Observations of the Horizontal Interactions between the Internal Wave Field and the Mesoscale Flow[1]

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ABSTRACT

Momentum and energy transfers from the mesoscale horizontal velocity shear to the internal wave field have been deduced from an analysis of a closely spaced, 25 km, moored current-meter array. The correlation between the low-frequency horizontal shear and internal-wave-field continuum effective stress implies a significant horizontal eddy viscosity of O (10^6 cm^2 s^{-1}), somewhat larger than predicted by Müller (1976). A simple steady-state energy balance for the internal wave field using the observed correlation between the internal wave kinetic energy and the square of the low-frequency shear implies a 10-day relaxation time for the internal-wave field and a combined vertical viscosity and horizontal diffusivity not significantly different from zero. These estimates are within the experimental uncertainty of previous observational analyses.

1. Introduction

The interaction between the large-scale (mesoscale) flow and internal waves has been a topic of much recent work. To the internal wave enthusiast, it is one of several possible energy sources for the internal wave field. For those interested in mesoscale eddy dynamics, it represents a possible sink for energy and enstrophy, which cascade to small scales on account of quasi-geostrophic turbulence (Rhines, 1977). At present, there are several different opinions as to the strength of this interaction (Müller, 1976; Frankignoul, 1976; Frankignoul and Joyce, 1979; Ruddick and Joyce, 1979).

One mechanism for the proposed interaction is based on an equilibrium internal wave state that is independent of the low-frequency motion. Müller (1976) has shown that the distortion of the wave field by the shear causes momentum transfer from the mean flow to the internal wave field as it relaxes back to the equilibrium state. The strength of the interaction can be evaluated analytically using the WKB approximation in the form of horizontal and vertical eddy coefficients relating stresses in the wave field to the mean shear. The magnitude of the viscosity coefficients depends on the relaxation time of the wave field and the assumed frequency dependence of the equilibrium spectrum. Müller (1976) estimated the horizontal and vertical eddy viscosities from the Garrett-and-Munk (1972) spectrum to be 7×10^4 cm^2 s^{-1} and 4×10^3 cm^2 s^{-1}, respectively. The order of magnitude of these quantities is not changed by the frequency and wavenumber modifications in any of the later Garrett–Munk spectral models (Garrett and Munk, 1975; Cairns and Williams, 1976).

A recent analysis of moored-current-meter measurements gave a much lower vertical eddy viscosity than predicted by Müller (1976). Using temperature and velocity records from POLYMODE arrays I and II, Ruddick and Joyce (1979) found only a weak vertical momentum transfer between the internal-wave field and the quasi-geostrophic flow. From the records in the array I setting where the mean speeds were low (~10 cm s^{-1}), they computed a vertical eddy viscosity coefficient (0 ± 200 cm^2 s^{-1}) several orders of magnitude less than the predicted value. The results from Array II where the mean speeds (~25 cm s^{-1}) were beyond the range applicable in the Müller theory also were inconsistent with the stress-shear relation. Ruddick and Joyce determined that the magnitude and sign of the vertical eddy viscosity coefficients could be explained only by wave generation in regions of large shear in the thermocline.

Similar conclusions were reached by Frankignoul and Joyce (1979) on the basis of the data from the Internal Wave Experiment (IWEX). Although the data record length (~40 days) from IWEX is clearly insufficient for determining modulaton of the internal-wave field by the low-frequency flow, the data set from this very stable tri-mooring is the most accurate for calculating internal-wave vertical velocities from temperature measurements. Frankignoul and Joyce determined that the vertical viscosity

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could be at most O (10^2 cm^2 s^{-1}) in absolute magnitude, significantly less than the O (10^3 cm^2 s^{-1}) predicted by Müller. The large uncertainty in these two vertical-eddy-viscosity estimates is due mainly to errors introduced in calculating vertical velocities from the temperature time series (Ruddick, 1977; Ruddick and Joyce, 1979). These errors involve the linearization of the heat balance equation and the assumption of a linear equation of state. It is unlikely that the magnitude of this uncertainty can be reduced without direct means of measuring vertical velocity.

