Deconstructing and reconstructing the base-10 structure of arabic numbers

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Abstract

When comparing 2-digit Arabic numbers, not only the relevant semantic relations between numbers are computed, but also irrelevant ones, like the unit-decade compatibility effect (Nuerk, Weger, & Willmes, 2001). As pointed out by Nuerk and Willmes (this issue), the magnitude representations of decade and unit digit may be activated beside the semantic representation of the two-digit magnitude. The magnitude representation of units and decades may produce congruity effects, which have not been explored yet. Here we examine different forms of digit congruity produced by the mode of presentation of the stimuli, and how the congruity of decade and unit digits may interfere with the normal two-digit number processing. In two experiments participants were asked to identify the larger of two two-digit Arabic numbers. Unit- and decade digits were presented in different combinations within a short time interval, so that the two-digit structure was perceptually disturbed. Regression analyses revealed that in both Experiments 1 and 2 the semantic congruity of the digits being presented simultaneously led to a bias in response times which strongly interfered with the decade distance and compatibility effects. Our results suggest that the semantic comparison of 2-digit Arabic numbers may also be determined by congruity relations between the digits of these numbers. Implications for presenting one two-digit number in a comparison with a fixed standard are discussed.

Key words: two-digit number processing, unit-decade compatibility effect, second-order congruity

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Introduction

Semantic two-digit number representation is conceptualized in very different ways by different cognitive models (Dehaene, Dupoux & Mehler, 1990; McCloskey, 1992; Nuerk & Willmes, this issue). Dehaene and colleagues have claimed that two-digit number comparison activates an overall distance representation which is better represented by the logarithm of the distance and is insensitive to the decimal structure of Arabic numbers. In contrast, the model of McCloskey postulated that two-digit number semantics is represented in a base-10 structure (e.g. "57" = {5}*10EXP1 + {7}*10EXP0). In a new hybrid model, Nuerk and Willmes (this issue, see Nuerk et al., 2001) have combined features from both models of Dehaene and coworkers (1990) and McCloskey (1992) into a single structure. Nuerk and Willmes propose that when comparing two-digit numbers, the overall and logarithmically compressed magnitude representation is activated together with the decomposed representations of units and decades (e.g. "57" = {57}, {5} and {7}). An important piece of evidence for decomposed representations of units and decades is the compatibility effect (Nuerk et al., 2001; Nuerk, Weger, & Willmes, 2002, 2004a; Nuerk & Willmes, this issue; Nuerk, Knops, & Willmes, submitted). The compatibility effect shows that the time for comparing number magnitude of two-digit numbers is slower when the smaller number contains the larger unit (e.g. 81 26) than when the larger number contains the larger unit (e.g. 76 21). The compatibility effect cannot be accounted for by overall distance alone, but rather by the activation and comparison of decomposed unit and decade representations.

The hybrid model postulates interactions between the overall and the decomposed representations of units and decades at the semantic level (cf. Nuerk & Willmes, this issue, Figure 1). Furthermore, the hybrid model are predicting that the overall and the decomposed representations of two-digit numbers may be activated or inhibited in accord with task characteristics and demands.

Here the occurrence of interactions between the overall and the decomposed magnitude representations was explored. The activation of the different representations was manipulated in two SOA experiments, where unit and decade digits were shortly masked. While some digits are masked, the overall magnitude might not get activated, but the decomposed magnitudes may be activated. Figures 1A and 1B show how numbers were masked in both Experiments 1 and 2, respectively. In Experiment 1 unit and decade digit from different numbers were masked. According to the hybrid model, this manipulation would decrease the relevance of overall magnitude and increase the relevance of unit and decade magnitudes for performance. In Experiment 2, however, the impact of overall magnitude on performance might be larger than in Experiment 1, since overall magnitude of one number can always be activated from the beginning of each trial on.

Another interesting consequence of digit masking concerns the moment when the comparison of units and decades starts. Although the hybrid model does not make direct predictions about the starting point of unit and decade comparison, two possible accounts may be derived: the first one follows the holistic model of Dehaene and colleagues (1990). The holistic model (Dehaene et al., 1990, p. 635) predicts that "...the subjects will wait until the decade digit appears..." before activating the magnitude representations of two-digit numbers. If participants wait until the masked digits appear before activating and comparing overall and decomposed magnitudes, the compatibility effect should be comparable in both Figures 1A and 1B:

Examples of trials from Experiment 1 (1A) and Experiment 2 (1B). In both experiments, a visual mask was presented for 100 ms and was followed by the stimuli, which were presented for a maximum of 1400 ms. Following each trial the screen was empty for 500 ms.



