

## Propagation of a 14.7-Day Wave along the Northern Coast of the Guinea Gulf

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### ABSTRACT

Long time series of sea level and sea surface temperatures measured at different coastal stations along the northern coast of the Gulf of Guinea are analyzed statistically. The results indicate that pronounced fortnightly oscillations in sea level are composed of two waves: one is the lunar fortnightly tide  $M_2$  (13.661-day period) which has a constant phase all along the coast. The other wave has a period of 14.765 days which is the period of the luni-solar fortnightly tide  $M_{2f}$ ; this wave propagates westward along the east-west oriented coastline with a mean phase speed of  $53 \text{ cm s}^{-1}$  and a wavelength of 675 km. These waves have important effects on the thermal structure and give rise to strong vertical oscillations of the sub-surface isotherms throughout the year. The sea surface temperature, however, has pronounced oscillations around the  $M_{2f}$  frequency during the upwelling season (June–September) only. The 14.765-day wave is of tidal origin and is due to a nonlinear interaction of the  $M_2$  and  $S_2$  (barotropic or baroclinic) tides but the generation mechanism is obscure.

### 1. Introduction

Extensive oceanographic studies over the past 10 years, mainly by the French Office O.R.S.T.O.M. in the Ivory Coast, and by the Fishery Research Unit of Ghana, have increased our knowledge of the general hydrography, circulation and seasonal variations in the Gulf of Guinea considerably. (Varlet, 1958; Longhurst, 1962; Morlière, 1970; Lemasson and Rébert, 1973; Houghton, 1976). There are several reasons why the interaction between the ocean and atmosphere in the Gulf of Guinea is of particular interest.

First, the atmospheric forcing in the area is very regular and is not subject to sudden and dramatic changes as is the case in higher latitudes. As a result tide gage data from this meteorologically quiet area do not merely mirror noisy meteorological forcing but could provide information about the dynamics of the ocean, about the fundamental modes of oscillations of the tropical Atlantic, and possibly about the effects of coastal geometry on equatorial waves (Philander, 1977).

Second, the coastal upwelling observed each year along the northern coast of the Gulf during the Northern Hemisphere summer exhibits a number of perplexing features. These upwellings were noted for the first time by Schott (1944) and considerable efforts have since been made to understand their mechanism (see Houghton, 1976). A simple model of Ekman upwelling driven by the local winds is

irreconcilable with the observations. Although the winds over the Gulf are monsoonal, their intensity does not change drastically at the coast during the summer monsoon, and it is probable that the coastal phenomena could be part of large-scale equatorial processes. During the FINE (FGGE/Index/NOR-PAX Equatorial) workshop at the Scripps Institution of Oceanography (27 June–12 August 1977) Moore *et al.* (1978) proposed that remote forcing by the increase of the wind stress in the western part of the equatorial Atlantic could explain these coastal upwellings. O'Brien *et al.* (1978) have developed these ideas in a simple baroclinic numerical model of upwelling. Another approach to the problem is given by Philander (1978) who shows that the strong cross-equatorial winds between June and September can cause coastal upwellings along the northern coast of the Gulf.

Third, the northern coast of the Gulf of Guinea is an interesting region to extend the studies of trapped shelf waves which have been observed along north-south oriented coastlines off Oregon (Mooers and Smith, 1968; Cutchin and Smith, 1973), off the North Carolina Coast (Mysak and Hamon, 1969), off Florida (Brooks and Mooers, 1977), and off the Australian eastern and western coasts (Hamon, 1966; Mysak, 1967). These traveling waves are energized by atmospheric forcing. Cartwright (1969) gives the only example of a diurnal tidally induced continental shelf wave.

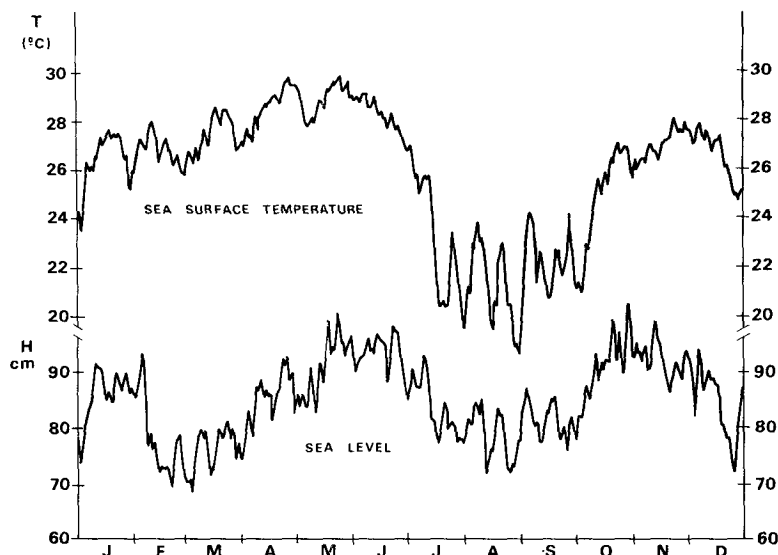


FIG. 1. Daily sea surface temperature and sea level during 1974 at Tema.

The analyses of long time series available at different coastal stations along the northern coast of the Gulf of Guinea show the presence of many low-frequency oscillations. Most of them are induced by tidal and atmospheric forcing (Picaut and Verstraete, 1976; Verstraete *et al.*, 1978).

A wave with a period of about 15 days that exists throughout the year is easily revealed by a visual inspection of the mean sea level (Fig. 1). By looking at the coastal sea surface temperature, this oscillation clearly appears during the upwelling season (July–September) when the thermocline is very close to the surface. Temperature measurements below the surface, with a thermistor chain off Abidjan, show that this wave is present throughout the year

(see Fig. 2). Its appearance at the surface in the non-upwelling season is masked by the mixed surface layer.

The purposes of this paper are 1) to document the existence of a westward propagating fortnightly wave along the west-east oriented coastline of the Gulf of Guinea and 2) to evaluate hypotheses concerning the generation of this wave.

## 2. Data analysis

### a. The data

All the data used for this study are listed in Table 1 and the locations are given by Fig. 3. They cover a period between 1951 and 1977 and were

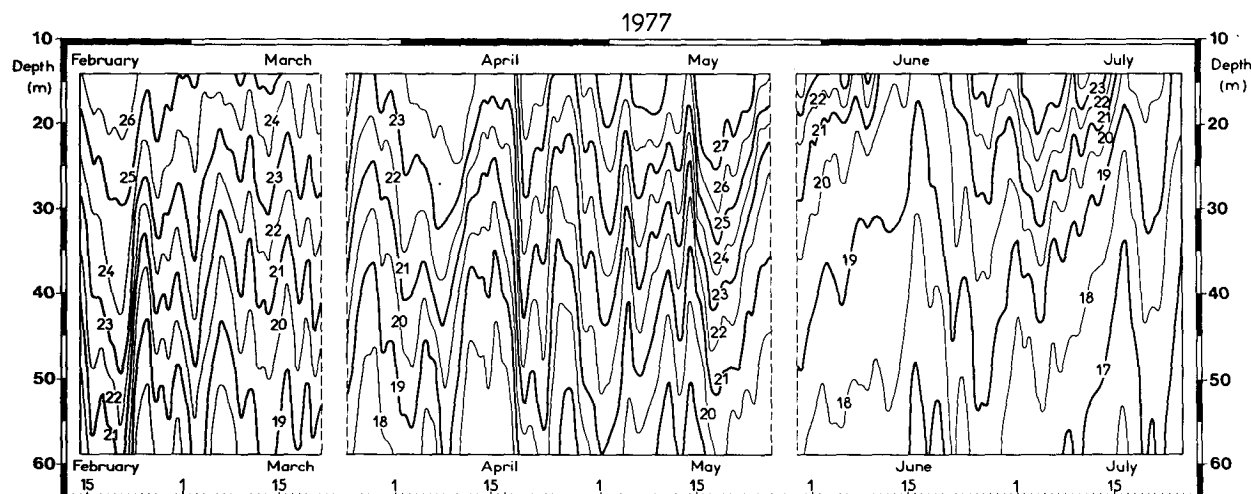


FIG. 2. Low-pass filtered isotherms (Dermerliac filter) at the Aanderaa thermistor chain mooring off Abidjan at a depth of 65 m, from 11 February to 25 July 1977.