The strength of this wave-mean shear interaction can alternatively be determined from the horizontal eddy viscosity which is not contaminated by the errors in calculating the vertical velocity. From theoretical considerations the discrepancy between predictions and measurements of the vertical eddy viscosity has been attributed to strong forcing of the internal wave field into vertical wavenumber symmetry (McComas and Bretherton, 1977). Asymmetries in the wave field caused by the mean vertical shear are rapidly smoothed, thereby reducing the effectiveness of the vertical momentum-transfer process. Since no comparable mechanism has been identified for the horizontal wavenumber of the internal wave field, one would expect the experimentally observed horizontal momentum transfer to correspond more closely with Müller's predictions. The difficulty in observing the horizontal eddy viscosity is not due to errors in estimating the internal wave stresses but rather in estimating the horizontal large-scale shear. Frankignoul (1976) calculated a value of O(10^5–10^7) cm^2 s^{-1} for the horizontal viscosity coefficient from the MODE-experiment data. However, the relatively short data length and the presence of high-energy inertial oscillations contributed to the large uncertainty in that analysis.

The nine-mooring current-meter array deployed for 15 months as part of the POLYMODE Local Dynamics Experiment (LDE) was specifically designed to estimate terms in the quasi-geostrophic potential-vorticity equation. As a result, accurate estimates of the large-scale horizontal shear were possible. Presented below are time series of the mesoscale shear and the effective horizontal stresses due to the internal-wave field. Statistical analyses of these time series demonstrate a stronger horizontal interaction than predicted by Müller (1976). Before presenting and discussing the observations of the interactions between the mesoscale flow and internal wave field, we will discuss the appropriate equations and derivation of the effective internal-wave stress.

2. Theoretical background

It has been suggested (Müller, 1976) that there is a significant flux of momentum and energy from mesoscale eddies (large-scale mean flow) into the internal wave field. Müller finds that the interaction between an equilibrium spectrum, such as that of Garrett and Munk (1972), and the mesoscale flow can be locally parameterized by vertical [O (10^3 cm^2 s^{-1}] and horizontal [O (10^5) cm^2 s^{-1}] eddy viscosities. The strength of this interaction is measured in terms of wave-momentum fluxes which oppose the mesoscale shear.

To examine the effects of the wave-field momentum fluxes, one can examine the quasi-geostrophic potential vorticity equation for the mesoscale flow:

$$\left(\frac{\partial}{\partial t} + \bar{u}_\alpha \frac{\partial}{\partial x_\alpha}\right)\left[\frac{\partial^2}{\partial x_\beta} + \frac{\partial}{\partial x_3}\left(\frac{f^2}{N^2}\frac{\partial}{\partial x_3}\right)\right]\psi = -\epsilon_{\alpha\beta 3}\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_j} T_{\beta j}, \quad (1)$$

where

$T_{\beta j} = \overline{u_\beta' u_j'} + P_w \delta_{\beta j} - \epsilon_{\alpha\beta 3}\frac{f}{N^2}\overline{b' u_j'}$

$\overline{u_\alpha}$ mean geostrophic horizontal velocity

ψ streamfunction such that $\overline{u_1} = -\frac{\partial\psi}{\partial x_2}$, $\overline{u_2} = \frac{\partial\psi}{\partial x_1}$

u_j' perturbation velocities

b' perturbation buoyancy field $= -\frac{g}{\rho_0} \times [\rho'(x,t) - \rho_e(x_3)]$

$\bar{b}$ mean buoyancy field

$\rho_e(x_3)$ equilibrium density stratification

$N^2(x_3)$ Brunt-Väisälä frequency $= -\frac{g}{\rho_0}\frac{\partial\rho_e}{\partial x_3}$

P_w mean wave-induced pressure α; β varies from 1 to 2, j varies from 1 to 3

$\delta_{\beta j}$ Kronecker delta

$\epsilon_{\alpha\beta j}$ alternating unit tensor.

