

## Conservation Calculations in Natural Coordinates (with an Example from the Baltic)

GARY SHAFFER

*Institute of Oceanography, University of Gothenburg, Sweden*

(Manuscript received 30 May 1977, in final form 12 October 1978)

### ABSTRACT

Conservation expressions for a fluid property  $\phi_i^*$  (e.g., salt, heat) and mass are derived for a semi-enclosed, stratified region as functions of a concentration  $\phi_i$  (e.g., salinity, temperature) and time. A method is developed for calculating the mean advective and diffusive cross-isosurface transports of  $\phi_i^*$ ,  $\bar{A}d(\phi_i)$  and  $\bar{F}(\phi_i)$  from time-averaged "local" fluxes at points on a vertical control plane at the opening of the region. This is a particularly useful form for using automatic recording instrument data. As an example of this type of calculation, cross-isohaline advective and diffusive salt transports were calculated from recording current meter and synoptic profiling measurements in a small coastal canyon in the northwest Baltic proper. Certain requirements for the use of this method are discussed in the light of the steps taken and assumptions made in the example.

### 1. Introduction

Knowledge of distribution and intensities of vertical exchange is the key to many problems of theoretical and practical importance within oceanography. These may range over a large spectrum of time and space scales from, for example, the local problem of pollutant concentration resulting from a subsurface outlet to the oceanic thermocline problem (e.g., Welander 1971). Actually it is more meaningful physically to talk about exchange across and along isopycnal surfaces rather than vertical and horizontal exchange since density stratification forms a barrier for fluid exchange. With no mixing across isopycnal surfaces, parcels of fluid are constrained to move along them. Such mixing can only occur when and where sufficient energy exists to overcome the potential energy of the density stratification and feed the turbulence. The density  $\sigma_t$  is thus a natural physical choice as an independent variable, particularly in regions of large horizontal density gradients. The use of vertical and horizontal (with respect to geopotentials) turbulent exchange coefficients may lead to ambiguous results (Veronis, 1977).

The common method for estimating turbulent fluxes in the ocean is to multiply concentration gradients by turbulent diffusion coefficients, which may be estimated using tracer techniques, energy arguments, etc. (Rooth and Ostlund, 1972; Turner, 1973). Fluxes estimated in this way are highly approximate. Even if the diffusion coefficients were representative of the turbulent intensity and reasonably well determinable, turbulence in strati-

fied fluids as well as the instantaneous concentration gradients are highly variable in space and time (Turner, 1973).

Walin (1977) has presented an alternative approach based on salt and mass conservation to calculate mean cross-isohaline advective and diffusive fluxes in estuaries, i.e., a description in the natural coordinate, salinity  $S$ . Information on the distribution of the salty inflow as a function of  $S$  averaged over a sufficiently long time and the area occupied by each isohaline within the estuary form a sufficient data set for calculating these fluxes.

In the present paper a modified version of this approach is developed for arbitrary substances and semi-enclosed regions and methods of calculating fluxes in natural coordinates from standard data are discussed, in particular, automatic recording instrument data. As an example of this type of calculation, cross-isohaline advective and diffusive transports of salt are estimated from recording current meters (RCM's) and profiling data in a small coastal canyon in the Baltic and compared with calculations from the whole Baltic. Finally the reasoning and assumptions on which the results depend are discussed critically in the spirit of formulating criteria for the method's satisfactory application in general.