TABLE 1. List of the coastal stations and the collected data.

State	Location	Parameter	Data length	Time origin	Remarks	
Benin	Cotonou	SST	16 years	1 Apr 1958	Nansen bottle; 0, 5, 8 m depths; T, S; 0800 GMT; wharf	
Togo	Kpémé	SST	6 years	1 Jan 1970	Nansen bottle; T, S; 0800, 1200, 1600 GMT; wharf	
		SST	3 years	1 Jan 1951	Nansen bottle; T, S; 0800, 1200, 1600 GMT; wharf	
	Lomé	SST	2 years	1 Jan 1967	Nansen bottle; T, S; 0800 GMT; wharf	
		SL	2 years	9 Jan 1966	High and low tides; 4% of gaps	
		SL	6 years	6 Feb 1970	High and low tides; 5% of gaps	
Ghana	Keta	SST	9 years	1 May 1968	Bucket sample; T, S; 0800 GMT; beach	
		SST	14 years	1 Jan 1963	Bucket sample; T, S; 0800 GMT; harbor breakwater	
	Tema	SL	8 years	1 Jan 1969	Mean of 24 h observations	
		SL	1 year	1 Jan 1976	Hourly observations	
	Winneba	SST	7 years	1 Jan 1970	Bucket sample; T, S; 0800 GMT; beach	
		SST	9 years	1 May 1968	Bucket sample; T, S; 0800 GMT; beach	
	Takoradi	SL	8 years	1 Jan 1969	Mean of 24 h observations	
		SL	2 years	1 Jan 1976	Hourly observations	
	Cape Three Points	SST	3 years	1 Jan 1974	Bucket sample; T, S; 0800 GMT; beach	
		SST	5 years	1 May 1968	Bucket sample; T, S; 0800 GMT; beach; station closed down and replaced by Cape Three Points	
	Axim	SST	9 years	1 May 1968	Bucket sample; T, S; 0800 GMT; beach	
		Half Assini	SST	8 years	1 Jan 1969	Bucket sample; T, S; 0800 GMT; beach
	Ivory coast	Abidjan	T	12 years	29 Mar 1966	Twice a week; Nansen bottle; 0, 5, 10, 15, 20 m depths; T, S; 0930 GMT; in 30 m water depth
			T	165 days	11 Feb 1977	Thermistor chain; 14-59 m depths; $\Delta t = 20$ mn; in 66 m water depth
		SL	10 years	9 Jan 1967	High and low tides	
		SL	1 year	1 Jan 1971	Hourly measurements	
		SL	27 months	1 Jan 1974	Hourly measurements	
Sassandra		SL	304 days	5 Jan 1965	High and low tides	
		SL	50 months	1 Apr 1973	High and low tides; 10% of gaps	
San Pedro		SST	21 months	1 Apr 1958	Bucket sample; T; 0650, 1200, 1800 GMT; beach	
		SST	10 months	18 Aug 1968	Bucket sample; T; 0700 GMT; beach; 20% of gaps	
Liberia		Monrovia	SL	338 days	22 Sep 1953	Hourly measurements

collected by the Centre de Recherches Océanographiques (ORSTOM) and Service Hydrographique in Ivory Coast, the Fishery Research Unit and the Survey Department in Ghana, the Service Hydrographique and ORSTOM in Togo and the Service des Pêches in Bénin.

The sea surface temperatures were measured from a bucket sample at the beach or with a Nansen bottle on the wharf, two or three times per day. In these analyses we used the measurements near 0800 GMT.

The hourly measurements of sea level at Mon-

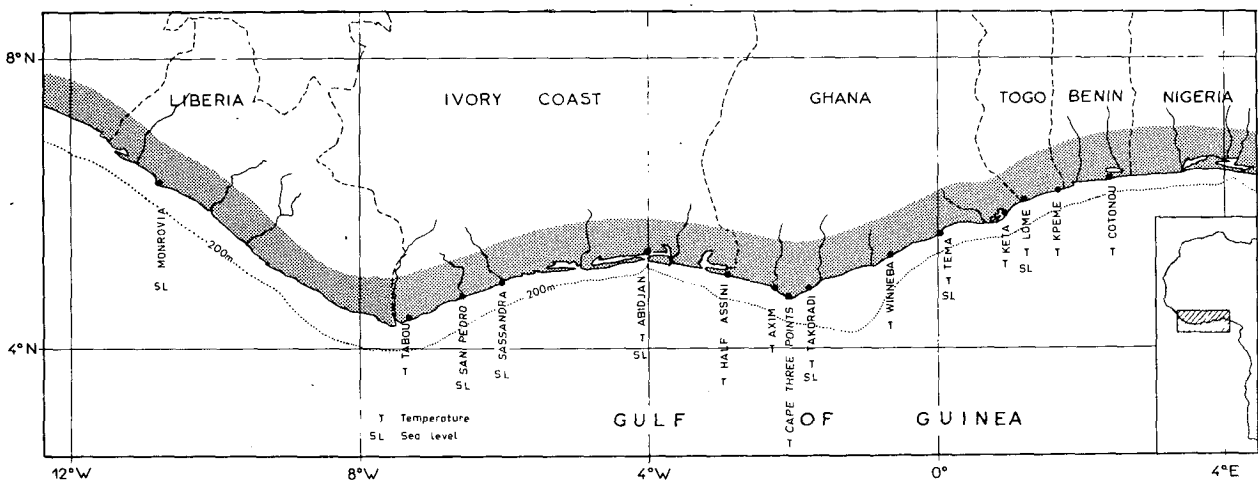


FIG. 3. Location of the coastal stations along the northern coast of the Guinea Gulf.

rovia were generously given by G. Philander. In order to simplify the digitizing of analog records we have extracted the high and low tides from the tidal records at San Pedro, Sassandra, Abidjan and Lomé, and have taken as the daily mean sea level the mid-tide level. For Tema and Takoradi, on the other hand, we collected the mean of 24 consecutive hourly observations of the water levels.

These two types of daily mean sea level could suffer from aliasing of the fundamental tidal constituents (Godin, 1972; Demerliac, 1973). Therefore, we also obtained hourly observations for three years at Abidjan, one year at Tema, and two years at Takoradi, in order to estimate the possible errors introduced by aliasing.

At Lomé, Tema, Takoradi and San Pedro, the tide gages are positioned in or near the harbor entrance where there are no perturbations from any lagoons or rivers. At Abidjan the tide gage is at the entrance of the Vridi Canal and one must be on the lookout for spurious variations from the lagoon during the rainy season. At Sassandra the tide gage had been working on the wharf for less than one year. We have no information about the position of the tide gage in Monrovia Harbor.

*b. Determination of the period of the oscillations*

Usually the analysis of the sea level first requires corrections for the isostatic variations due to the atmospheric pressure (Hamon, 1966). In a recent paper (Picaut and Verstraete, 1976), we have shown that such a correction is unnecessary in the present area.

The present analysis of the sea level time series that exceed five years gives evidence of several significant peaks, particularly near the periods of 45, 14.7 and 13.7 days. With the longest time series, the one from Abidjan, we are able to separate the two last peaks and find maxima of energy at periods of 14.73 and 13.73 days. These values are very close to those of M<sub>sf</sub> and M<sub>f</sub><sup>1</sup> (Fig. 4). To obtain the period of the two peaks more accurately, we made numerous spectral analyses of the sea level at Abidjan with a variable length of the autocorrelation interval. If the variations are small and the mean lengths much shorter than those of the time series, the spectral density found in all these analyses are of the same order and may overlap each other. This fact allows us to put the results on the same graph (Fig. 5). The autocorrelation length varied from 298 to 306 days. On this graph the usual bandwidth for a simple spectral analysis is given above the two peaks. With the refined treatment, our resolution is eight times higher. The two maxima are now found to be exactly at 13.66 and 14.77 days. This

<sup>1</sup> M<sub>f</sub> is the lunar fortnightly tide (13.661 days) and M<sub>sf</sub> the luni-solar synodic fortnightly tide (14.765 days).