The overbar indicates a time-average over several inertial periods. Thus mean quantities denote subinertial, mesoscale variables, while perturbations are due to the internal wave field. It has been pointed out earlier (Müller, 1976; Ruddick and Joyce, 1979) that the effects of the internal wave field on the quasi-geostrophic flow are best formulated as the effective stress, $T_{\beta j}$, which combines both Reynolds stresses and buoyancy fluxes. The buoyancy fluxes $\overline{b'u'_j}$ tilt the density surfaces in the x_j direction and through the thermal wind relationship, cause momentum fluxes perpendicular to x_j. Thus, the buoyancy fluxes, in combination with the Coriolis force, can cause horizontal as well as vertical momentum fluxes. Using the dispersion relationship for internal waves, one can then define a factor for each frequency band within the internal wave field which converts the measured Reynolds stresses, $\overline{u'_i u'_j}$, into the effective stress, T_{ij} (Ruddick, 1977). For the horizontal Reynolds stresses, this becomes

$$T_{\alpha\beta} = \int_f^N \left(\frac{\omega_0^2 - f^2}{\omega_0^2 + f^2}\right)\hat{u}_\alpha'\hat{u}_\beta' d\omega_0, \quad (2)$$

where $\hat{u}_\alpha'\hat{u}_\beta'$ is the cospectrum of the internal wave velocities and ω_0 is the intrinsic wave frequency. Since the Reynolds stresses and buoyancy fluxes nearly cancel at the lowest internal wave band, it was found that taking the integral from f to N (total band) instead of from $2f$ to N (continuum band) increases the uncertainty of the estimates without altering the values: that is, although the contributions to the wave stress from f to $2f$ are a significant fraction of the total, they are not correlated with the large-scale shears and so only introduce noise in the eddy viscosity estimate. Thus, following Ruddick and Joyce (1979), we have approximated the integral by the continuum band alone.

It is therefore appropriate to look for a correlation between the effective momentum flux and the mesoscale horizontal shear as an estimate of the horizontal eddy viscosity:

$$\nu_h = \frac{T_{12} + T_{21}}{-\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)} = \frac{2T_{12}}{-\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)} . \quad (3)$$

In addition to being a source for wave field momentum, the mesoscale flow also is a possible source of internal wave energy. The internal wave energy balance can be written as (Frankignoul, 1976)

$$\frac{dE}{dt} = \left(\frac{\partial}{\partial t} + \overline{u_\alpha}\frac{\partial}{\partial x_\alpha}\right)E = S_0 - \frac{E}{\tau}, \quad (4)$$

where E is the total internal wave energy per unit mass, τ is the relation time of the wave field and S_0 is the sum of the sources which can include interactions with the large-scale shear, atmospheric forcing or topographic generation. In mid-ocean regions where the large-scale currents are ≤ 20 cm s^{-1} so that critical layers are unlikely to occur, an appropriate form for the source term is given by Müller (1976),

$$S_0 = \nu_h \frac{\partial \bar{u}_\alpha}{\partial x_\beta}\frac{\partial \bar{u}_\alpha}{\partial x_\beta} + \nu_v \frac{\partial \bar{u}_\alpha}{\partial x_3}\frac{\partial \bar{u}_\alpha}{\partial x_3} + \frac{K_h}{N^2}\frac{\partial \bar{b}}{\partial x_\alpha}\frac{\partial \bar{b}}{\partial x_\alpha}, \quad (5)$$

where K_h is the horizontal eddy diffusivity. The thermal-wind equation can be used to eliminate b:

$$S_0 = \nu_h \frac{\partial \bar{u}_\alpha}{\partial x_\beta}\frac{\partial \bar{u}_\alpha}{\partial x_\beta} + \left(\nu_v + K_h\frac{f^2}{N^2}\right)\frac{\partial \bar{u}_\alpha}{\partial x_3}\frac{\partial \bar{u}_\alpha}{\partial x_3} . \quad (6)$$

