The Interaction of Wind-Generated Sea Waves with Tidal Currents

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ABSTRACT

The interaction between wind-generated surface waves and tidal currents can be described in terms either of the energy balance of the system or of the conservation of wave action, assuming that the tidal currents are weak. Analytical solutions for the variation in surface wave amplitude and wavenumber are shown for the case where the surface waves are in deep water and are propagating parallel to the direction of tidal wave propagation, using the energy balance approach.

Wave analysis from two adjacent sites in the southern North Sea show that wave heights during two 16-day periods were modulated at a period of 12–13 h and that higher waves were occurring when the waves were propagating in the same direction as the tidal current. A simplified model for the tidal regime in this area was used to compute the theoretical modulation of the wave amplitude from the energy balance equations and these were compared with the observed wave heights. Very good correspondence was found between the phases of the modulations, but the observed wave height variations exceeded those predicted by over 50%. The limitations of this analysis and some possible causes of this underestimation are discussed.

1. Introduction

It has been well known among mariners for many centuries that the form of wind-generated surface waves is affected by the prevailing tidal conditions, and that changes in such wave fields are most marked in shallow waters, in channels with high tidal currents and in estuarine regions. What are less well known are the magnitude of such changes and the mechanisms controlling them. Several theories have been proposed to predict these changes but little experimental work appears to have been carried out to verify the predictions of these theories.

Two laboratory experiments have been undertaken (Francis and Dudgeon, 1967; Plate and Trawle, 1970) to investigate the generation of wind waves on flowing water but neither were designed to give information on the interaction between waves and a current that varied either with time or with distance. Neither experiment showed any alteration in the mechanisms by which waves were generated on the water surface; changes in the wave amplitude or wave celerity that did occur could be accounted for either by the change in the relative wind speed and fetch (Vincent, 1975), or by bottom-induced effects.

It can be argued in a very simple manner that the wind-generated surface waves in a region of significant tidal currents must inevitably exhibit some form of modulation at the tidal period because of the periodic change in the effective wind speed. Tominaga (1964a,b) went on to suggest that a resonance coupling also occurs between the wind and the current and that this enhances the waves at times of countercurrents (i.e., wind against currents). He developed this theory mathematically basing his work on the wave-generating mechanisms of Miles (1960) and Phillips (1957). The experimental work of Snyder and Cox (1966) found the Miles' and Phillip's mechanisms to be inadequate, by an order of magnitude, to account for the observed rate of growth of wind-generated gravity waves. However, this does not preclude the existence of such a coupling between wind and waves.

In a series of papers, Longuet-Higgins and Stewart (1960, 1961, 1964) made an exhaustive theoretical survey of the interaction between two surface waves. They included the concept of the radiation stress where the tidal wave transfers energy to the surface waves when the water surface is in a state of convergence, and, conversely, the surface waves transfer energy to the tidal wave when the water surface is diverging. This introduced an additional term into the energy balance equations of the system. The approach used is essentially a weak-current approach where the velocity field of the short waves is sufficiently small to be considered as a perturbation of the velocity field of the longer wave. The two velocity fields are assumed additive and the interaction appears as a second-order effect.

An example is given by Longuet-Higgins and Stewart (1960) for a typical situation of waves of period $T = 10$ s on a tidal stream of period 12.4 h

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in 100 m of water and they use a perturbation analysis technique in solving for the interaction. The fractional change in the amplitude of the surface waves when modified by their interaction with the tidal wave is given by

$$\frac{a_1'}{a_1} = 1 + a_2k_2\frac{3\mu - 2\lambda}{(2\mu - \lambda)^2}\sin(k_2x - \sigma_2t), \quad (1)$$

where $\lambda = \sigma_2/\sigma_1$ and $\mu = \tanh k_2h$.