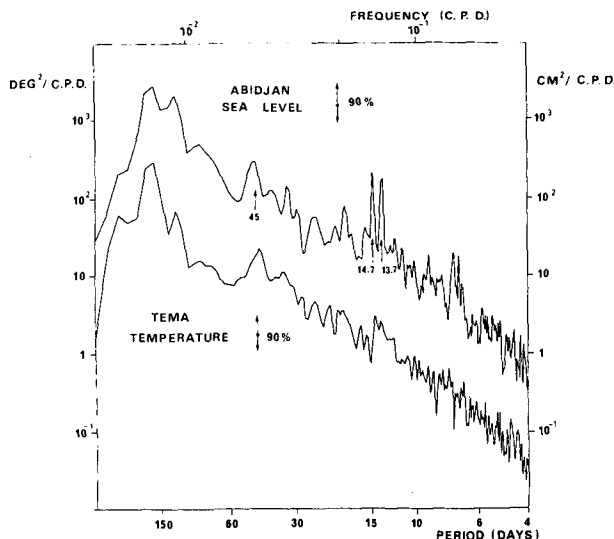


FIG. 4. Autospectra of mid-tide sea level at Abidjan (10 years) and sea surface temperature at Tema (13 years).

establishes that these two peaks correspond exactly to the M<sub>f</sub> (13.6608 days) and M<sub>sf</sub> (14.7653 days) periods.

If these two oscillations are of purely tidal origin, they must be represented by two lines above a continuous noise. The M<sub>f</sub> tide is not an integral subharmonic of the M<sub>sf</sub> tide, so we choose a record length such that the M<sub>f</sub> and M<sub>sf</sub> frequencies are as close as possible to harmonics of the record length. Fig. 6 gives the result of the Fourier analysis with 2922 daily mean sea level data points at Abidjan. The two lines at 14.757 and 13.654 days are far above the continuous noise, with an amplitude of 1.20 and 1.14 cm, respectively. Such a small noise level may be explained by the stability of meteorological conditions in the Gulf of Guinea and probably by the good quality of the measurements at Abidjan.

*c. Elimination of possible aliasing*

With a sampling frequency of  $q$  and in the presence of an oscillation with frequency  $f$ , apparent oscillations appear at the frequencies  $q - f, q + f, 2q - f, 2q + f, \dots$

Table 2 gives the mean result of a harmonic analysis of hourly measurements of the sea level at Monrovia, Abidjan, Takoradi and Tema, for the most important tidal constituents of periods less than one month.

The analyses of sea surface height are made with two types of "daily mean sea level": the mid-tide level and the 24 hourly average. Table 3 gives the residue coefficients for the main tidal constituents with these two types of "daily mean sea level". The average lets through 3.5% of the M<sub>2</sub> tide and with a sampling frequency of 1 cycle per day (cpd)

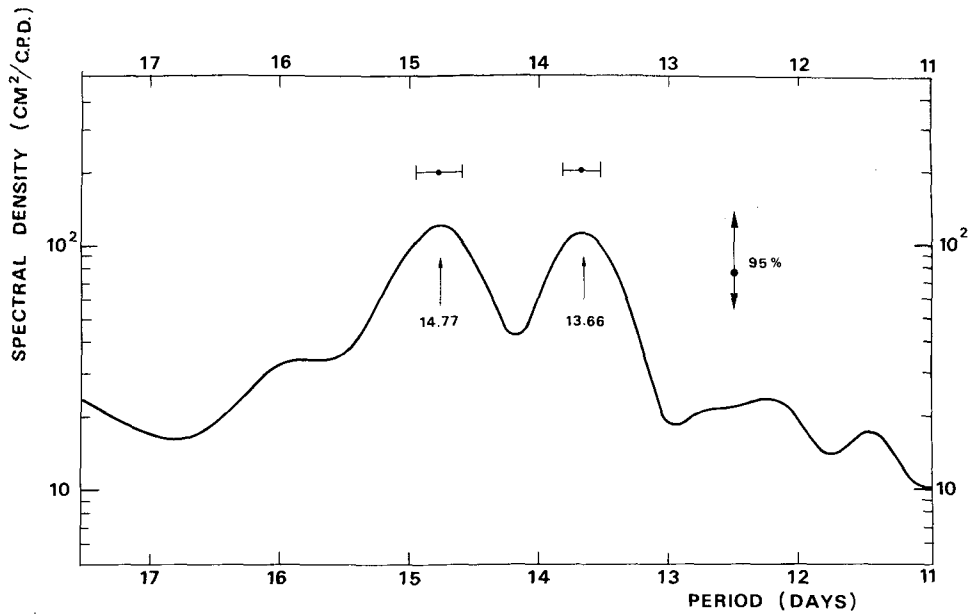


FIG. 5. Detailed spectral analysis of Abidjan sea level.

may give a fictitious Msf oscillation as large as the real phenomenon (Table 2). The mid-tide level has been estimated for each day by the mean of the four nearest low and high tides of the day, so the effective sampling is 0.966 cpd; with 5.9% of the  $K_2$  tide it may give an aliased oscillation at the Mf frequency about eight times smaller than the real one.

In order to control these factors, we have used, on all the hourly measurements of sea level at Abidjan, a special filter that spans 72 h (Demerliac, 1973), and have calculated the corresponding daily mean sea level. Table 4 presents the harmonic analysis of the daily mean sea level, the 24 hourly average, the mid-tide level and the hourly meas-

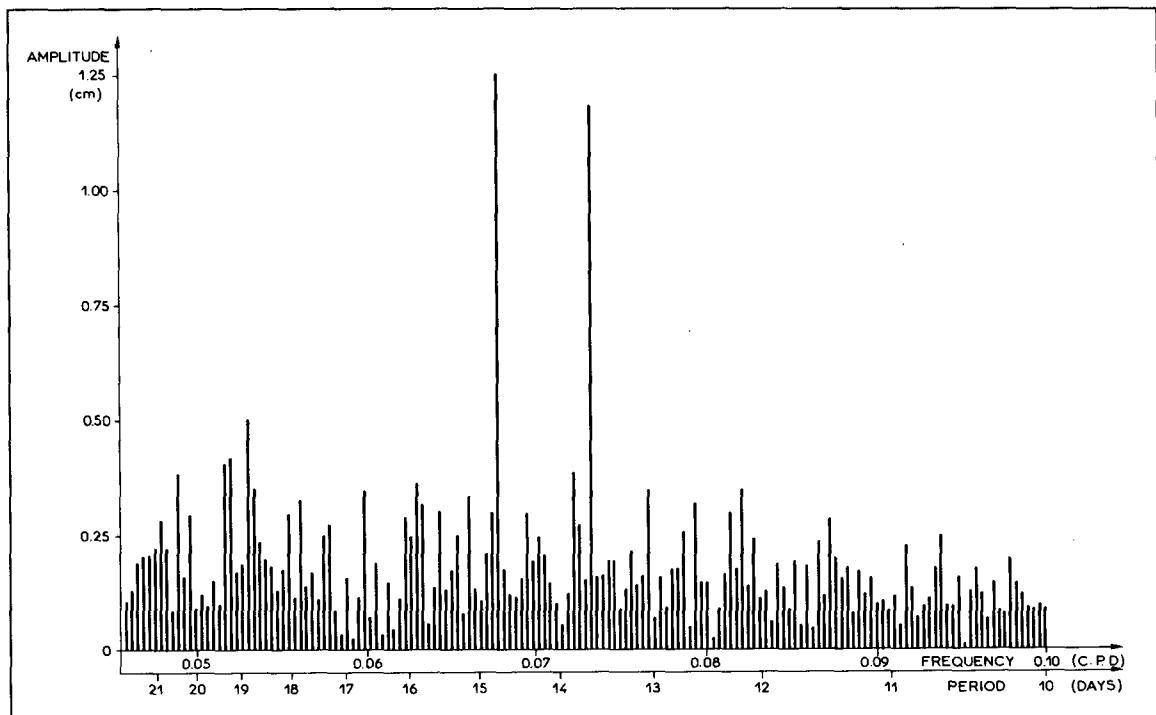


FIG. 6. Fourier analysis of Abidjan mid-tide sea level (8 years).

urements. It is clear that the average perturbs the Msf result considerably, whereas the mid-tide level does not disturb the Mf result significantly.