If the internal wave field can be described by an equilibrium spectrum, then the left-hand side of Eq. (4) vanishes and the total internal wave energy should be correlated with the mesoscale shears:

$$\frac{E}{\tau} = \nu_h \frac{\partial \bar{u}_\alpha}{\partial x_\beta}\frac{\partial \bar{u}_\alpha}{\partial x_\beta} + \left(\nu_v + K_h\frac{f^2}{N^2}\right)\frac{\partial \bar{u}_\alpha}{\partial x_3}\frac{\partial \bar{u}_\alpha}{\partial x_3} . \quad (7)$$

Although a more accurate model would take account of different relaxation times due to the horizontal and vertical mean shears, Eq. (7) provides a rough estimate of the strength of the interactions between the mesoscale flow and the internal wave field.

3. Data analysis

The POLYMODE Local Dynamics Experiment (LDE) array was deployed for 15 months starting in May 1978. The array consisted of nine moorings arranged in two crosses centered around the central mooring at 31°N, 69°30′W. Only the inner cross, with nominal spacing of 25 km, was used in this analysis for estimating the mesoscale shear. The array was designed to measure the terms in the potential-vorticity equation and the inner cross was deployed to sample optimally the mesoscale shear (McWilliams and Owens, 1981). A more complete description of the low-frequency flow is given by Owens *et al.* (1981). We will give only a cursory description of the data analysis since much of the methodology is the same as that originated by Frankignoul (1976) and extended by Ruddick and Joyce (1979). This simplicity in analyses was chosen so that our conclusions can be accurately compared.

The total low-frequency horizontal shear, $(\partial \bar{u}_1/\partial x_2) + (\partial \bar{u}_2/\partial x_1)$, is estimated from each of the four triangles of the inner cross using simple finite differences. However, by averaging over the cross one obtains a centered difference which has greater accuracy (Roache, 1972). The inner cross was instrumented at nominal depths of 600 and 825 m, but instrument failures precluded computations of the shear for the entire 15-month period for which wave stresses are available. Centered differences could be calculated only for 180 and 108 days for the 600- and 825-m levels, respectively, while estimates using only two of the four possible triangles extended the two shear time series to 224 and 432 days. These less accurate shear estimates do not give values significantly different from the centered differences, but the additional data reduce the statistical uncertainties in the correlation between the shear and wave-stress estimates.

All instruments used in these calculations were vector-averaging current meters (VACM's) with a 15-min sampling rate. The low-frequency shears were calculated from records that were low-passed using a Gaussian filter with a 24 h half-width and subsampled at 2-day intervals. Using objective analysis, McWilliams and Owens (1981) estimated an error of ~20% for the shear measurements. Another measure of the quality of the horizontal-derivative estimates is given by the ratio (Bryden, 1976)

$$R = \frac{\left\langle \left| \frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_2}{\partial x_2} \right| \right\rangle}{\left\langle \left| \frac{\partial \bar{u}_1}{\partial x_1} \right| \right\rangle + \left\langle \left| \frac{\partial \bar{u}_2}{\partial x_2} \right| \right\rangle},$$

where angle brackets denote a record-length average. R should be small if the flow is quasi-geostrophic. When summed over the record lengths for each triangle, R varies from 0.3 to 0.5. Thus, the horizontal shear estimates have the highest signal-to-noise ratio available from oceanic data, which should allow us to observe any possible correlations with the wave stresses.