The notation used follows that of Longuet-Higgins and Stewart (1960). Subscript 1 refers to the deep water surface waves and subscript 2 to the tidal wave. A prime indicates the parameter is modified by the wave-current interaction. a, k, σ and h are the wave amplitude, the wavenumber, the wave angular frequency and the water depth, respectively. Rectangular Cartesian coordinates are used throughout with waves propagating in the x direction. Expression (1) is valid only if $a_2k_1 \ll \tanh k_2h$ which, for the example given by Longuet-Higgins and Stewart (1960), requires the tidal wave amplitude a_2 to be very small. For the interaction theory to be generally applicable to the tidal waves and surface waves encountered in shallow seas, it must allow the tidal wave to have considerable amplitude.

This particular problem can be solved by considering the energy budget equation of Longuet-Higgins and Stewart (1961), i.e.,

$$\frac{\partial E_1'}{\partial t} + \frac{\partial}{\partial x}[E_1'(c_g' + U)] + S_x\frac{\partial U}{\partial x} = 0, \quad (2)$$

where $E_1' = \tfrac{1}{2}\rho g(a_1')^2$. E_1' and c_g' are the wave energy and wave group speed of the surface waves and both will vary depending on the position of the surface waves on the tidal wave; U is the speed of the tidal stream which, for simplicity, is in the same direction as the direction of wave propagation (the x direction); S_x is the component of the radiation stress tensor in the x direction; g is the acceleration due to gravity and ρ the water density. Vertical accelerations other than g are neglected.

The variation of wave speed c_1' is obtained directly from the conservation of waves on a current:

$$\frac{\partial k_1'}{\partial t} + \frac{\partial}{\partial x}[k_1'(c_1' + U)] = 0 \quad (3)$$

and from the dispersion equation for a deep water wave

$$c_1' = (g/k_1')^{1/2} \quad (4)$$

by eliminating k_1', giving

$$\frac{\partial c_1'}{\partial t} + \frac{c_1'}{2}\frac{\partial c_1'}{\partial x} + U\frac{\partial c_1'}{\partial x} - \frac{c_1'}{2}\frac{\partial U}{\partial x} = 0. \quad (5)$$

When, to a first approximation, the surface elevation of a progressive tidal wave has the form

$$\zeta_2 = a_2\sin(k_2x - \sigma_2t) \quad (6)$$

and this wave is in shallow water depth h, where $a_2/h \ll 1$, then

$$U = \frac{a_2\sigma_2}{k_2h}\sin\psi_2, \quad (7)$$

where $\psi_2 = k_2x - \sigma_2t$. A solution of the form

$$c_1' \approx c_1\left[1 - \frac{a_2\sin\psi_2}{2h(1 - c_g'/c_2)}\right] \quad (8)$$

can be shown to satisfy Eq. (5) providing (Vincent, 1975)

$$\frac{a_2\sin\psi_2}{2h(1 - c_g'/c_2)} \ll 1.$$

For deep water waves $S_x = \tfrac{1}{2}E_1'$ and $c_g' = \tfrac{1}{2}c_1'$ and (2) can be written

$$\frac{\partial E_1'}{\partial t} + \frac{\partial}{\partial x}[E_1'(\tfrac{1}{2}c_1' + U)] + \tfrac{1}{2}E'\frac{\partial U}{\partial x} = 0 \quad (9)$$

and, to the same order of approximation,

$$E_1' \approx E_1\left[1 + \frac{3a_2\sin\psi_2}{2h(1 - c_g'/c_2)}\right], \quad (10)$$

$$a_1'/a_1 \approx 1 + \frac{3a_2\sin\psi_2}{4h(1 - c_g'/c_2)}. \quad (11)$$

These results are very similar to those obtained by Longuet-Higgins and Stewart (1960) which restricted both waves to deep water. Eqs. (8) and (11) are valid when the long wave is a shallow water wave and allows the wave group speed to be comparable with the tidal wave speed.

Reversing the direction of propagation of the tidal wave essentially leaves the interaction unchanged, with the exception of the sign of the c_g'/c_2 term in the denominator, so that

$$\frac{a_1'}{a_1} \approx 1 + \frac{3a_2\sin\bar{\psi}_2}{4h(1 + c_g'/c_2)}. \quad (12)$$

The significance of the term $(1 \pm c_g'/c_2)$ is that it shows the difference in the wave amplitude modulation due to the differing lengths of time that the surface waves remain in regions of surface convergence or divergence, depending on the relative velocities of the two waves.