Experiments 1 and 2 and with that found in previous studies presenting all digits simultaneously. If both overall and decomposed magnitude representations of two-digit numbers are activated only after the whole numbers are presented, the effects of semantic number processing on performance may be largely comparable regardless of the SOA manipulation. Nevertheless, the comparisons between unit and decade start before all digits are presented. In this case, congruity between the magnitudes of digits being presented together may be a relevant predictor of performance in both Experiments 1 and 2. The early activation of decomposed representations in SOA experiments is compatible with the hybrid model, but not with the old holistic model of Dehaene and colleagues (1990).

Before passing to the empirical investigation, a word on how congruity between units and decades would be estimated of need. When the result of a unit-decade digit comparison is compatible with the comparison of decades, the unit-decade distance is positive and otherwise negative. This logic follows the same principle as the normal compatibility effect. Let the digits 6 and 4 from two-digit numbers 61_94 be masked: in this example, the distance of the digits 1 and 9 would be computed as + 8, because 9 is the larger decade digit and also larger than the unit digit 1, which is presented at the same SOA. In contrast, the distance between the digits 6 and 4 would be coded as -2, because the decade digit 6 is smaller than the other decade digit 9, but larger than the unit digit 4. In sum, the coding is identical to the coding of unit distance in previous studies, however, it is not the distance between the two units, but the distance of the digits presented together that is computed.

Experiment 1

In Experiment 1 the activation of the overall representation of two-digit numbers was visually degraded in comparison to the activation of the decomposed representations of units and decades, because it can only be assessed after all four digits have been presented on the screen. The hybrid model predicts that in Experiment 1 the effect of overall distance on performance will be smaller than when all digits have been presented simultaneously. Furthermore, if the comparison of units and decades starts before whole numbers are present, the congruities of the (masked and unmasked) digits being presented together may be significant predictors of performance.

Participants

Sixteen students from the Technical University Aachen, 8 female, between 19 and 28 years (mean=24 years) participated in Experiment 1. All of them were native speakers of German and have normal or corrected to normal vision.

Stimuli, design and procedure

In the magnitude comparison task, participants were asked to choose the larger of two two-digit numbers and press a response key. The same set of 240 stimulus pairs as in Nuerk and colleagues (2001) was used, which consist of pairs of two-digit numbers between 21 and 98. As shown in Figure 1A, a unit and a decade digit from different numbers were masked visually for 100 ms. In a 2 x 2 x 2 within-participant design decade distance was varied (small: 1-3; large: 4-7), unit distance (small: 1-4; large: 5-8) and unit-decade compatibility (compatible vs. incompatible). There were two conditions of visual masking (decade above & unit below, DA-UB, vs. decade below & unit above, DB-UA). Both masking conditions were merged because of the lack of similar analyses in the literature with which one could have compared these results. Overall distance, decade distance, unit distance and problem size were matched both absolutely and logarithmically between all stimulus categories.

Results

The main effect of decades was significant (F(1,15)=20.80; p<.001), but responding to larger decade-distances was only 23 ms faster than to smaller ones. Numerically the decade distance effect was small when compared with previous studies (Nuerk et al., 2001, 2002, 2004a, this issue). Figure 2 shows RT for small and large decade distance, as well as for compatible and incompatible trials. The main effect of compatibility was also significant

(F(1,15)=6.51; p=.022), however, much smaller than in previous studies (Nuerk et al., 2001; Nuerk et al., 2004a, Ratinckx, Nuer, van Dijk, & Willmes, in press; Nuerk et al., 2004b). Responding was 5 ms faster in average in compatible trials than in incompatible ones. No interaction reached significance.

Both decade- and compatibility effect were numerically small in comparison with other published studies (Nuerk et al., 2001, 2002a, 2004a; Ratinckx et al., in press).

In order to investigate more closely the effect of congruity between numbers presented together, a multiple regression analysis over items was carried out (see the original article of Nuerk et al., 2001), and included the distances of masked and unmasked digits as predictors (see description of the computations in the introduction). The regression analyses started with the linear regression model being most successful for the original data of Nuerk et al. (2001) for the same stimuli. This regression model included logarithmic distance, (compatible or incompatible) unit distance, problem size, and absolute distance (see Table 1). The model performed badly. It explained less than 10% of the variance (as compared to adjusted 71% in Nuerk et al., 2001). In the second step, the distance of masked digits (masked digit distance) and of unmasked digits (unmasked digit distance) were included to examine if this extended model would account better for the data. Multiple R^2 was .235. Both variables added a significant amount of variance to the original model (masked digit distance t(479)=3.42, p<.001; unmasked digit distance (t(479)=-1.68; p<.05 one-sided) in the expected direction. Responses became faster when the decade digit comparison was compatible with the masked or unmasked digit distance. In the third and final step of this investigation, using a backward elimination approach, some of the 6 (4 original and 2 new) variables were necessary to explain as much variance as possible and some could be dismissed without significant loss of variance being accounted for. The final model only comprised the three variables unit distance, masked digit distance and unmasked digit distance with $R^2 = .232$. The variables logarithmic distance, problem size and absolute unit distance were dropped without any loss of variance explained. Thus, the results of Experiment 1 could best be ac-