Velocity records for the internal-wave field at the central mooring were obtained by high-pass filtering the raw current-meter data to remove frequencies below the local inertial frequency (0.0429 cph) using a Linette filter (Linette, 1961) with half-power point at 0.0346 cph and less than 10% leakage from the lower frequencies. The filtered data were divided into 4-day pieces, overlapping by 50%, and Fourier-transformed under a Hanning window. The resulting cospectral estimates were multiplied by

$$\frac{\omega_0^2 - f^2}{\omega_0^2 + f^2}$$

to obtain the effective wave stress and then summed over the frequency ranges 0.0429–2.0 cph (total band) and 0.0938–2.0 cph (continuum band). The raw wave-stress estimates are dominated by near-inertial and tidal oscillations. However, since these frequencies should not contribute much to the effective stress [<10% for a Garrett–Munk spectral model (Ruddick and Joyce, 1979)], we have used the continuum band estimate of the wave stress. The statistical errors for the estimates in the wave-stress time series are 1.3 $cm^2\ s^{-2}$ at 825 m and 1.1 at 600 m depth [based on a white-noise hypothesis as given by Jenkins and Watts (1969)]. At both depths the temporal variations of the effective stress are significantly larger than the statistical uncertainties. It should be pointed out that although this statistical error is greater than that seen by Ruddick and Joyce (1979), the percentage error is smaller since the horizontal stresses are larger than the vertical stresses. The only other significant contribution to errors in the wave-stress estimate is the Doppler shift by the mean flow.

In addition to the continuum-band estimates of the effective wave stress, time series of the internal wave kinetic energy were also calculated in order to estimate the terms in the internal wave energy balance. The low-frequency vertical shear could be estimated as a centered difference only at 600 m because of the failure of Geodyne-850 current meters deployed below 825 m on the central mooring. The vertical shear at 600 m was calculated from the difference between the horizontal velocities at 375 and 825 m on the central mooring.

4. Results

a. Momentum fluxes

Since modulations of the internal wave field should occur on the time scale of the mesoscale flow, we have low-pass filtered the time series using a Linette filter to remove variability at periods shorter than 10.4 days. The low-passed time series, shown in Fig. 1, show some visual correlation between mesoscale shear and wave-field effective stress. Correlation coefficients between the mesoscale horizontal shear and negative of continuum-band wave-field effective stress and 95% confidence limits are

$$R = 0.48 \pm 0.44 \quad \text{at} \quad 600\ \text{m},$$

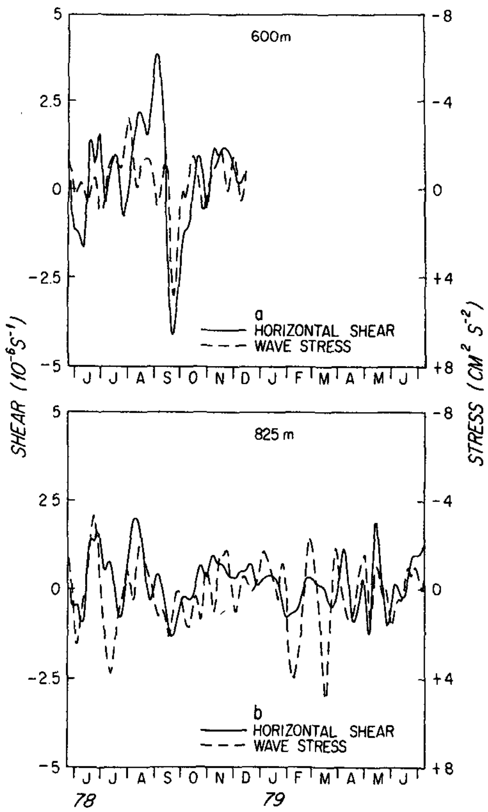

FIG. 1. Time series of mean horizontal shear and negative continuum-band horizontal wave stress at (a) 600 m and (b) 825 m depth. The series have been low-passed to remove frequencies above 0.004 cph.