When the tidal wave is a standing wave or has a standing wave component, then treating the tidal wave as two progressive waves with amplitudes a_2 and b_2 traveling in opposite directions, the short wave amplitude due to the interaction will be

$$\frac{a_1'}{a_1} \approx 1 + \frac{3a_2\sin\Psi_2}{4h(1 - c_g'/c_2)} + \frac{3b_2\sin\bar{\Psi}_2}{4h(1 + c_g'/c_2)}. \quad (13)$$

The maximum short-wave amplitude is therefore to be found at the crest of the tidal wave. In contrast to the interaction between surface waves and a non-time-dependent current (where the surface waves increase in amplitude as they progress into a region of increasing countercurrent), the current speed does not determine whether the short waves will be amplified or attenuated. For example, when the surface waves are propagating in the same direction as a progressive tidal wave, then the maximum amplitude of the surface waves occurs at the crest of the tidal wave where the tidal current is in that same direction.

It is the rate of change of current speed with distance ($\partial U/\partial x$) that is important, as is also shown from the example of the interaction of surface waves with a standing tidal wave (Fig. 1), where $a_2 = b_2$. Maximum and minimum surface wave amplitudes occur at the crest and trough of the tidal wave (C and A in Fig. 1), where the current speed is zero for all time but where $\partial U/\partial x$ has its greatest and least values. Similarly, at the nodal points B and D where the current has its maximum, the short-wave amplitude is unmodified (strictly only when $c_g' \ll c_2$). At B and D $\partial U/\partial x = 0$, for all times.

From Eq. (6), it can be simply shown that the wavelength of the surface waves change, such that when the wave amplitude is at its maximum the wavelength is at a minimum. The surface waves thus become steeper at the crest of a tidal wave and this steepening often adds to the visual impression of larger waves. A 10% modulation of the surface wave amplitude, together with a similar order of magnitude wavelength modulation, would produce a 20% modulation of the wave steepness over a tidal cycle.

An alternative approach to this energy balance technique is to use the concept of the conservation of wave action (Whitham, 1965). This has many advantages when applied to moving media in general (Bretherton and Garrett, 1968; Hayes, 1970). The conservation of wave action implies that the wave action flux B of the surface waves will be constant along the tidal wave, i.e.,

$$B = \rho g a_1^2(U + c_g)/2\sigma_1. \tag{14}$$

For a weak current where all the changes are small, the change in the amplitude of the surface waves becomes

$$\frac{da_1}{a_1} = -(3U + 2c_1)dU/(2U + c_1)^2, \tag{15}$$

where U is the tidal wave speed c_2 and dU is the change in the tidal current speed (Peregrine, 1976). While Eq. (15) is much more applicable to the general problems of the interaction of deep water short waves with currents, for the description of the particular situation encountered in the southern North Sea, the energy balance technique was found more convenient to use.

A full and concise review of the general interaction between surface waves and water currents on most time and space scales is given by Peregrine (1976).

2. Wave observations from the North Sea

Two 16-day periods of data from wave records available from two sites in the southern North Sea (Fig. 2) were analyzed. The measurements were made using Baylor wave staffs mounted on gas production platforms in the Leman Bank field. The platforms, designated A and B, were situated near the eastern end of the complex region of long narrow banks to the northeast of the East Anglian coast. The banks form a series of sand ridges parallel to the coastline and are orientated approximately NW–SE. The major axis of the tidal current ellipse in this region is in the same direction. The sea floor depth is 35 m from which sand ridges rise to

