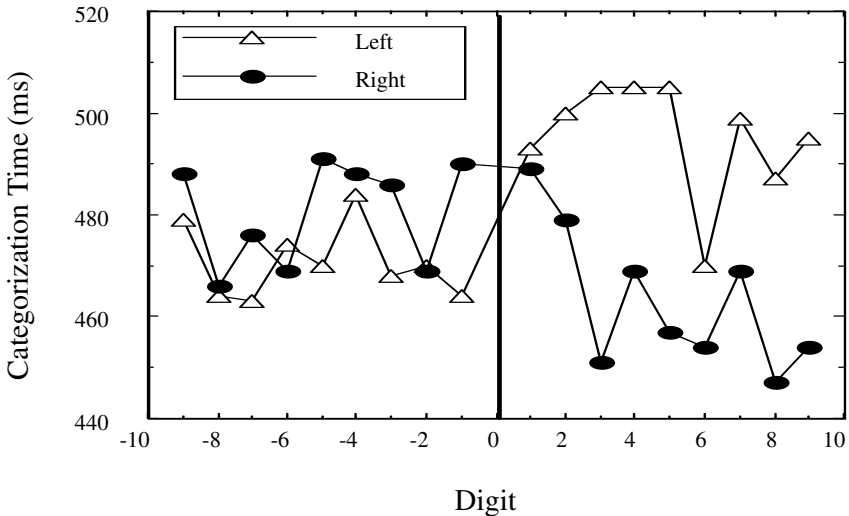
A total of 5,040 trials were recorded, but 4% of these were either erroneous responses or categorization latencies outside the temporal window from 100 ms to 1,300 ms. These were discarded from further analyses. Next, a 2 (response side) x 2 (digit range) x 4 (digit magnitude) x 2 (digit parity) ANOVA evaluated categorization latencies, excluding digits 9 and -9 for the sake of this analysis. In addition, each participant's average categorization latencies were regressed on digit magnitudes, separately for each full digit range and using absolute digit values for the x-axes throughout. This way, a negative slope would indicate for both ranges that decision latencies increased as digit magnitude approached the reference value of zero.

Results

Figure 2 shows the results of Experiment 2. The average time to correctly categorize a digit was 477 ms. The main effect of side was significant, $F(1, 13) = 10.76$, $p < .01$. Responses were given on average within 482 ms with the left button and within 472 ms with the right button. The main effect of range was not significant, $F(1, 13) = 1.10$, $p > .31$, but the main effect of magnitude was reliable, $F(3, 39) = 3.11$, $p < .05$. This result was further explored with regression analyses (see below) to test for the presence of a distance effect. There was no reliable main effect of parity, $F(1, 13) = 2.86$, $p > .11$. The interaction of side with range was not quite significant, $F(1, 13) = 2.47$, $p < .14$, but decisions were 10 ms faster with the left than with the right button for negative digits, $F(1, 13) = 0.76$, $p > .42$, and 31 ms faster with the right than with the left button for positive digits, $F(1, 13) = 5.01$, $p < .05$. All other interactions were not significant, with p-values $> .22$. Thus, there was no reliable MARC effect.

Figure 2:

Categorization times in Experiment 2 for positive and negative digits. Data are averaged across all 14 participants and separately plotted for left (white triangles) and right hand responses (black circles).



Consider next the distance effect. When regressing average categorization times on the full range of digits, the average regression weight for the positive range was -2.45 (range -9.01 to 3.26 across participants). This value differed from zero, $t(13) = 2.56$, $p < .03$, indicating a distance effect. The average regression weight for the negative digit range was -0.27 (range -6.79 to 3.03 across participants) which did not differ from zero, $t(13) = 0.34$, $p > .74$. Finally, the two slopes did not differ from each other, $t(13) = 1.48$, $p > .16$.

Consider finally the SNARC effect. When regressing dRT on digits, the average regression weight for the positive digit range was -2.28 (range -10.40 to 9.32 across participants). This value did not differ from zero, $t(13) = 1.39$, $p > .18$. The average regression weight for the negative digit range was -1.31 (range -10.59 to 15.06 across participants) which also did not differ from zero, $t(13) = 0.73$, $p > .47$. These two slopes did also not differ from each other, $t(13) = 0.36$, $p > .72$. Thus, there was no SNARC effect over and above the left-right association for negative and positive numbers, respectively.

Discussion

The manipulation of response assignments led to a reliable congruity effect with positive but not with negative numbers. This seems to support the idea of a purely positive mental number line that originates at zero and extends to the right side of personal space. However, finding this result for positive numbers implies that negative numbers did provide a cognitive contrast. The distance effect indicates that participants did not disregard magnitude when

categorizing digits as larger or smaller than zero but processed the sign and the digit in a holistic manner. Although this might be taken to suggest that the results of Experiment 1 were not due to disregarding the negative digit's sign, the differences between tasks and their associated strategies make this conclusion tentative. Moreover, the distance effect was limited to positive digits, suggesting that positive and negative numbers are processed differently. This result was not due to a difference in overall categorization speed but reflected considerable inter-individual differences, suggesting that participants used different strategies to solve the task. The absent spatial congruity effect with negative compared to positive digits further strengthens this conclusion. Positive digits were categorized faster with the right hand and negative digits were categorized slightly faster with the left hand, although a direct analysis of the SNARC effect did not produce reliable results for either digit range. Finally, there was no association between digit parity and left-right space in this magnitude classification task. Thus, we were not able to document a MARC effect outside the parity judgment task.

Conclusions

The present results suggest that negative numbers do not automatically elicit spatial associations. In this respect, negative numbers differ from positive numbers, which are automatically associated with space, even if magnitude information is irrelevant for the task. The MARC effect, in contrast, did occur for negative as well as positive numbers but seems to be limited to tasks where parity is task-relevant.

Some aspects of the present results provide moderate support for the general idea of separate cognitive representations for positive and negative numbers. First, notice that similar decision speeds were found for positive and negative numbers in both of the present experiments. This observation supports the claim that negative numbers do not need to be interpreted indirectly through accessing their positive counterparts. The increased processing times for negative number pairs in Fischer's (2003b) study might thus not reflect additional cognitive operations on the individual digits, but could alternatively be attributed to some cognitive comparison process that requires more time when the two numbers to be compared are both negative. This tentative proposal does, however, rely on comparisons across very different tasks.

Participants associated negative numbers with the left side of space and positive numbers with the right side of space in Experiment 2. This replicates previous findings that "smaller" responses are made faster with the left hand and "larger" responses with the right hand (Dehaene, Dupoux, & Mehler, 1990). This shows that negative numbers are indeed *represented* as being located to the left of positive numbers. Although the congruity effect itself was only statistically reliable with positive numbers this observation suggests that the negative numbers were cognitively available to provide a spatial contrast. Note that, again, this indication of a mental number line with an extension to the left for negative numbers was obtained using a task that required magnitude information.

Performances of our participants with negative numbers were more variable than with positive numbers, suggesting that different individuals process negative numbers differently. These different strategies seem to result in spatial associations less often for negative than for positive numbers. Different levels of mathematical training may lead to different process-

ing strategies for negative numbers, but our results do not allow us to identify such strategies. This may be due to insufficient differences between our two groups in Experiment 1, and future research should control mathematical experience with negative numbers more effectively.

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