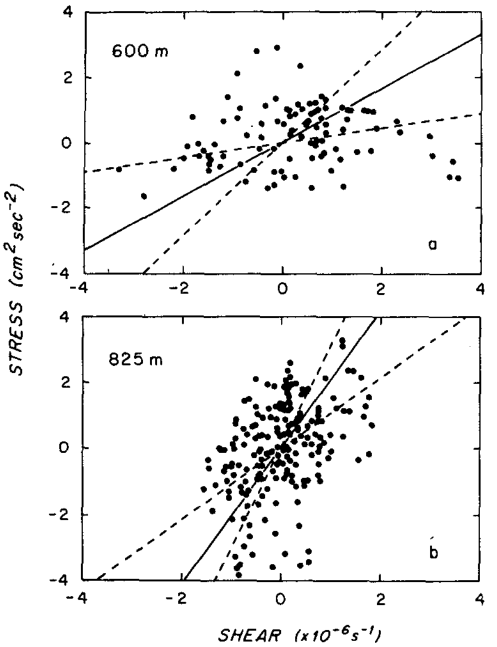

FIG. 2. Scatterplots of negative continuum-band wave stress versus mean horizontal shear at (a) 600 and (b) 825 m depth. The slope of the solid regression line with 95% confidence limits (dashed lines) determines the magnitude of the horizontal eddy viscosity coefficient.

$$R = 0.43 \pm 0.30 \quad \text{at} \quad 825 \text{ m}.$$

These confidence limits have been computed by assuming that the time series are autocorrelated with an independent estimate every 10 days. Thus there is a correlation between the mesoscale horizontal shear and wave-field stress indicative of a significant horizontal eddy viscosity. In contrast, Ruddick and Joyce (1979) found no such significant correlation between the vertical shear and vertical stresses, even when they also low-passed the time series to increase the signal-to-noise ratio.

Scatterplots of the negative of the continuum-band effective wave stress are plotted against the horizontal shear in Fig. 2. Regression lines based on the assumption that both variables are subject to error (Ricker, 1973), and dashed lines denoting the 95% confidence limits for the slope of the line, are also plotted. At both depths the horizontal viscosity coefficient

$$\nu_h = 2 \times 10^6 \pm 6 \times 10^5 \text{ cm}^2 \text{ s}^{-1} \quad \text{at 600 m},$$

$$\nu_h = 4 \times 10^6 \pm 1 \times 10^6 \text{ cm}^2 \text{ s}^{-1} \quad \text{at 825 m},$$

is on the order of 10^6 cm² s⁻¹ and significantly different from zero.

b. Internal wave energy fluxes

In the previous section we found evidence that the mesoscale flow transfers momentum to the internal wave field. To investigate possible energy fluxes, time series of the internal wave-field horizontal kinetic energy in the total (from f to N) and the squares of the mesoscale vertical and horizontal shears have been computed at 600 m depth (Fig. 3). At 825 m it was not possible to compute the centered-difference vertical shear estimates because of instrument failures. The two time series for internal wave kinetic energy and mesoscale horizontal shear show signals similar to those at 600 m. The total internal wave kinetic energy E_h has a background level of ~12 cm² s⁻² with fluctuations roughly corresponding in time to variations in the squares of the shears. The large difference between the time series of E_h and the mean shears at the end of October 1978 is caused by high-energy packets of inertial waves that propagated through the array. Using the Garrett–Munk (1972) model of the internal wave spectrum, the total internal wave energy E can be calculated from the horizontal kinetic energy E_h as

$$E = \tfrac{4}{3} E_h.$$

Both the relaxation time τ and the combined vertical viscosity and diffusivity can then be estimated from Eq. (7) using the correlation coefficients between the wave kinetic energy and the shears. Two equations for these unknowns are obtained by multiply-