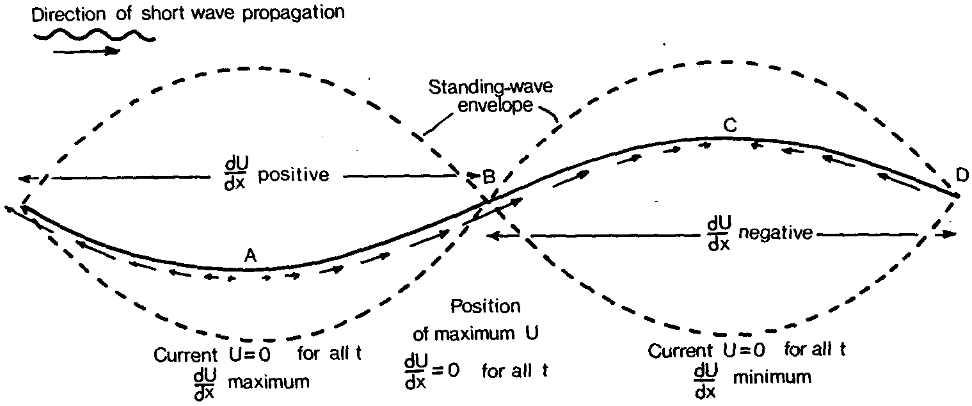

FIG. 1. The variations of current velocity U and the spacial rate of change of current velocity $\partial U/\partial x$ along a standing tidal wave. Surface waves propagating from the left are attenuated in the trough where $\partial U/\partial x$ is positive and are amplified on the crest where $\partial U/\partial x$ is negative. At the nodal points B and D where currents are at a maximum the surface wave amplitude is unchanged.

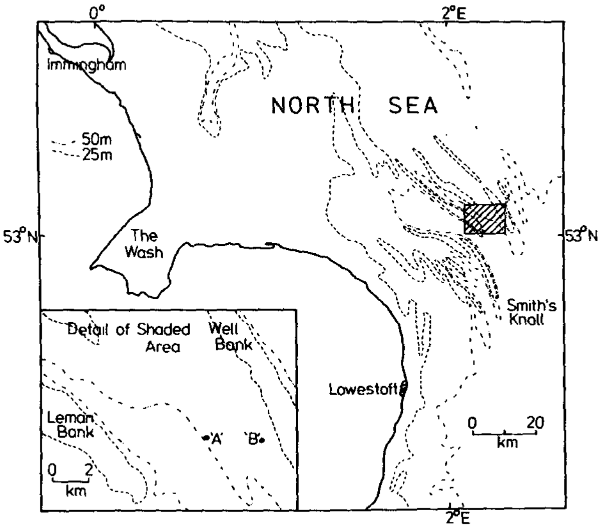

FIG. 2. The bathymetry of the southern North Sea showing the positions of the platforms A and B and the primary ports of Immingham and Lowestoft.

within 15 m of the surface at some points. Platform A is in 35 m of water, almost midway between the Leman and Well Banks; while B is in 28 m of water, 5 km away to the east, on the slope of the Well Bank.

Wave recordings were made at each platform every 3 h and each recording lasted 15 min. Output was in a graphical form and the paper charts required changing every 3 days. The 16-day periods that were chosen for analysis (1–16 June and 1–16 July 1969) were selected because of the calm-storm-calm wave sequence during each period. Recordings from platform A during 1–16 June period were not used because of a 44 h break in the records coinciding with the height of the storm. Current meter data were available for most of the two periods from Plessey internally recording current meters deployed from the platforms. Wind speed and direction were obtained from a Monroe recording anemograph on a nearby drilling rig.

Each 15 min recording of waves was processed using the statistical analysis method described by Tucker (1961), and the significant wave height $H_{1/3}$ of each record was computed from the root-mean-square (rms) wave height H_{rms} using $H_{1/3} = 1.41\ H_{rms}$ (Cartwright and Longuet-Higgins, 1956). During the storm period when the winds were strong and from the north, the values of H_{rms} appeared to be higher when the currents were toward the south. If the short-wave propagation direction is taken to be in the same direction as the wind which, although a good estimate at times of strong winds, is a doubtful assumption at times of light winds, then the highest waves were occurring at times of co-currents, i.e., wind and current in the same direction. This was tested statistically (Table 1) by dividing the waves into four classes depending on wind strength and then, within each class, taking the mean and standard deviation for the waves measured at times of co-currents and at times of countercurrents. In only the upper two classes were the differences significant in a difference of the means test at a 5% significance level.