Figure 2: Compatibility effect for small- and large decade distances in Experiment 1.

| | Experiment 1 | | | Experiment 2 | | |
|-------------------------|--------------|----------|---------|--------------|----------|-------|
| Original model of | R2=.092 | | | R2=.416 | | |
| Nuerk et al. (2001) | Standard | t | Raw | Standard | t | Raw |
| with 4 predictors | Beta | | corr | Beta | | corr |
| Logarithmic distance | 298 | -6.79** | 297** | 630 | -17.92** | 620** |
| Problem size | 045 | -1.03 | 031 | 155 | -4.41** | 126** |
| Unit distance | 041 | 95 | 048 | .087 | 2.48* | .073 |
| Absolute unit distance | 003 | 06 | 014 | .009 | .25 | 009 |
| Model of Nuerk et al. | | R2=.235 | | | R2=.426 | |
| with unmasked and | Standard | t | Raw | Standard | t | Raw |
| masked digit distance | Beta | | corr | Beta | | corr |
| Logarithmic distance | .157 | .16 | 297** | 205 | 1.30 | 620** |
| Problem size | 039 | 04 | 031 | 149 | -4.27** | 126** |
| Unit distance | .807 | 2.42 | 048 | 890 | -2.51* | .073 |
| Absolute unit distance | 012 | 28 | 014 | .001 | .02 | .009 |
| Unmasked digit | 477 | -1.68* | .140** | 750 | -2.84** | 257** |
| distance | | | | | | |
| Masked digit distance | 971 | -3.42** | 398** | 706 | -2.67** | 215** |
| Final reduced model | R2=.232 | | R2=.424 | | | |
| | Standard | t | Raw | Standard | t | Raw |
| | Beta | | corr | Beta | | corr |
| Masked digit distance | 732 | -10.73** | 398** | -1.041 | -16.35** | 215** |
| Unmasked digit distance | 238 | -3.49** | .140** | -1.085 | -17.05** | 257** |
| Unit distance | .527 | 6.25** | 048 | -1.339 | -15.77** | .073 |

Table 1: Overview about the regression models in Experiment 1 and 2 (see text for elaboration).

Standard beta: standardized beta predictor, t: t-value with df = 479; **: p < .01; *p < .05; raw corr: Raw correlation when no effect of another predicotr has been partialled out. Note that overall logarithmic distance is not a predictor any more in both final models and that the variance is completely explained by digit distance measures.

counted for by digit distances. Overall two-digit distance measures like logarithmic distance or problem size did not survive the backward elimination procedure in this diagonal masking experiment.

Discussion

The bad fit of overall distance predictors for the RT data in a multiple linear regression suggests that overall magnitude was less activated than in previous studies (e.g. Nuerk et al., 2001). This pattern of results corroborates the predictions of the hybrid model regarding less activation of overall magnitude and more activation of the decomposed representations when the units and decades are shortly masked. Furthermore, congruity between digits being presented together was a significant predictor of performance in the item analysis. In the final

regression model only digit distance measures determined performance, but not overall twodigit number measures like logarithmic distance and problem size as in the original experiment of Nuerk and colleagues (2001). These results suggest that the comparison of units and decades starts before the complete two-digit numbers are shown in contrast to the predictions of the holistic model of Dehaene and colleagues (1999).

Experiment 2

The smaller activation of overall magnitude and the effect of congruity between digits being presented together may be due to an unspecific effect of the duration of the visual mask and not to specifically masking unit and decade digits. In order to control for that, unit and decade digit from the same number were presented together (see Figure 1B) in Experiment 2. If the effect of masking was unspecific, the disruption of performance should be similar to the one observed in Experiment 1 and measures of overall magnitude should not predict performance well. However, if the effect of masking was specific, the measures of overall magnitude should be significant predictors of performance (Nuerk et al., 2001).

Participants

Eight native German students from the Technical University Aachen, 5 female, between 21 and 28 years old (mean=23 years) participated in Experiment 2. None of them took part in Experiment 1.