### 2. Conservation relations

#### a. Basic formulations

We consider a semi-enclosed region containing a stratified fluid "bounded" from the fluid by a verti-

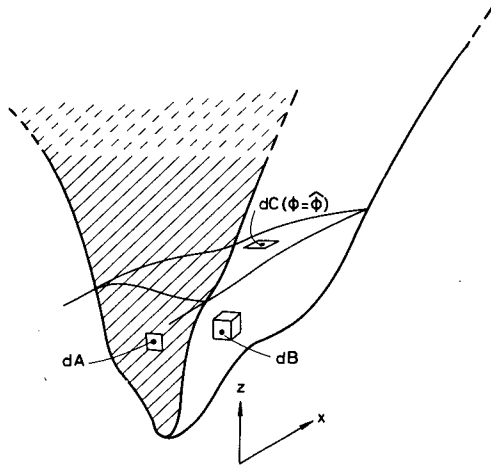


FIG. 1. Sketch of a possible semi-enclosed region to which the conservation relations developed in Section 2 could be applied.

cal conceptual plane  $A$  and containing a volume  $B$  (Fig. 1). At any moment the transport of a fluid property  $\phi_i^*$  (salt, heat, dissolved oxygen, etc.) up through  $C_j$ , a surface of constant  $\phi_j$ ; the concentration of  $\phi_j^*$ , i.e., salinity, temperature, etc., is equal to the inflow of  $\phi_i^*$  through  $A$  under  $C_j$  minus the time rate of change of the total amount of  $\phi_i^*$  in  $B$  under  $C_j$  plus the production of  $\phi_i^*$  in  $B$  under  $C_j$ . Using algebraic symbols, we have

$$H = I - \frac{\partial J}{\partial t} + Q. \quad (1)$$

Since (1) holds for every equiscalar surface of  $\phi_j$  contained within the semi-enclosed region at any time, the variables in (1) can be considered continuous functions of  $\phi_j$  and  $t$ . After averaging (1) over the period  $\tau$ , we have

$$\bar{H}(\phi_j) = \bar{I}(\phi_j) - \frac{\Delta J(\phi_j)}{\tau} + \bar{Q}(\phi_j), \quad (2)$$

where  $\Delta J$  is the change in storage between the beginning and end of  $\tau$  and the overbars denote time averaging.

Eq. (2) may be simplified under certain conditions. Of perhaps greatest practical interest is the case when

$$\tau \gg \tau_0(\phi_j) \equiv \Delta J/\bar{I} \quad \text{and} \quad \bar{I} \gg \bar{Q},$$

the quasi-steady, quasi-conservative case. Then

$$\bar{H}(\phi_j) \approx \bar{I}(\phi_j). \quad (3)$$

Volume conservation which follows from mass conservation in our Boussinesq fluid may be written

$$H_w = I_w - \frac{\partial J_w}{\partial t}. \quad (4)$$

Averaging over  $\tau$  gives

$$\bar{H}_w(\phi_j) = \bar{I}_w(\phi_j) - \Delta J_w(\phi_j)/\tau \quad (5)$$

and in the quasi-steady case

$$\bar{H}_w(\phi_j) \approx \bar{I}_w(\phi_j). \quad (6)$$

The total cross-equiscalar surface transport consists of an advective part,  $Ad$ , and a diffusive part  $F$ . For  $\phi = \phi_i = \phi_j$ , the advective transport of  $\phi^*$  through  $C$  is equal to the volume transport times  $\phi$  on  $C$ , or

$$Ad(\phi) = H_w(\phi) \cdot \phi. \quad (7)$$

The diffusive transport is then defined as the total transport minus the advective transport or

$$F(\phi) = H(\phi) - H_w(\phi) \cdot \phi. \quad (8)$$

Instantaneous fluxes are obtained by dividing by the area each  $C_j(\phi_j)$  covers at any moment in the semi-enclosed region.

Time-averaged transports are obtained from (7) and (8) in a straight-forward manner. The time-averaged fluxes can be obtained in a similar manner but often may have to be estimated from mean transports and areas. Then some estimates of the correlation between instantaneous transports and areas are needed.

For  $\phi_i \neq \phi_j$ , weighted means of  $\phi_i$  over  $C_j$  (and  $\tau$ ) are needed to calculate the instantaneous (and mean) transports using expressions like (7) and (8) (see Walin, 1977). These weighted means should take the effect of various correlations (which occur in this case) into consideration, thus requiring, in general, extra information on the dynamics and distributions within the semi-enclosed regions.