We have spectrally analysed the 27 consecutive months of hourly sea level at Abidjan, after using the Demerliac filter. The comparison with mid-tide level spectra shows a very good agreement between these two types of data (Fig. 7). Hence the mid-tide level is much better adapted for the study of low-frequency phenomena than the 24 hourly average, which may give significant aliasing.

Puzzling results with this "daily mean sea level" (see Section 4) show clearly the importance of these precautions. We can now be sure that the Mf and Msf oscillations of the sea level are real phenomena.

During the entire year, we find strong semi-diurnal internal waves all along the continental shelf (Fig. 13). During the upwelling season (July–September) it is possible for the thermocline to rise very close to the surface so that daily sea surface temperature measurements could give fictitious Msf results.

In the present study we use two types of sea surface temperature data. One set of data has been taken from the beach, the other at the end of the wharfs or the piers in water 5–8 m deep.

In both cases, the internal waves would have broken some distance away thus eliminating possible aliasing. During the 1974 Ghana upwelling season, Houghton (1976) found a very high correlation between Tema Harbor daily measurements of temperature at 0830 GMT and low-pass filtered data of hourly temperature measurements at a depth of 12 m on the shelf. The filter he used is a cosine one that spans 75 h. This reduces the  $M_2$  considerably.

Special attention has been given to the sea surface temperature from the wharf of Lomé. Comparison of the spectrum for three years measurements at 0800, 1200, 1600 GMT, and for the mean measurements reveals no significant difference in the period range 4–100 days.

We conclude that fortnightly oscillations of the sea surface temperature are real. Visual inspection of the Demerliac filtered temperature as measured with a thermistor chain off Abidjan (Fig. 2) shows clearly such an oscillation, especially in March and April 1977.

TABLE 2. Mean amplitude of the main tidal components at Tema, Takoradi, Abidjan and Monrovia.

Constituent	$M_2$	$S_2$	$K_1$	$N_2$	$K_2$	$P_1$	$O_1$
Period (h)	12.42	12.00	23.93	12.66	11.97	24.07	25.82
Amplitude (cm)	38.13	13.28	9.94	8.49	3.75	2.83	2.11

Constituent	Mf	Msf	Mm
Period (days)	13.66	14.76	27.55
Amplitude (cm)	1.80	0.88	1.00

### 3. Origin of the Mf and Msf oscillations

When tidal phenomena propagate on the shelf, they are modified by the bottom and coastal topography so that higher or compounded harmonic waves are created (Le Provost, 1976). These secondary components are the result of the presence of nonlinear advection terms  $uu_x, uv_x, vu_y, vv_y$  and the quadratic friction terms. If the tidal forcing is a pure harmonic of frequency  $\omega$ ,  $u = u_0 e^{i\omega t}$ , the quadratic term  $uu_x$  contains a frequency  $2\omega$ ; also, if the tidal forcing contains two frequencies  $\omega_1$  and  $\omega_2$ , the quadratic term in  $uu_x$  will have higher harmonics at  $2\omega_1$  and  $2\omega_2$  and two compounded harmonics at  $\omega_1 - \omega_2$  and  $\omega_1 + \omega_2$ . Such higher frequency waves created by nonlinearity or by friction are common in the present area. Detailed harmonic analysis of the hourly measurements at Monrovia, Abidjan and Tema shows the presence of 10 waves of this type, with an amplitude higher than 0.5 cm. In the lower range, the Mf oscillation could be due to such an interaction between the  $K_2$  and  $M_2$  tides and the Msf due to an interaction between the  $M_2$  and  $S_2$  tides.

A question remains: are the observed Mf and Msf tides in response to the driving tidal potential or are they due to a nonlinear interaction between the  $M_2$ ,  $S_2$  and  $K_2$  tides?

While the semidiurnal and diurnal tidal constituents have been studied extensively, this is not so in the case of the semi-monthly tides. Recent studies of the lunar fortnightly tide in the Pacific (Wunsch, 1967) and in the world ocean (Maksimov, 1966;

TABLE 3. Residues of different tidal filters.

Method	Constituent						
	$M_2$	$S_2$	$K_1$	$N_2$	$K_2$	$P_1$	$O_1$
24 h average	-0.03516	0.00000	-0.00274	-0.05445	0.00276	0.00275	0.07537
Mid-tide	0.00000	0.05461	-0.04079	-0.02945	0.05905	-0.03485	0.04329
Doodson filter	-0.00058	0.00000	0.00015	0.00171	0.00033	-0.00013	0.00299
Demerliac filter	-0.00004	0.00000	0.00000	-0.00014	0.00000	0.00000	0.00041

TABLE 4. Mean results of the harmonic analysis at Abidjan with four different types of sea level data.

Type of data	Constituent			
	Msf		Mf	
	A (cm)	G (deg)	A (cm)	G (deg)
24 hourly average	0.66	130	1.27	11
Mid-tide level	0.91	189	0.98	8
24 h average after using the Dermerliac filter	0.79	195	1.25	11
Hourly measurements	0.81	195	1.31	12

Lisitzin, 1974; Kagan *et al.*, 1976) show that this tide is not exactly an equilibrium tide. Nevertheless, the deviation from the equilibrium tide seems to be a minimum near the equator.

The long-period equilibrium tides can be computed from the following equations:

$$W = V(1 - 3 \sin^2 \phi) \cos \psi,$$

$$\Delta H = \frac{W}{g} (1 + k - h),$$

where  $W$  is the potential of the disturbing force,  $V$  the relative coefficient,  $\phi$  the geographical latitude,  $\psi$  the argument,  $\Delta H$  the disturbance in mean sea level,  $g$  the acceleration of the earth's gravity and  $(1 + k - h)$  a factor representing the resilient and elastic properties of the earth's yielding crust, usually estimated to be 0.67.

This equilibrium equation gives for the Mf and Msf constituents:

$$\Delta H_{Mf} = 1.402(1 - 3 \sin^2 \phi) \cos 2s,$$

$$\Delta H_{Msf} = 0.123(1 - 3 \sin^2 \phi) \cos(2s - 2h).$$

With  $\phi = 5^\circ$  in the present area, we obtain

$$\Delta H_{Mf} = 1.37 \text{ cm},$$

$$\Delta H_{Msf} = 0.12 \text{ cm}.$$

The mean results of the harmonic analysis of Monrovia, Abidjan, Takoradi and Tema give (Table 2) 1.80 cm for Mf and 0.88 cm for Msf.

The amplification factor of 1.3 for Mf is in good agreement with the observation of Maksimov (1966). Kagan *et al.* (1976) explain the deviation by the dynamic tidal theory. On the other hand, we have seen that Mf could be due to the interaction of the tidal constituents  $K_2$  and  $M_2$ , but the amplitude of  $K_2$  is 10 times less than that of  $M_2$  (Table 2). The symmetric compound tide  $MK_4$  ( $M_2 + K_2$ ) is very weak in the present area and it would probably exist if ( $M_2 - K_2$ ) were present.

All these facts tend to prove that observed Mf oscillation is of purely astronomical origin.

An amplification factor of 7.4 for the Msf oscillation is inconsistent with the equilibrium theory and even with the dynamic one.  $M_2$  and  $S_2$  are the most important tides in our area (Table 2) and we find a wave  $MS_4$  ( $M_2 + S_2$ ) of the same order as the observed Msf wave. The observed 14.76-day wave is probably induced by a nonlinear interaction on the shelf between the semidiurnal tides  $M_2$  and  $S_2$ .