The series of H_{rms} for each 16-day period constituted a time series with a sampling interval of 3 h and a fast Fourier transform (Cooley and Tukey, 1965) was used to obtain the distribution of energy through the frequency spectrum. As would be expected from the response of the sea to the wind, the majority of the energy was at low frequencies (Fig. 3), the energy dropping quickly to a trough at between one and two cycles a day. The major high-frequency peak was close to the semi-diurnal frequency and was the dominant feature of all three spectra. The spectra presented in Fig. 3 have been smoothed by Hanning once and have been scaled to show the high-frequency energy to more advantage.

3. Comparing observed with predicted wave heights

The tidal regime in the southern North Sea is a complex rotating system, the tidal wave moving counterclockwise around an amphidromic point in the Southern Bight. A full description of the tides, the subsequent application of the wave interaction theory, and the calculation of the amplitude of short

TABLE 1. Difference in the mean wave height for countercurrent (−) and co-current (+) conditions compared with the 5% level of significance in a difference of the means test.

	Wind speed							
	5–7 m s^{-1}		7–9 m s^{-1}		10–12 m s^{-1}		13–17 m s^{-1}	
	+	−	+	−	+	−	+	−
Number of observations	35	42	23	25	15	13	15	18
Mean wave height $\bar{H}_{rms}$	3.05	2.68	2.93	2.72	4.94	4.11	7.59	6.43
Standard deviation	1.04	0.96	1.16	1.01	1.22	0.96	1.52	1.50
5% level for difference of means	0.40		0.54		0.72		0.88	
$(\bar{H}_{rms})_+ - (\bar{H}_{rms})_-$	0.37		0.21		0.83		1.16	

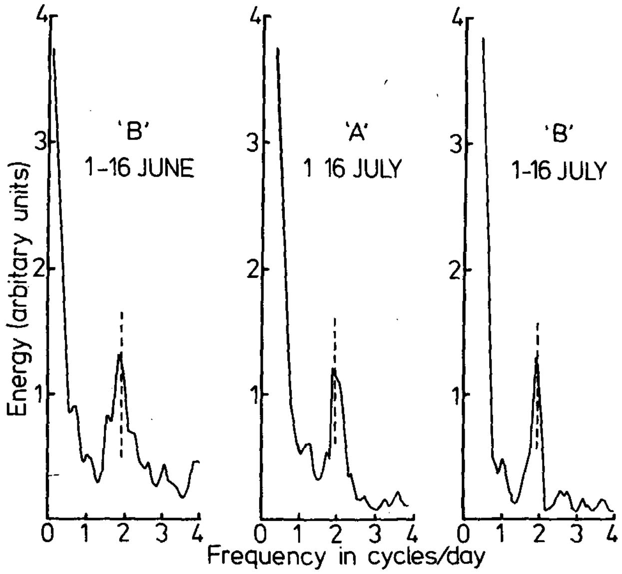

FIG. 3. Energy spectra from the time series of maximum wave heights for the periods 1–16 June (platform B only) and 1–16 July (platforms A and B).

waves moving into the Southern Bight from the north would require a large numerical model and a stepwise solution of the interaction. For such an approach, the wave action approach and Eq. (10) would be appropriate. However, a much more simplistic technique is to approximate the tides in the area concerned to a linear tidal system.

Consider the region of the southern North Sea close to the East Anglian Coast and from Immingham in the north to Lowestoft in the south (Fig. 2). A first approximation to the real tide can be made by considering Lowestoft, which has a relatively small tidal range, and Immingham, with a much larger tidal range, to be at the positions of minimum and maximum tidal amplitude caused by a linear southward traveling wave and its reflection traveling northward. Both Lowestoft and Immingham are primary ports, and the times of high and low water and the tidal range at each are thoroughly documented. Taking the mean tidal range at Immingham to be 6.5 m and that at Lowestoft to be 2 m, then the amplitude of the two tidal waves would be 2.1 m for the southward traveling wave and 1.1 m for the northward reflection.

This is at best a crude estimation, but for the region of the platforms A and B it seems a reasonable first approximation. The tidal currents calculated from this model compare well with the current data (Fig. 4).