Stimuli, design and procedure

In Experiment 2 the same stimuli were presented as in Experiment 1. The only variation is compared to the setup of Experiment 1 was that instead of masking one decade digit and a unit digit from different numbers, decade and unit digits from one of the numbers were masked for 100 ms.

Results

Responding to larger decade distances was 66 ms faster than to smaller ones. The compatibility effect for small and large units is shown in Figure 3 for large and small decade distances. No main-effect of compatibility (F(1,7)=3.51; p=.103, two-sided) was observed. Surprisingly, the compatibility effect tended to be inverted, as RT was 9 ms faster in incompatible trials than in compatible ones. The interaction decade-distance by compatibility effect for small decade distance, and no RT difference for large decade distances. The three-way interaction decade-distance by unit-distance by compatibility (F(1,7)=12.52; p=.009) was also significant. The inversion of compatibility was 31 ms for the small-decade, small-unit condition whereas it was only 3 ms for other conditions. Figure 3:

Compatibility effect for small and large decade and unit distances in Experiment 2. The compatibility effect was large when decade and unit distances were both small. In all other conditions it was very small or negligible.



To further explore this digit congruity, the same multiple regression analyses were used as in Experiment 1 (see Table 1). The original model of Nuerk and coworkers (2001) explained the data much better than in Experiment 1. The model explained 41.6% of the variance. However, there was one important difference as compared to the original model. Responses became slower rather than faster with positive (compatible) unit distance.

As before, in the second step of the analysis the masked digit distance and the unmasked digit distance were included to examine if this extended model could account for the data in a better way. This was not the case. The multiple R^2 just rose to 42.6%. None of the variables added explained variance to the original model. However, when all variables were included the beta weights changed considerably (see Table 1). As in Experiment 1, the backward elimination approach was used to find out which of these 6 (4 original and 2 new) variables was necessary and which variable could be dismissed. The final model had only four variables: unit distance, masked digit distance, unmasked digit distance and problem size. All digit distance measures had negative beta weights, indicating that the higher (and more congruent or compatible) the digit distances were, the faster was performance. The variables logarithmic distance and absolute unit distance were dropped without any loss in the proportion of variance explained (R^2 = 42.4%). Therefore, the RT data of Experiment 2 may be explained by digit distances and an overall problem size measure.

So, one is left with two possible models explaining very similar amounts of variance: (i) the original model including two-digit overall logarithmic distance and unit distance with an inverted beta weight and (ii) the model without overall logarithmic distance including three digit distance measures and problem size.

Since the compatibility effect was numerically inverted in Experiment 2, an ANCOVA over items was calculated including decade-distance, unit-distance and unit-decade compatibility as experimental factors and masked and unmasked digit distances as two covariates. Both covariates had a significant contribution to RT variability (unmasked: F(1, 470)=13.84, p<.001; masked: F(1, 470) = 10.72, p=.001). Furthermore, the main effect of decades was significant (F(1,470)=31.11; p<.001) because large decade distances were responded to 44 ms faster than small ones. The main effect of compatibility was also significant (F(1,470)=7.31; p<.01). Interestingly, the compatibility effect was not inverted any more. Compatible trials were responded to 37 ms faster than incompatible ones; i.e. the usual compatibility effect was observed. Furthermore, the interactions decade-distance by compatibility and unit-distance by compatibility were significant (F(1,470)=13.14; p<.001). The compatibility-effect was larger for small decade-distance (47 ms vs. 27 ms) and for large unit-distances (63 ms vs. 11 ms), which agrees nicely with the previous literature (Nuerk et al., 2001, 2002, 2004a; Ratinckx et al., in press).

Discussion

The results of Experiment 2 suggest that the disruption of overall magnitude in Experiment 1 was not due to the duration of masking, but due to its visual pattern. In Experiment 2 visual masking was not detrimental to the encoding of the overall magnitude representation, given that measures of overall distance were significant predictors of performance. These findings confirm the predictions of the hybrid model regarding the interaction between overall and decomposed magnitude representations. Furthermore, in Experiment 2 the evidence from Experiment 1 was replicated in that digit comparison may start before both two-digit numbers are presented on the screen. Nevertheless, the regression analyses have shown that the congruity between masked and unmasked digits was also a significant predictor of performance. The regression models including overall distance fitted the data as well as the regression model including only measures of digit congruity. Furthermore, the compatibility effect was inverted, mainly when decade and unit distance were small (-31 ms). These results may be explained by the computation of the digit congruity between unit and decade digit. When both decade and unit distance are small, the relation of both decade digits with the units tend to be the same. This may have increased the time to decide which decade digit was the larger. Interestingly, the ANCOVA analysis revealed a regular compatibility effect after partialling out digit (in)congruity. Furthermore, the regular interactions of compatibility with unit distance and decade were also found (Nuerk et al., 2001). More consequences of these findings for the hybrid model and the compatibility effect will be discussed in the General Discussion.