Offshore, the analysis of tidal observations at Ascension Island in the equatorial Atlantic (Cartwright, 1971, and private communication) gives a significant amplitude (0.9 cm) at the Mf frequency but not at the Msf frequency. In the equatorial Pacific, Wunsch (1967) observes the Mf tide at many islands but the Msf only at Canton Island.

#### 4. Wave propagation

Houghton and Beer (1976) presented evidence of a westward propagating wave with a mean phase speed of  $64 \text{ cm s}^{-1}$  and a period around 14.5-day (800 km wavelength) in the region east of Cape Three Points during the upwelling season of 1974. In our IUGG paper (Picaut and Verstraete, 1975) we have shown preliminary results of the spectral analysis of some sea surface time series between Tabou and Cotonou. The temperature data indicate that in the neighborhood of the fortnightly frequency, there is an energetic wave with a westward phase speed of roughly  $75 \text{ cm s}^{-1}$ . On the other hand, the sea level data analyzed by Houghton and Beer show no phase propagation between Tema and Takoradi. This perplexing result is confirmed by our calculations, although we do find westward phase propagation from Ghana to the Ivory Coast in the sea level at a period of 14.7 days.

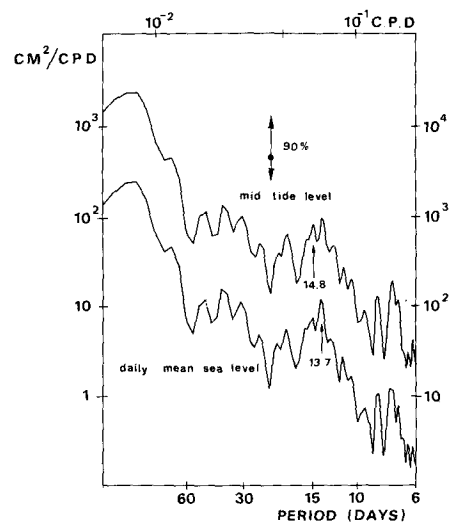


FIG. 7. Autospectra of mid-tide sea level and daily mean sea level. Daily mean sea level obtained by Demerliac filtering of hourly measurements at Abidjan (27 months).

We have tried to make a more detailed cross-spectral analysis by using data from all the coastal stations in Fig. 3. For the sea level at Tema, Takoradi and Abidjan, we find a very strong coherence for the Mf and Msf frequencies; unfortunately, the coherence drops at Lomé and San Pedro. This can be explained by the poor quality of these two sets of data. Furthermore, the time series of Sassandra and Monrovia are too short to give precise information around the fortnightly frequency with spectral analysis. A harmonic analysis seems to be more convenient for this problem. For this purpose, with B. Simon, we adapted a program developed by the French Hydrographic Service (Simon, 1974) for the analysis of daily mean sea level. Table 5 gives the mean results of yearly sea level harmonic analysis.

In Section 2 we have seen that the long-period tide results given by harmonic analysis, on the mid-tide level, are very close to those given by hourly measurement (Table 4). On the other hand, the 24 hourly average gives an entirely fictitious Msf oscillation. So for the Msf result of Tema and Takoradi, we use the hourly measurements for one year at Tema and two years at Takoradi. The mean results with the 24 h/average give an amplitude  $A$  of 2.00 cm and a Greenwich phase  $G$  of  $67^\circ$  for Tema and  $A = 1.66$  cm,  $G = 55^\circ$  for Takoradi; with such values, the phase difference between Tema and Takoradi is nearly null (no propagation) instead of  $51^\circ$  (Table 5) for the real values (westward propagation of  $121 \text{ cm s}^{-1}$ ). So for the Msf analysis we use the hourly measurement at Tema and Takoradi.

In Table 5 the mean amplitude and phase for Mf and Msf are given, in addition to error estimates. These error estimates are based on the concept of estimation of a deterministic sinusoidal signal embedded in a random noise background. For further explanations see Wunsch (1967) and Middleton (1947). The high noise level at San Pedro is due to the presence of some gaps in the time series and probably is due to a poor offset of the tide recorder. This could perturb the long-period analysis (Desnoës, 1975). Error estimates are also large for short

time series (Sassandra, Monrovia). Nevertheless, Table 5 shows clearly that the Mf oscillation does not propagate. On the other hand, the phase difference for the Msf oscillation increases regularly from Lome to Monrovia. In Fig. 8, we observe a westward propagation of  $53 \text{ cm s}^{-1}$ , which gives a wavelength of 675 km.

In the case of the sea surface temperature we must distinguish between the whole year and the upwelling season (June to early October) when the thermocline rises to the surface so that the temperature fluctuations can reach the surface (Fig. 1). We did the following analyses on all our sea surface temperatures:

- Autospectral and cross-spectral analyses.
- Fourier analyses on the longest series.
- Harmonic analyses on the whole year and on the upwelling season.
- Cross-correlation centered on the upwelling season.

Cross-spectral analysis reveals generally the presence of a maximum of energy density around the fortnightly frequency but rarely at the exact frequencies of Mf and Msf. The coherence between the stations is generally strong on the eastern side of Cape Three Points but is much lower on the western side. This is probably due to a cape effect which gives a permanent upwelling downstream (east) of Cape Three Points and an accumulation of warm water upstream (Marchal and Picaut, 1977) so that the oscillations may not appear on the upstream side.

A Fourier analysis of the sea surface temperature also fails to separate the Mf and Msf signals. (See Fig. 9). Fig. 10 shows the phase propagation of the fortnightly oscillations. It is approximately  $42 \text{ cm s}^{-1}$ . Data for the entire year, and for the upwelling season only, give essentially the same result.

The background noise, associated with mixing in a shallow surface layer, presumably blurs the distinction between the Mf and Msf signals in sea surface temperature. Harmonic analysis of the subsurface thermistor chain data (166 days) shows the Msf amplitude to be twice as large as the Mf amplitude.

TABLE 5. Mean results of yearly sea level harmonic analysis.

Station	Lome	Tema	Takoradi	Abidjan	Sassandra	San Pedro	Monrovia
Time duration (years)	5	1	2	10	0.83	4	0.93
Msf	A (cm)	$0.83 \pm 0.32$	$0.46 \pm 0.21$	$0.73 \pm 0.22$	$1.24 \pm 0.13$	$1.46 \pm 0.57$	$0.90 \pm 0.49$
	G (deg)	$214 \pm 28$	$336 \pm 32$	$27 \pm 23$	$206 \pm 10$	$61 \pm 29$	$20 \pm 37$
Time duration (years)	5	7	8	10	0.83	4	0.93
Mf	A (cm)	$1.34 \pm 0.42$	$1.69 \pm 0.22$	$1.89 \pm 0.20$	$0.99 \pm 0.17$	$2.19 \pm 0.46$	$1.80 \pm 0.57$
	G (deg)	$0 \pm 24$	$0 \pm 11$	$357 \pm 10$	$359 \pm 14$	$11 \pm 22$	$8 \pm 24$



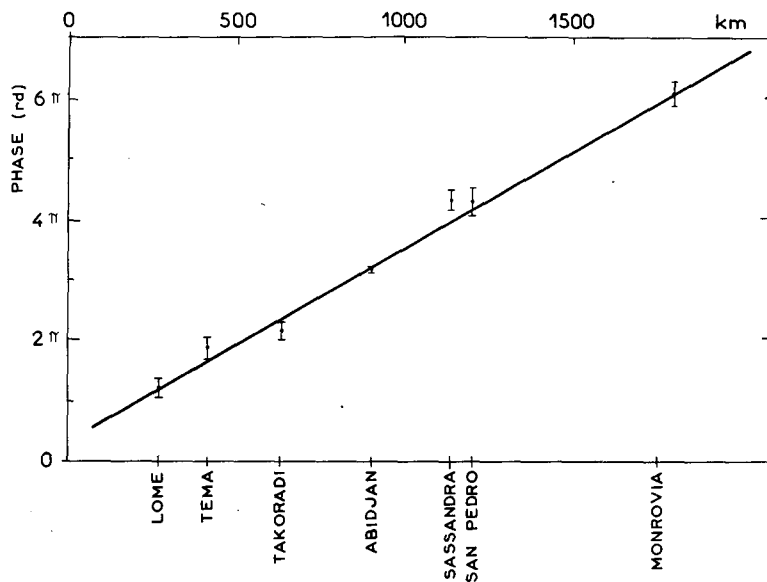


FIG. 8. Sea level phase variation at the Msf frequency along the coast.