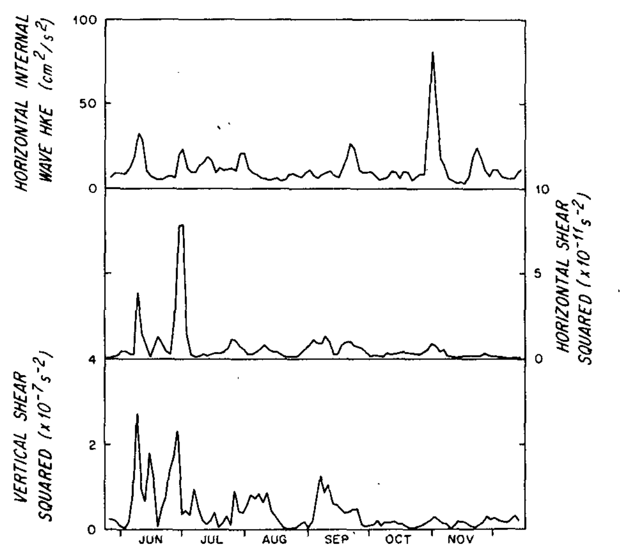

FIG. 3. (a) Total-band horizontal internal wave kinetic energy. (b) Mean-square vertical shear

$$\left(\frac{\partial \bar{u}_1}{\partial x_3}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_3}\right)^2.$$

(c) Mean-square horizontal shear

$$\left(\frac{\partial \bar{u}_1}{\partial x_2}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_1}\right)^2$$

at 600 m depth.

ing Eq. (7) by the horizontal and vertical shears and averaging the results over the records:

$$\frac{4}{3}\frac{\langle E_h\rangle}{\tau}\rho_{EH} = \nu_h\left\langle\frac{\partial\bar{u}_\alpha}{\partial x_\beta}\frac{\partial\bar{u}_\alpha}{\partial x_\beta}\right\rangle + \left(\nu_v + K_h\frac{f^2}{N^2}\right)\rho_{HV}\left\langle\frac{\partial\bar{u}_\alpha}{\partial x_3}\frac{\partial\bar{u}_\alpha}{\partial x_3}\right\rangle, \quad (8)$$

$$\frac{4}{3}\frac{\langle E_h\rangle}{\tau}\rho_{EV} = \nu_h\left\langle\frac{\partial\bar{u}_\alpha}{\partial x_\beta}\frac{\partial\bar{u}_\alpha}{\partial x_\beta}\right\rangle\rho_{HV} + \left(\nu_v + K_h\frac{f^2}{N^2}\right)\left\langle\frac{\partial\bar{u}_\alpha}{\partial x_3}\frac{\partial\bar{u}_\alpha}{\partial x_3}\right\rangle. \quad (9)$$

The computed correlations between wave energy and horizontal shear (ρ_{EH}), wave energy and vertical shear (ρ_{EV}), horizontal and vertical shear (ρ_{HV}), and the record-averages of the energy and the shears are substituted with $\nu_h = 2 \times 10^6$ cm^2 s^{-1} into these equations to obtain

$$\tau = 10 \text{ days},$$

$$\nu_v + f^2\frac{K_h}{N^2} = -11\frac{\text{cm}^2}{S}.$$

The combined vertical eddy viscosity and the diffusivity is not significantly different from zero, considering the possible errors in this calculation. This agrees with the results of Ruddick (1977) who estimated this quantity, the effective vertical viscosity, to be 0 ± 200 cm^2 s^{-1} from POLYMODE data and of Frankignoul and Joyce (1979) who calculated it to be -30 ± 48 cm^2 s^{-1} at 73 m and -168 ± 227 cm^2 s^{-1} at 1023 m from IWEX data. Measurement errors have obscured the sign of the vertical viscosity coefficient although in all these studies it is less than O(10^2) in absolute magnitude.

5. Conclusions

Horizontal velocity measurements from the thermocline in the LDE moored array show a significantly nonzero correlation between the quasi-geostrophic mean horizontal shear and the internal wave Reynolds stresses in the continuum frequency band between 0.1 and 2 cph. The horizontal eddy viscosity coefficient due to the internal-wave field was calculated to be $\nu_h = 2 \times 10^6$ cm^2 s^{-1} at 600 m and $\nu_h = 4 \times 10^6$ cm$_2$ s^{-1} at 825 m, with an error of ~30%. The order of magnitude of this quantity is consistent with the estimate of ν_h = O(10^5–10^7 cm^2 s^{-1}) made by Frankignoul (1976) from the MODE data. However, there is an indication from the relatively low correlation between the wave stress and the mean horizontal shear that the horizontal eddy viscosity coefficient is constant only to within an order of magnitude.