Under the assumption that  $\phi_i^*$  and  $\phi_j^*$  are transported by the same velocity and turbulence fields, we may write

$$\left. \begin{aligned} Ad_{\phi_i^*}(\phi_j) &\approx Ad_{\phi_j^*} \left[ \frac{[\phi_i]}{\phi_j} \right] \\ F_{\phi_i^*}(\phi_j) &\approx F_{\phi_j^*} \left[ \frac{\partial[\phi_i]}{\partial\phi_j} \right] \end{aligned} \right\}, \quad (9)$$

where  $[\phi_i]$  is the appropriate weighted mean of  $\phi_i$  over  $C_j$ .

#### b. Choice of variables

According to the general formulation many combinations of transport of a fluid property as a function of a "natural" coordinate are possible. In real situations one is interested, however, in those combinations which are useful to the problem at hand, physically meaningful and measurable. As we have seen the choice of  $\sigma_t$  as an independent variable is well motivated physically. Temperature and salinity are also appropriate where one or the other determine the density field alone.

The variable  $\phi_i^*$  may be oxygen or nutrients in an ecological study. Since such properties are nonconservative and may be expected to vary over isopycnal surfaces, extra information on their distributions and that of the transport processes are needed leading to the uncertainty of the results. Clearly, when the nature and intensity of the transport processes are to be studied, autotransport calculations ( $\phi_i = \phi_j$ ) give the best results.

*c. Methods of calculation*

Data for studying real systems are usually collected in geopotential space. Methods are needed for transforming such information to obtain good estimates of the variables in  $\phi_j$  space. Petrén and Walin (1976) estimated the function  $M(S)$  [our  $I_w(S)$ ] for salty water flow into the Baltic through the Bornholm Straits (Fig. 2) by averaging over a number of instantaneous  $M(S)$  from synoptic current and salinity sections across the strait. An alternative approach is to first take time averages and then space averages. Eq. (2) may be expressed

$$\begin{aligned} \bar{H}_{\phi_j}(\phi_j) &= \iint_A \frac{1}{n} \sum_{k=1}^n \hat{u}_k \hat{\phi}_{ik} \Gamma(\hat{\phi}_{jk} - \phi_j) dA' - \frac{1}{n \Delta t} \\ &\times \iiint_B [\hat{\phi}_{in} \Gamma(\hat{\phi}_{jn} - \phi_j) - \hat{\phi}_{i1} \Gamma(\hat{\phi}_{j1} - \phi_j)] dB' \\ &+ \iiint_B \frac{1}{n} \sum_{k=1}^n \hat{q}_{ik} \Gamma(\hat{\phi}_{jk} - \phi_j) dB', \quad (10) \end{aligned}$$

where  $n \Delta t = \tau$ ,  $\Delta t$  being the time interval of data collection.  $\hat{u}(\mathbf{x}, t)$  is the velocity normal to  $A$ , positive inward,  $\hat{\phi}_{i,j}(\mathbf{x}, t)$  are the distributions of  $\phi_{i,j}$  in geopotential space,  $\hat{q}_i(\mathbf{x}, t)$  is the distribution of the local production rate of  $\phi_i^*$ , and  $\Gamma$  is a stepfunction such that  $\Gamma = 1$  for  $\hat{\phi}_j \geq \phi_j$  and 0 otherwise. A similar expression for (5) can be obtained by setting  $\hat{\phi}_i = 1$  and  $\hat{q}_i = 0$  in (10).

Note that  $A$  and  $B$  are independent of time since they are taken to be bounded above the highest excursion of that  $C_j(\phi_j)$  just still contained in that semi-enclosed region during  $\tau$ . This therefore limits the range of  $\phi_j$  which can be considered.