To study the phase propagation more closely, we made a cross-correlation analysis of sea surface temperature data taken over 122 days during the upwelling season. In order to discern more clearly the possible fortnightly oscillation from other variations (45-day wave, seasonal variation, . . .) we have submitted all the time series to a bandpass filter centered on 15 days. About half of the 456 cross-correlation analyses reveal the presence of a

significant fortnightly oscillation. On each of these analyses, the day lag, corresponding to the maximum of cross-correlation, gives an estimate of the phase speed of the fortnightly wave. Despite a large dispersion of individual results (from 25 to 117 cm s<sup>-1</sup>), the mean value of 54 cm s<sup>-1</sup> is highly significant.

Note that the acceleration seems to appear in this westward propagation between Winneba and Takoradi. This result is corroborated by the Msf

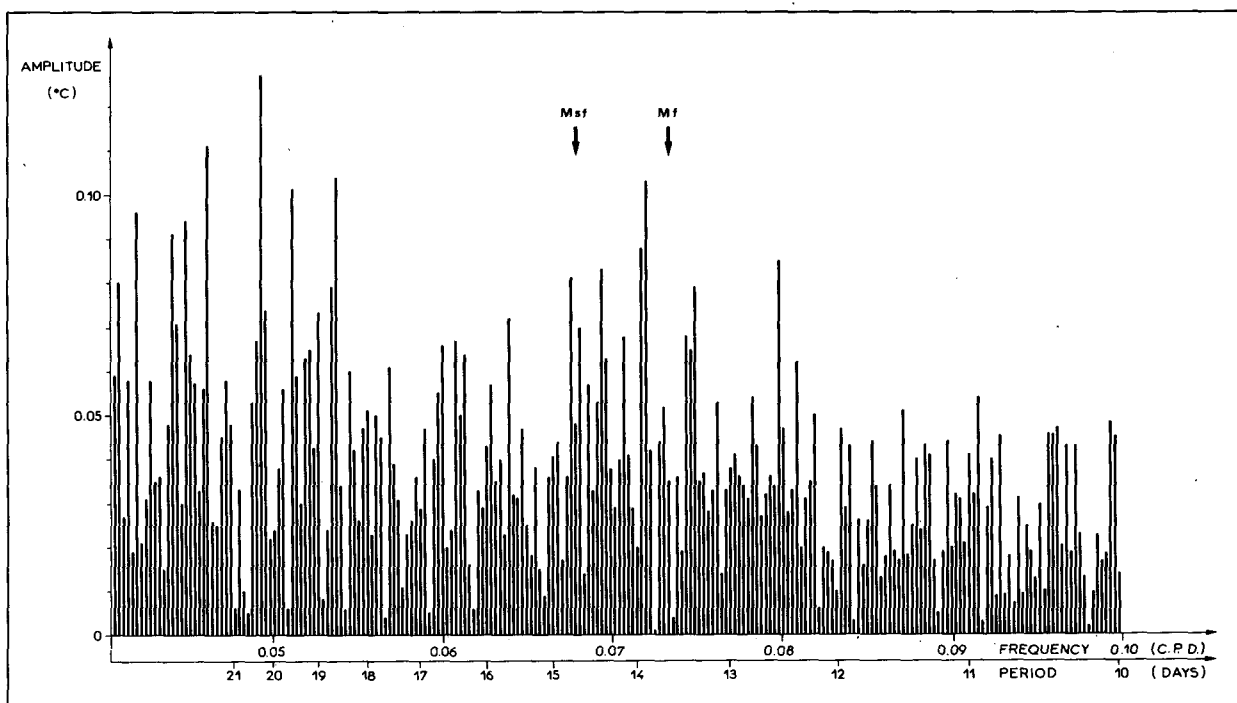


FIG. 9. Fourier analysis of sea surface temperature at Tema (14 years).

phase difference between Tema to Takoradi, for the sea level ( $121 \text{ cm s}^{-1}$ ). It might be explained by the increase of the shelf width in this area (Fig. 3).

Morliere and Rébert (1973) have shown that stratification on the shelf changes significantly in the upwelling season (Fig. 2). Table 6 gives the mean results of harmonic analyses for the whole year and for 147 days centered on the upwelling season [according Godin (1970), 147 days are enough to separate the  $M_f$  and  $M_{sf}$  frequencies]. No variation appears between these two types of analyses, so that the wave propagation seems not to be affected at all by the variation of stratification.

**5. Hypotheses concerning the generation of the wave**

The fortnightly wave can either be forced nonlocally, in which case it must satisfy the dispersion relation for a freely propagating wave, or it can be forced locally all along the coast, in which case its frequency and wavenumber (and hence phase speed) is determined by that of the forcing function.

*a. Nonlocally forced free waves*

Since the period of the wave is precisely that of the  $M_{sf}$  tide, its origin must be tidal. This eliminates atmospheric forcing as a generation mechanism and also eliminates the 14–18 meanders of the Undercurrent described by Duing *et al.* (1975) from the list of possible causes. A possibility that remains is

TABLE 6. Mean results of harmonic analyses for the whole year and around the upwelling season (147 days) with the sea level data

Station		Lome	Abidjan	San Pedro
Number of analyses		5	10	4
Yearly results	A (cm)	0.83	1.24	0.40
	G (deg)	241	206	59
Upwelling season results	A (cm)	0.59	1.43	0.95
	G (deg)	239	204	61

a nonlinear interaction of the barotropic  $M_2$  and  $S_2$  tides in a relatively small region. The “wave-maker” in this small region (which could be the shallow northeast corner of the Gulf of Guinea for example) could then generate waves that propagate freely westward along the coast. If this were indeed the generation mechanism, then the waves would attenuate with increasing distance from the region of forcing. Our data do not show a systematic attenuation of the amplitude. Nonetheless, we consider the free waves that could possibly propagate westward along the coast.

1) INTERNAL KELVIN WAVE

Philander (1977) has shown that a wave with a period of 14.7 days and a wavelength of 650 km does not satisfy the dispersion relation for a free internal Kelvin wave. Houghton (private communication) has evidence that the amplitude of the observed wave attenuates above the thermocline so that it cannot be an internal Kelvin wave.

2) TOPOGRAPHIC SHELF WAVE

Of the various theories related to nondivergent waves associated with sharp changes in depth, we refer to the Robinson (1964) and Mysak (1968) models because their choice of the bottom topography fits conveniently the bathymetry of the Guinea Gulf continental shelf. We have calculated the dispersion relation for numerous sections of the shelf between Monrovia and the Gulf of Biafra. There is only one large area where we find  $(\omega, k)$  in agreement with a 14.76-day period and a 675 km wavelength, the North shelf area of the Gulf of Biafra. A second possible area is off Takoradi. All the other sections give a theoretical speed at the  $M_{sf}$  frequency about 50% slower than the observed speed which is unsatisfactory.