The data suggest that in addition to a direct momentum transfer, there is also a time modulation of the internal wave continuum-band horizontal kinetic energy by the squared horizontal and vertical shears. The results are consistent with the mechanism proposed by Müller and Olbers (1975) for the source of energy in the internal wave field. The distortion of the internal wave field by the mean shear causes momentum and energy transfer to the wave field, which then returns to its equilibrium energy level within a relaxation time τ. By applying the measured correlations between the internal wave kinetic energy and the mean horizontal and vertical shears to a simple model for the internal wave energy balance, we estimate this relaxation time to be ~10 days. Theoretical calculations (Olbers, 1976) predict a value of τ less than 10 days. Our indirect estimate is in agreement with the results of Frankignoul and Joyce (1979) who obtained τ – 10 days from the IWEX data, but smaller than that estimated by Ruddick and Joyce (1979) who found $\tau \sim 20$ days from the POLYMODE data.

The length of this data set has enabled us to establish what we believe are consistent order-of-magnitude estimates of the parameters describing the interaction of the quasi-geostrophic mean flow with the internal wave field. The data suggest that there is momentum and energy transfer from the mean flow to the internal wave continuum with a horizontal eddy viscosity of O (10^6 cm^2 s^{-1}) and a combined vertical viscosity and diffusivity of O (0 cm^2 s^{-1}). The rate of energy dissipation (per unit volume) of the quasi-geostrophic flow due to the internal-wave field is given by Müller (1977) as

$$\dot{E} = \left[-\rho_0\nu_h\frac{\partial\bar{u}_\beta}{\partial x_\alpha}\frac{\partial\bar{u}_\beta}{\partial x_\alpha} - \rho_0\left(\nu_v + k_h\frac{f^2}{N^2}\right)\frac{\partial\bar{u}_\alpha}{\partial x_3}\frac{\partial\bar{u}_\alpha}{\partial x_3}\right].$$

Using horizontal and vertical shear variances at 600 m of 6.1×10^{-12} s^{-2} and 3.9×10^{-8} s^{-2}, we estimate the mesoscale energy dissipation due to internal waves to be

$$E = -1.2 \times 10^{-5} \text{ ergs cm}^{-3} \text{ s}.$$

Given 75 ergs cm^{-3} (Owens *et al.*, 1981) for mesoscale kinetic energy, an *e*-folding time due to the internal waves would be three years ignoring potential energy, or six years if one assumed a potential energy comparable to the kinetic energy. A further comparison of this dissipation rate with the net local conversion of mean to eddy energy, 1.8×10^{-5} ergs cm^{-3} s (Bryden, 1981), demonstrates that internal waves are a significant dissipative mechanism for mesoscale eddies.

Although we believe that the horizontal eddy viscosity coefficient due to the interaction of the internal-wave field with the mesoscale flow is O (10^6 cm^2 s^{-1}), other experimental evidence questions the validity of this result. The small-scale energetic eddies that propagate through the LDE area with peak velocities of 25 cm s^{-1} and horizontal scales of ~15 km (Ebbesmeyer *et al.*, 1978; H. T. Rossby, S. Riser, and S. McDowell, personal communication; Sanford, 1978; Taft *et al.*, 1978) would have shears an order-of-magnitude larger than the rms shears seen in Fig. 1. The *e*-folding time (16 days) predicted from the above internal-wave-induced eddy viscosity for these eddies is inconsistent with their anomalous water properties, which indicate lifetimes on the order of several years. It is possible that this discrepancy is due to the small scale of these eddies because the diffusion mechanism (McComas and Bretherton, 1977) that contributes most to the momentum transfer depends on a large scale separation between the mesoscale and the internal-wave field. This explanation, however, is purely speculative. Although we are confident that we have observed a significant interaction between mesoscale eddies and the internal wave field, we suggest that the reader use caution in extrapolating these results.

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