Using this simple model it is now possible to calculate the maximum values of the normalized wave amplitude a_1'/a_1 and the normalized wavenumber k_1'/k_1 at every point along the tidal wave (Fig. 5). This varies from 1.1 and 1.13, respectively, at Immingham to 1.05 and 1.07 at Lowestoft, assuming a southward traveling surface wave with a 6 s period. At the position corresponding to the platforms A and B, a_1'/a_1 is 1.07 and k_1'/k_1 is 1.09. A wave 5 m in height would vary over a tidal cycle between 5.35 and 4.65 m. The relative times of occurrence of the maximum tidal height, current speed and interaction modulation are shown in detail for this site in Fig. 6. The amplified waves will occur at high water and when the tidal current is to mainly the south. The waveheights at A and B would be expected to show a ±7% variation over a tidal cycle, although the exact value will depend on the group speed of the short waves and on the direction of

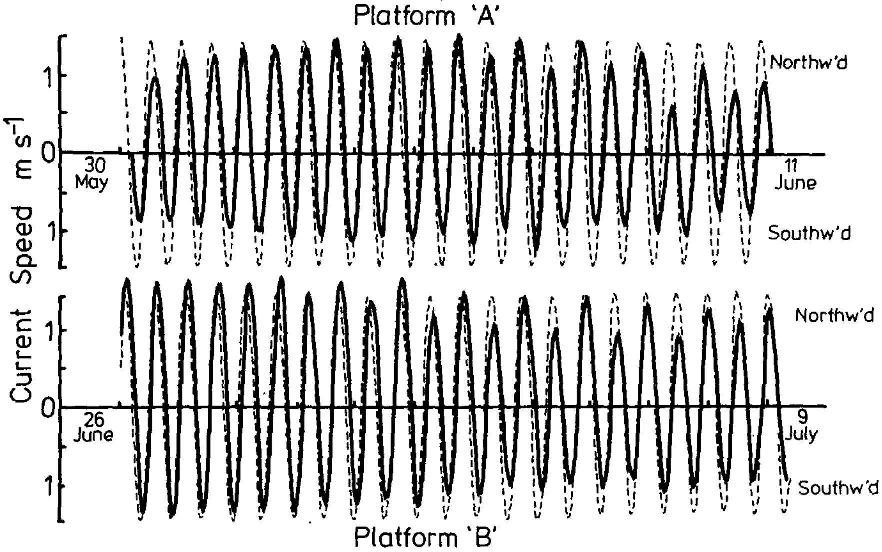

FIG. 4. A comparison between the current speeds measured 5 m above the sea bed at A and B and the current computed from the simplified tidal model.

the short-wave propagation relative to the axis of the rectilinear tidal system. For both storms, the winds were from the north and the short waves could be assumed to be propagating parallel to the tidal waves. The short-wave group speed was calculated from the zero-crossing period T_z taken from the statistical analysis of each record.

A direct comparison between the wave heights computed from this simple interaction model and the observed wave heights is difficult because of the large low-frequency component in the time-series of H_{rms}. The majority of this low-frequency variation must be the response of the wind waves to the wind field, and the low energy occurring at between 1 and 2 cycles per day (cpd) suggests that the majority of the wave height variations due to the wind are on time scales greater than the tidal period. This was confirmed by spectral analyses of the wind velocities which showed no significant peaks (at the 95% level) in the power spectra at periods of less than the duirnal period. For these analyses, both the easterly component and the northerly component of the wind velocity were analyzed each with a 3 h sampling interval and a record length of 16 days over the periods of the wave measurements. Each of the three time-series of observed wave heights was therefore filtered using a high-pass filter with a cutoff about 1.5 cpd, leaving a time series H^*_{rms} dominated by the variations with a period of around 12–13 h.

The wave heights H^*_{rms} are compared with the wave height variations H computed from the interaction equation (8) for the first storm period (June) in Fig. 7 and for the second storm period (July) in Fig. 8. Each period covers the 5 days around the height of the storm, and in Fig. 8 the values of H^*_{rms} shown are the average from A and B.