This analysis is for nondivergent waves on an  $f$ -plane. In two recent papers, Mysak (1978a,b) investigates the properties of zonally propagating long-period equatorial waves on an equatorial beta plane with topography. He assumes that the motions are barotropic and nondivergent and that the iso-

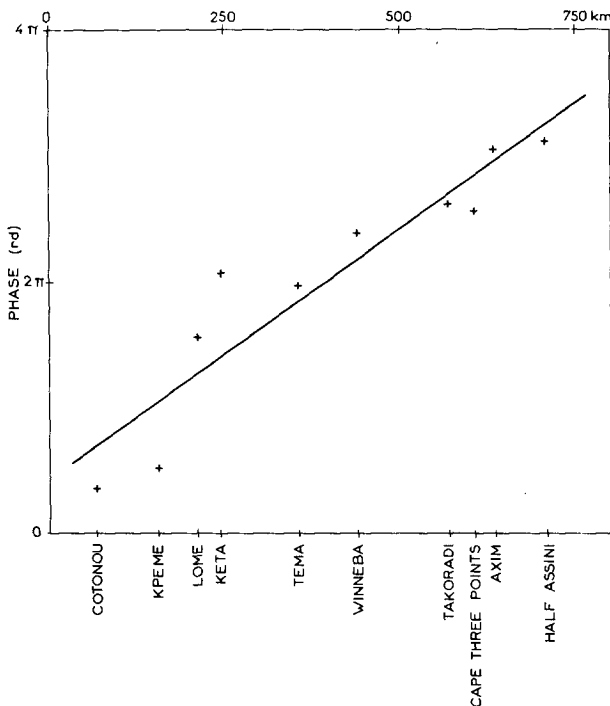


FIG. 10. Sea surface temperature mean phase variation along the coast at the  $M_f$  and  $M_{sf}$  frequencies.

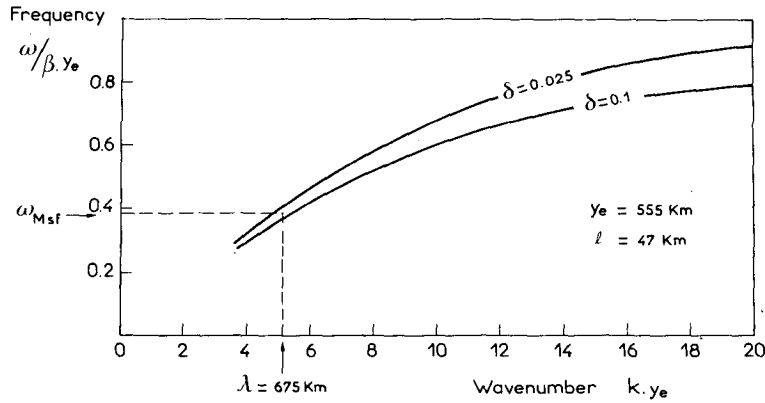


FIG. 11. Dispersion relation for an equatorial topographic wave.

baths are parallel to the equator. He applies his model to the present 14.7-day wave for two types of shelf: a step shelf and an exponential shelf with a mean shelf width of 100 and 92 km, respectively. He obtains a theoretical wavelength of 1300 km for the first profile and 580 km for the second one. However, these topographic values do not fit well the mean shelf of the Ghana and Ivory Coast.

Following Mysak (1978a), we made the calculations for the step shelf profile

$$H(y) = \begin{cases} H_1, & y_e < y < y_e + l \\ H_2, & -\infty < y < y_e \end{cases}$$

where  $y_e$  is the distance from the equator of the shelf

edge,  $l$  the shelf width,  $H_1$  the shelf depth and  $H_2$  the ocean depth.

In terms of the nondimensional frequency  $\Omega = \omega/\beta y_e$  and wavenumber  $\kappa = k y_e$ , the dispersion relation found is

$$\Omega = \frac{-\kappa(1 - \delta) \sinh \chi r}{\chi(\delta \sinh \chi r + \cosh \chi r)},$$

where

$$\chi = \left( \kappa^2 + \frac{\kappa}{\Omega} \right)^{1/2}, \quad \delta = H_1/H_2,$$

$$r = l/y_e, \quad \beta = 2\Omega_E/R,$$

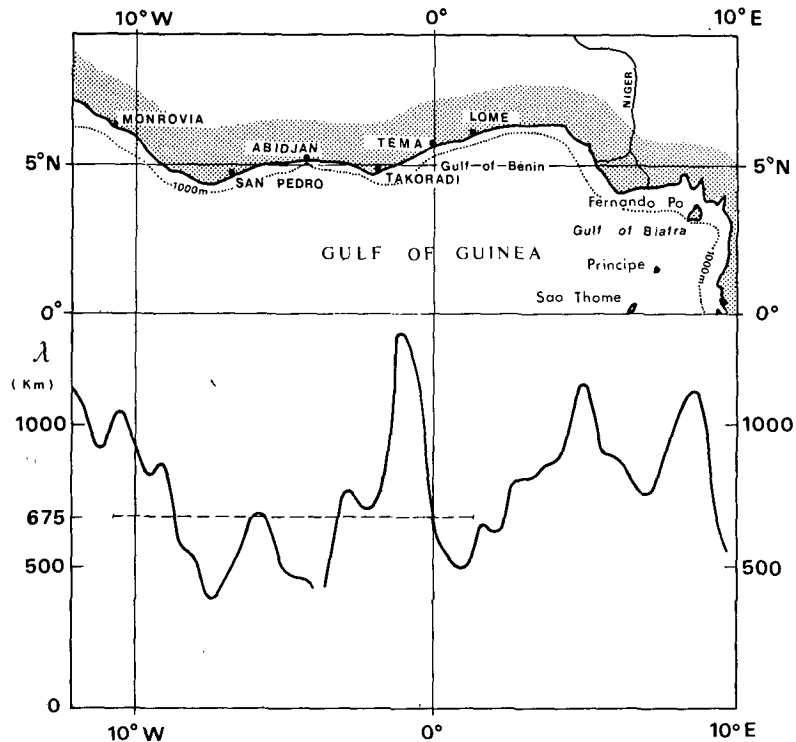


FIG. 12. The theoretical wavelength in the Gulf of Guinea at the Msf period.

and  $\Omega_E$  and  $R$ , respectively, the rate of the earth's rotation and the earth's radius. We have used the 1000 m depth contour to estimate the width of the shelf and  $H_2 = 4000$  m for the ocean bottom. Between Monrovia and Lomé we estimate the mean width  $l$  to 47 km and the mean  $y_e$  to 555 km ( $\varphi = 5^\circ N$ ). Fig. 11 gives the corresponding dis-

person relation. We first note that the variations of the shelf depth are unimportant.

At the Msf frequency, with  $\delta = H_1/H_2 = 0.025$ , we obtain a theoretical wavelength of 722 km, which is very close to the mean observed 675 km wavelength between Monrovia and Lomé.