In practice a choice between these two approaches to estimate the variables in  $\phi_j$  space could be made based on the characteristics of space versus time variation in the study region and the resources and material available. Clearly, complementary information on the time (space) variations of the dominant modes of variability will be needed in general if the first (second) approach is chosen.

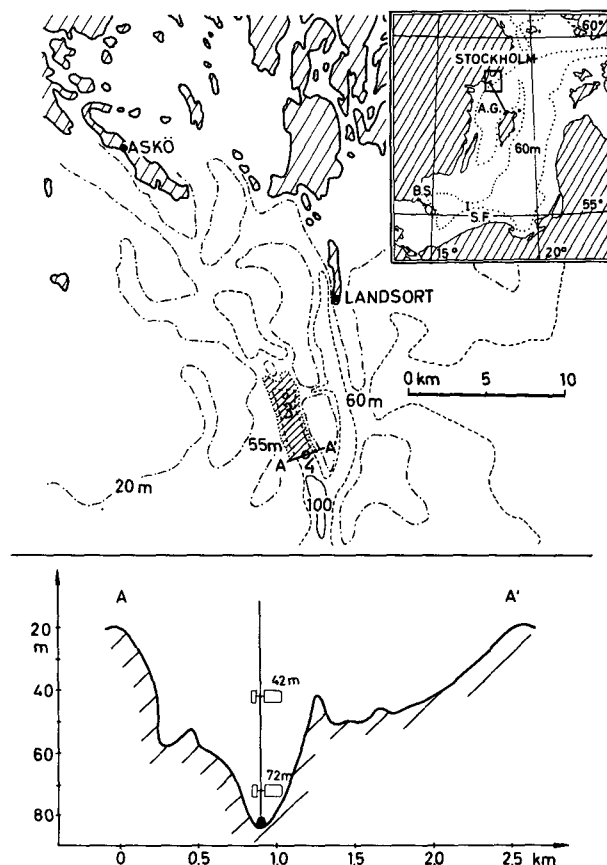


FIG. 2. The study area in the Northwest Baltic proper. The hatched region is the coastal canyon considered in our calculations. Station 4 is the location of the RCM mooring, stations 3 and 4 the location of the synoptic velocity and salinity profiles. B.S., S.F. and A.G. are the Bornholm strait, Stolpe Furrow and Askö-Gotland sections, respectively.

**3. An example—Cross-isohaline salt fluxes in a coastal canyon**

*a. Motivation and field program*

The Baltic as a coupled estuarine-large basin system is quite unique. The estuarine circulation driven by freshwater outflow and mixing results in a permanent salt stratification. This is characterized by a halocline found in the mean between 60 and 80 m depth and salinities between 7.5 and 10.0‰ within which density is determined entirely by salinity.

The dynamics in a large basin like the Baltic are greatly influenced by the earth's rotation (e.g., Csanady, 1975). Coastal jets and large halocline displacements were found near the coast in the Baltic during a several-year coastal dynamics field program by the author. An example showing these features is presented in Figs. 3a and 3b (see Fig. 2 inset for location of the section).