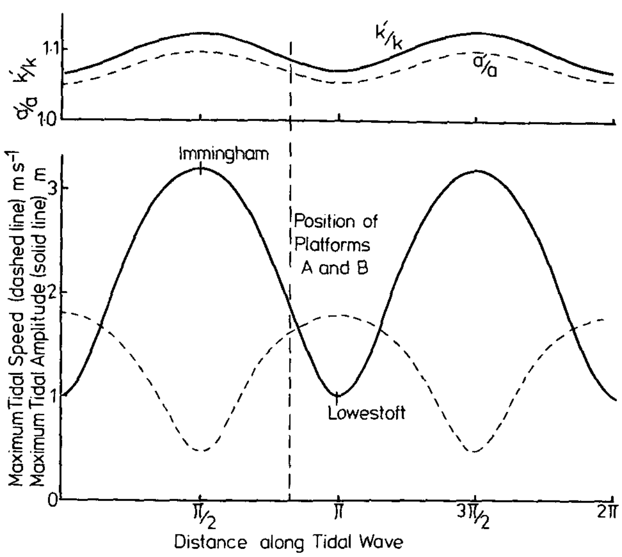

FIG. 5. Maximum values of the normalized surface wave amplitude a_1'/a_1 and wavenumber k_1'/k_1 along the simplified linear tidal wave system between Immingham and Lowestoft.

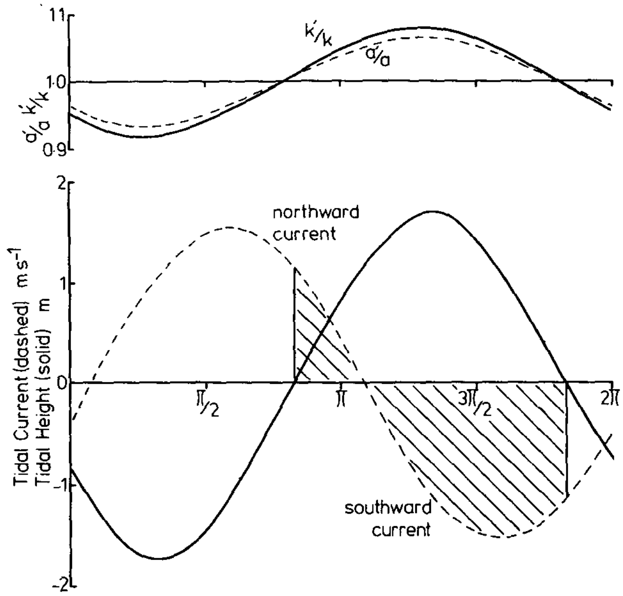

FIG. 6. The variation of the normalized surface wave amplitude a_1'/a_1 and wavenumber k_1'/k_1 with time at sites of A and B, showing the tidal current and tidal wave height computed from the linear model.

The observed wave heights clearly show that the strong tidal modulation corresponds well in phase with the wave height variations predicted by the simple wave-tide interaction model. The magnitude of the observed modulation, however, is greater than that predicted by ~50%. Regression analysis on the relationship between H^*_{rms} and H shows that, for the June period

$$H = 0.47\, H^*_{rms} - 0.095$$

and for the July period

$$H = 0.76\, H^*_{rms} - 0.173.$$

The correlation coefficients were 0.63 and 0.67, respectively, well above the level of 0.354 for significant correlation at the 1% level.

4. Discussion

The tidal model used for the southern North Sea was only approximate and this is clearly a source of error and may go some way accounting for the discrepancy between the observed and the predicted wave amplitude modulation. In particular, to the north of the site of A and B the match of the linear tidal model to the real tide becomes progressively worse. The storm waves, propagating from the north with a group velocity of ~5 m s^{-1}, will have traveled nearly 250 km in the previous 12 h and will have been modulated during this time by a tidal wave field very unlike the model.