On Fig. 12 we have plotted the theoretical wave-

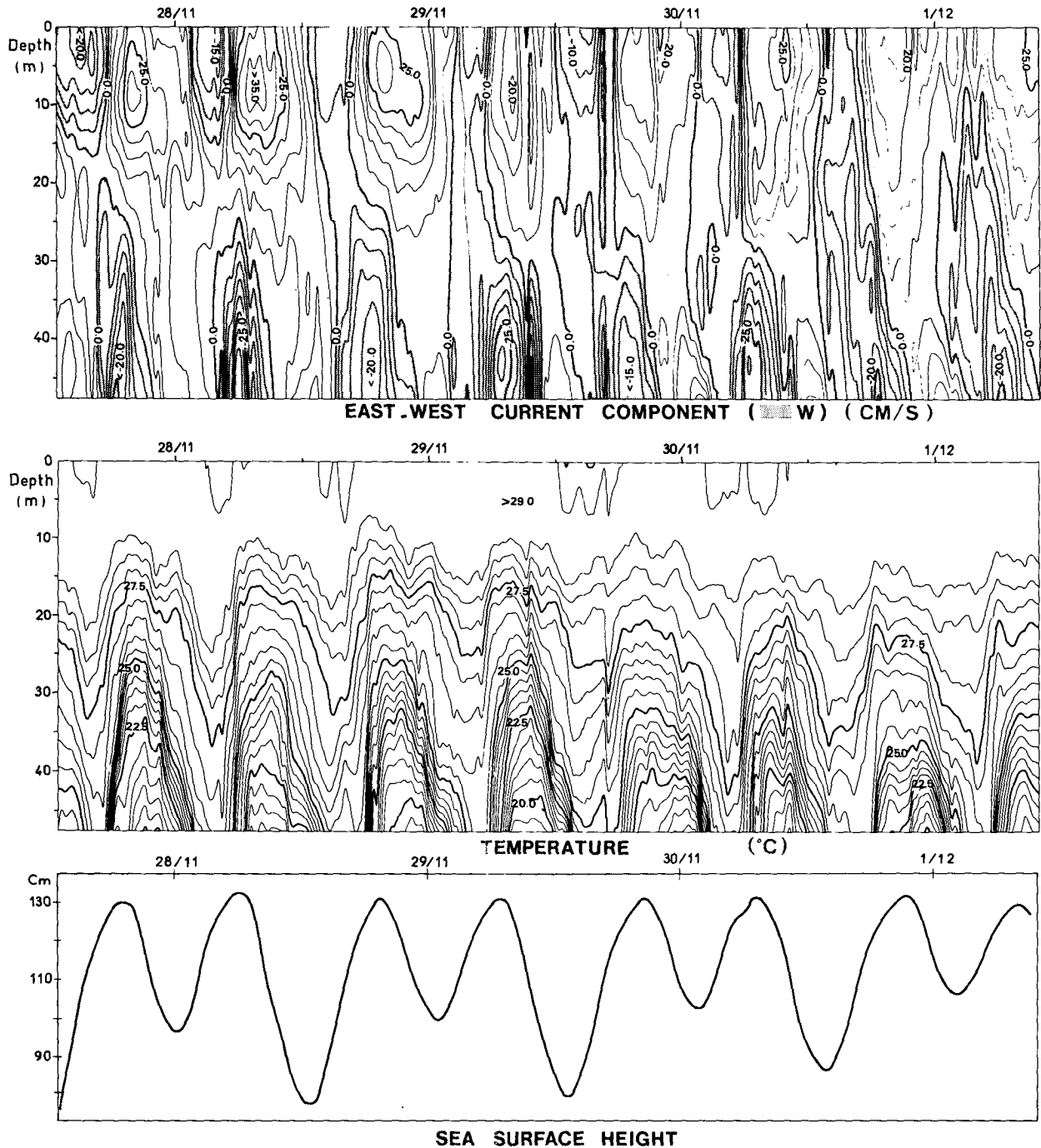


FIG. 13. High-resolution temperature and current profiles (298 profiles) in a water depth of 52 m during 4 days, off Abidjan. Sea surface height at the Abidjan tide gage.

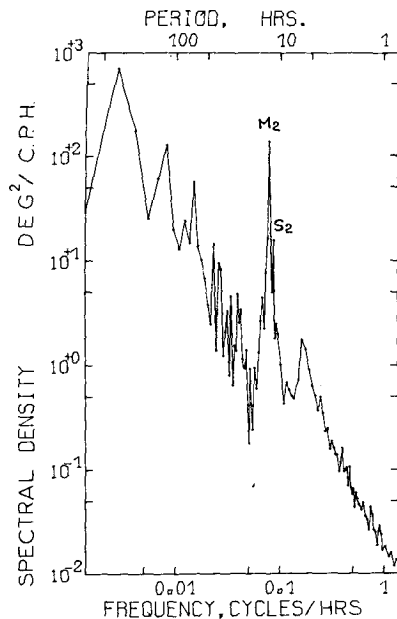


FIG. 14. Autospectrum of temperature at 45 m depth, at the thermistor chain mooring off Abidjan.

length at the  $M_{sf}$  frequency for 45 sections between  $12^{\circ}W$  and the Gulf of Biafra. This graph shows clearly that we must take into account the along-shore variation in the shelf profile.

Nevertheless, such a simple model leads us to speculate that the observed 14.7-day wave might be a freely propagating equatorial shelf wave.

#### b. Locally forced waves

The period of the observed wave (14.765 days) is such that it could be due to a nonlinear interaction of the  $M_2$  and  $S_2$  tides. If this interaction is forcing the observed wave all along the coast then the wavenumber  $K_1$  and  $K_2$  of the  $M_2$  and  $S_2$  tides, respectively, must be related to the wavenumber  $K_3$  of the observed wave by

$$K_1 \pm K_2 \pm K_3 = 0. \quad (1)$$

But the wavelengths of  $M_2$  and  $S_2$  are approximately 10 000 km, whereas the wavelength of  $M_{sf}$  equals about 675 km; the last wavelength is so small compared with the two others that the condition for a local forcing cannot be satisfied. It is impossible that the nonlinear interaction takes place along the coast, and we must rule out an interaction between the two barotropic waves  $M_2$  and  $S_2$ . The remaining possibility is a nonlinear interaction between the internal  $M_2$  and  $S_2$  tides.

Numerous and careful experiments in the area off Abidjan give evidence of very strong semidiurnal internal waves. These internal tides were observed both in the "warm season" and in the upwelling season. For example, one experiment with an an-

chored ship in a water depth of 52 m for four days during the warm season, obtained high-resolution temperature and current profiles simultaneously every 20 min. Some detailed results of this experiment (Picaut and Park, unpublished manuscript) are presented in Fig. 13, which depicts typical situations observed numerous times (eight experiments in 1973). We observe a very significant coherence between the vertical oscillations of the isotherms, the east-west component of current and a sea level record from the Abidjan tide gage located nearby.

In 1977, we succeeded in maintaining an Aanderaa thermistor chain of 50 m length on the shelf off Abidjan in water 65 m deep; the mooring design was similar to that of Pillsbury *et al.*, (1969) with subsurface buoys at 12 m depth. The instruments sampled data every 20 min and were maintained from 11 February to 25 July 1977. All the autospectra of 10 thermistor time series show two important peaks at the exact frequencies of  $M_2$  and  $S_2$  (Fig. 14). We observe also one significant peak around 6 h, which corresponds to the compound frequencies  $MS_4$  (6.103 h) and  $M_4$  (6.21 h) and one other at about 15 days. It is possible that the strong baroclinic internal tides observed could interact nonlinearly and transfer energy from the  $M_2$  and  $S_2$  tidal periods to the periods  $M_4$ ,  $MS_4$  and  $M_{sf}$ . Park (private communication) has shown that these oscillations are mainly of the first baroclinic mode. The stratification on the shelf is such that a first baroclinic mode semidiurnal internal tide has a theoretical wavelength between 14 and 21 km (depending whether or not it is the upwelling season). Despite the fact that the relation (1) between the wavenumbers is nearly satisfied, it seems difficult that such short waves will interact nonlinearly to give rise to 675 km wave.

Further measurements to study the generation and propagation of the internal tides are necessary to clarify relation between the internal tides and the  $M_{sf}$  signal.

#### 6. Concluding remarks

In this study we show that the half-monthly oscillations observed at seven tide gage stations along the northern coast of the Guinea Gulf are composed of two waves. One is the direct effect of the lunar fortnightly tide  $M_f$ . It has constant phase all along the coast and probably has a weak influence on the thermal structure. The other oscillation in the sea level has a period that corresponds exactly to that of the  $M_{sf}$  tide. It probably results from a nonlinear interaction between the  $M_2$ ,  $S_2$  (barotropic or internal) tides. (Strong internal tides with the exact frequencies  $M_2$  and  $S_2$  are observed on the shelf throughout the year.) The 14.7-day wave propagates all along the coast, with a mean phase speed of  $53 \text{ cm s}^{-1}$  and a wavelength of 675 km;

this phase speed does not vary with the stratification. This wave has an important effect on the thermal structure and gives rise to strong oscillations in a broad frequency band around the original frequency  $M_{sf}$ . The associated vertical oscillations of the subsurface isotherms are found to be present throughout the year (see Fig. 2). If this wave is coastally trapped then its presence throughout the year is consistent with the observation by Bakun, *et al.* (1973) of an offshore temperature maximum all the year round.

The amplitude of the sea surface temperature oscillations associated with this wave might reach 2.5°C in the upwelling season. This is to be compared with the 1.5°C oscillations induced by the atmospherically forced 40–50 day wave (Picaut and Verstraete, 1976).

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