General discussion and conclusions

The hybrid model of Nuerk and Willmes proposes that both overall and decomposed representations of magnitude may be activated in two-digit number comparison tasks. Depending on task requirements, overall or decomposed magnitude representations may be more or less activated. When comparing two-digit Arabic numbers presented simultaneously on the screen, both overall and decomposed representations seem to be activated. Several studies by Nuerk and colleagues (2001, 2002, 2004a, 2004b, this issue; Ratinckx et al., in press) have shown that both measures of overall magnitude (logarithmic overall distance) and decomposed unit and decade representations (unit distance, unit decade compatibility) are significant predictors of performance.

In Experiment 1, the activation of overall magnitude seems to be modulated by masking some digits for a short time. The effect of this manipulation on performance was very clear. Measures of overall magnitude did not fit the RT data well (see Table 1). In contrast, the two measures of digit congruity were significant predictors of performance. These results show that the decomposed representations of units and decades may be activated more strongly when the activation of overall magnitude is impaired for a short interval by visual masking. This is in line with the predictions of the hybrid model, but not with the holistic model of Dehaene and colleagues (1990), from which one may predict that the activation of number semantics is comparable when some digits of two-digit numbers are masked or not. The results of Experiment 2 complement those of Experiment 1. When decade and unit digits of one number were presented together, the overall magnitude representation of one two-digit number may be activated at the beginning of each trial. Consequently, in Experiment 2 measures of overall magnitude became better predictors of performance than in Experiment 1. These results are in line with the hybrid model, which predicts the activation of both overall and decomposed magnitude representations, when no temporal advantage for the decomposed representations is given. Furthermore, the results show that the spatial structure and not the duration of the spatial mask may have determined the effect of digit congruity on performance.

The second empirical question of the present study was the timing of the number comparison process. The hybrid model does not make direct predictions about the time of magnitude comparison, namely, whether magnitude comparison may start before whole numbers are presented on the screen or not. The hybrid model could accommodate both (i) an early comparison of decomposed unit and decade magnitudes, which may start before the whole numbers are presented on the screen, and (ii) a late comparison of both overall and decomposed magnitude representations, which starts only after whole numbers have been presented. The results of Experiments 1 and 2 suggest that the comparison of magnitudes may start before the whole numbers are presented, since measures of digit congruity were the most significant predictors of performance in both experiments. Interestingly, the consequences of computing digit congruity were very different in Experiments 1 and 2. In Experiment 1, the activation of overall magnitude representation was impaired, and measures of digit congruity were decisive as predictors of RT, but the compatibility effect remained significant. In contrast, in Experiment 2 digit congruity may have masked the compatibility effect. Interestingly, after partialling out the effect of digit congruity the normal compatibility effect as well as the common interactions with unit and decade distances (eg. Nuerk et al., 2001) could be found. Therefore, the results of Experiment 2 suggest that magnitude comparison was not limited to digit congruity but extended to the comparison of units and decades after the whole numbers were presented on the screen. For this reason one may conclude that the overall and decomposed magnitude representations may both influence RT, but the magnitude and direction of this influence may depend on the timing of activation of the different magnitude representations.

Finally, the present study offers another possible account for the failure to find unitbased (compatibility) effects in two-digit number comparisons with fixed standards (see Dehaene et al., 1990 and Nuerk & Willmes, this issue). If the distance between the two digits of a two-digit number were computed automatically, it would interfere with the compatibility effect in the same way as in Experiment 2. In compatibility studies without fixed standard, 4 digits are presented simultaneously. The relevant comparison is that of the decade digits. It seems that one is not able to compute a larger number of irrelevant comparisons in such a setting, but only the irrelevant (compatible or incompatible) distance of the two units of the two two-digit numbers. This unit distance produces the strongest interference which is captured in the compatibility effect.

To summarize, the reason why there are no unit-based effects in number comparison tested with a fixed standard may not be due to a non-existing activation of unit magnitude. Rather, the reason may be too much activation. Not only the relation of the unit magnitudes of standard number and target gets activated but also the irrelevant comparison between target unit and target decade digit seems to be computed. These two computations have opposite effects and consequently, the unit effect may be cancelled out, resulting in the null effect typically observed in magnitude comparison experiments with a fixed standard (see Nuerk et al., submitted; Nuerk & Willmes, this issue).

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