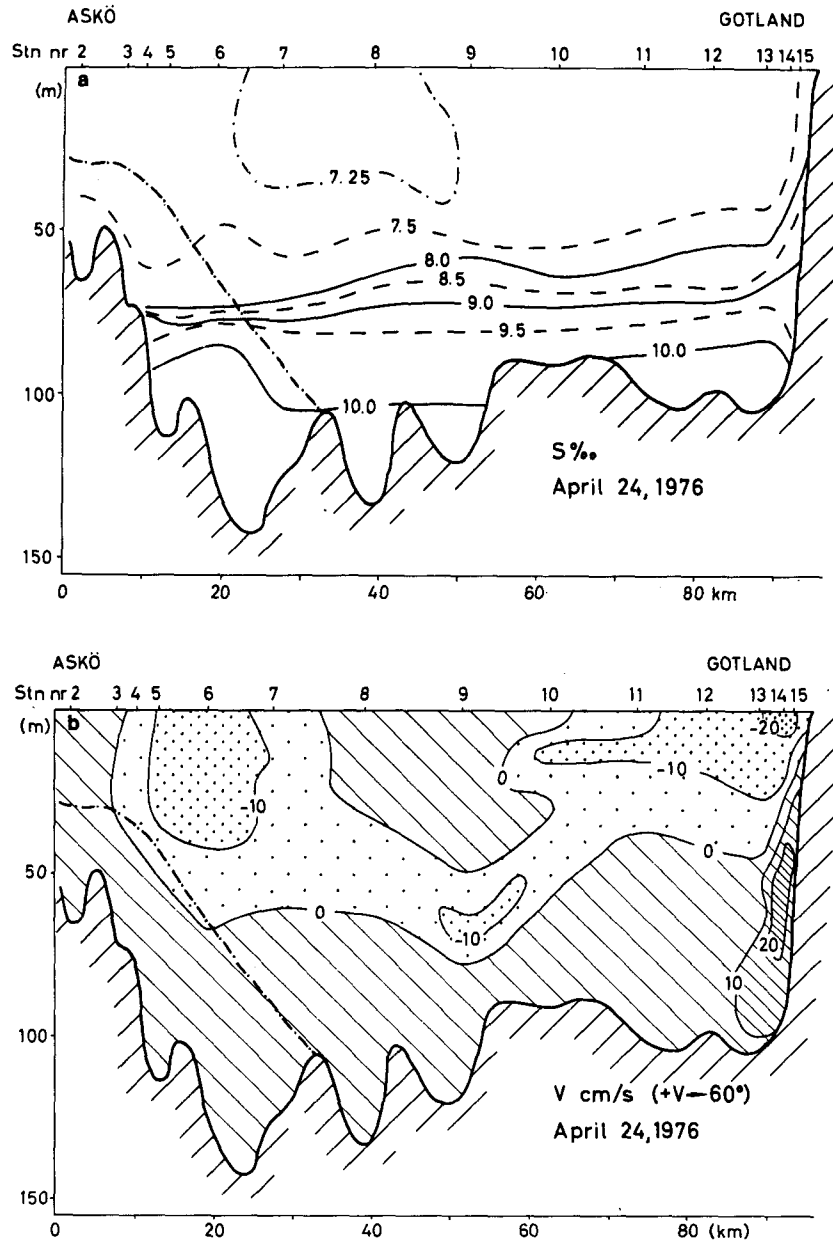


FIG. 3. Synoptic section of longshore velocity (a) and salinity (b) between Askö and Gotland. The section was run in 17 h, and was preceded by several days of moderate winds from the northeast.

These results led us to suspect that intense cross-isohaline exchange might be present in the coastal zone, thus playing a large role even in the estuarine circulation. Therefore, we wished to obtain good quantitative estimates of this exchange—in spite of the difficulties presented by the large vertical halocline displacements near the coast—to compare with mean exchange over the whole Baltic. With this in mind the formulations in Section 2 were developed and the following modest field program undertaken.

During the spring of 1976 two Aanderaa RCM4 recording current meters were moored in a small current canyon at 42 and 72 m depth in 83 m of water in the northwest Baltic proper (station 4 in Fig. 2). Each instrument measured mean speed, instantaneous direction, conductivity and temperature at 15 min intervals. During the intensive study period (28 March–6, May, henceforth, to be called P) eight synoptic sections of temperature *T*, salinity *S* and current (*V*) were run along the axis of the canyon

including stations 3 and 4 (Fig. 2). One of these sections was shown in Fig. 3.  $T$  and  $S$  profiles were obtained from CTD data.  $V$  profiles were obtained with strings of pendulum current meters (Haamer, 1974). The canyon studied had closed depth contours inshore of AA' at depths of 55 m or more.