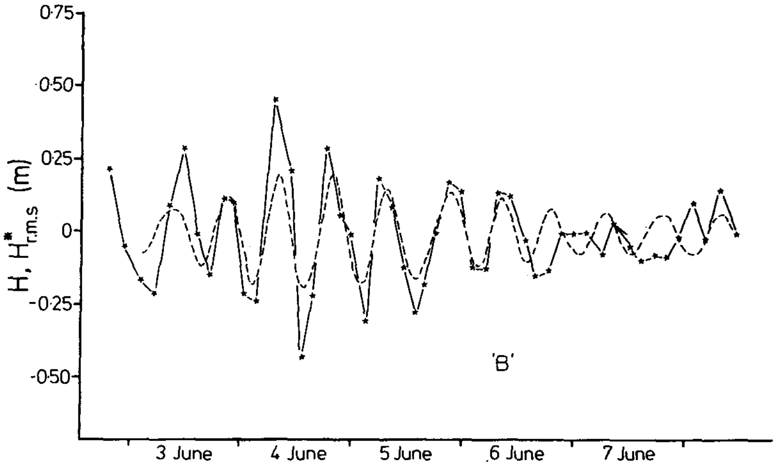

FIG. 7. The maximum wave heights H^*_{rms} (after low frequencies have been filtered out), shown by stars, and the maximum wave height variation predicted from the simplified tidal model and the energy balance equation (dashed line), for the 5-day period 3–7 June. Data from B only.

It is important to appreciate that the tidal modulation at one point (fixed with respect to the earth) is nonlocal and represents the sum of all the tidal influences along the entire path of propagation of the short waves. A full determination of the tidal modulation must therefore involve solving the radiative transfer equation for a large area of the southern North Sea using a spacially dependent tidal wave field.

Additionally, there is a further factor which may influence the waves measured at these sites and that is the bottom topography. There are indications that there is a systematic difference in the wave heights between A and B during the northerly storm periods which cannot be accounted for by instrumental error. $(H_{rms})_B \approx 1.2\ (H_{rms})_A$ relates wave heights during the northerly storms, while southerly storms show negligible difference. This could be due to refraction of waves passing the nearby Well Bank and causing wave convergence or divergence in the vicinity of the platforms, the degree of the modification varying slightly between A and B.

The bottom topography is also believed (Huthnance, 1973) to be responsible for the asymmetry of the currents in this area with currents flowing such that the residual tidal component is northerly on the southwest faces of the Banks and southerly on the northeast faces. The horizontal shear that this produces also results in the refraction of the short waves (Kenyon, 1971) and this may cause a modulation in the wave heights at the tidal period.

The interaction between short wind-generated waves is very complex to describe completely, requiring the accurate solution of the radiative transfer equations, or of the equations of the conservation of wave action, over an extended area of space-dependent tidal currents, combined with equations of geometrical optics. The results shown here indicate that a simple approach to the problem can show partial success, producing wave modula-

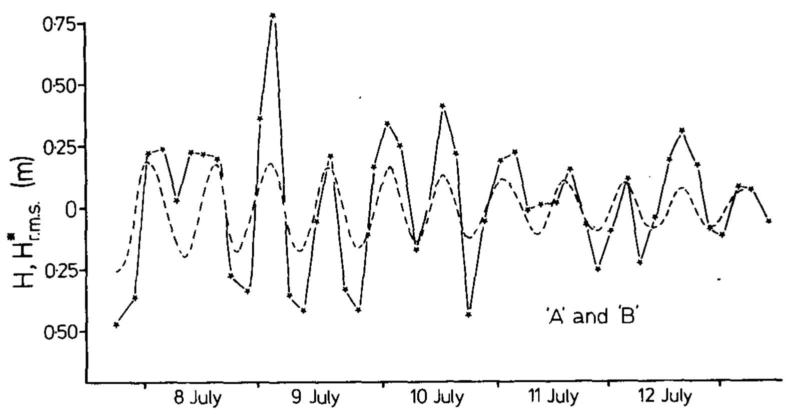

FIG. 8. As in Fig. 7 except for the 5-day period 8–12 July. Data from A and B.

tion with the correct phase, but with amplitude of the observed modulation considerably in excess of that predicted.

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