*b. Calculations of cross-isohaline transports*

We will consider the mean cross-isohaline flux of salt, i.e.,  $\phi_i = \phi_j = S$ . In (2) then we have  $\bar{Q}(S) = 0$  in our subsurface region. We assume that  $\tau \gg \tau_0(S)$  in the canyon over P which will be checked *a posteriori*. Then (3) and (6) hold and from (7), (8) and (10) we may write

$$\bar{A}d(S) \approx \bar{I}_w(S) \cdot S = \iint_A \bar{g}dA', \quad (11)$$

where

$$\bar{g}(S, x) \equiv \frac{1}{n} \sum_{k=1}^n \hat{u}_k S \Gamma(\hat{S}_k - S)$$

and

$$\bar{F}(S) \approx \bar{I}(S) - \bar{I}_w(S) \cdot S = \iint_A \bar{h}dA', \quad (12)$$

where

$$\bar{h}(S, x) = \frac{1}{n} \sum_{k=1}^n \hat{u}_k (\hat{S}_k - S) \Gamma(\hat{S}_k - S).$$

Thus, from the distribution of the "local" mean salt influxes,  $\bar{g}$  and  $\bar{h}$ , on  $A$  one can calculate  $\bar{A}d$  and  $\bar{F}$  in the quasi-steady state;  $\bar{g}$  and  $\bar{h}$  were calculated from the RCM data ( $\Delta t = 15$  min) at 42 and 72 m in the canyon over P (Figs. 4a and 4b, solid lines). The value of  $\bar{g}$  reflects net outflow at low salinities and inflow at high salinities at both depths. Note that for reversible wave motion (e.g., internal Kelvin waves)  $\bar{g}, \bar{h} = 0$  at all  $S$  and  $x$ . Whereas we need the "local" influxes at all points on  $A$  to be able to calculate the mean cross-isohaline transports, we have only measured the influxes at two. To proceed farther we must use additional information and physical reasoning to estimate the distributions of  $\bar{g}$  and  $\bar{h}$  on  $A$ . We first look at the characteristics of the flow measured by the RCM's and then at the synoptic profiling data.

Fig. 5 shows 14 h (inertial period) mean values of current and salinity from our RCM data. Daily mean values of wind-stress components normal and parallel to the coast are also presented based on 6 h observations at Landsort's lighthouse (Fig. 2). The wind-stress record is dominated by "events" of several days to one week duration. Current and salinity appear to respond as a rule as might be expected from simple Ekman upwelling-downwelling theory: onshore subsurface flow and high

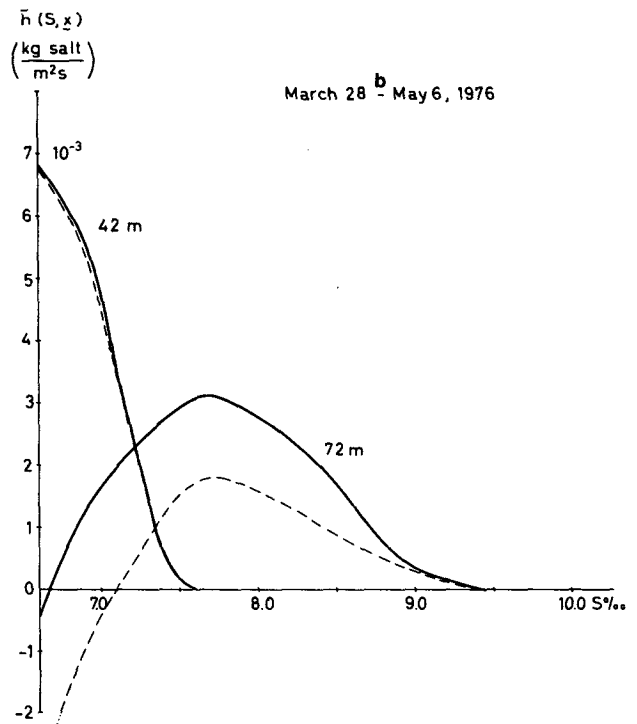
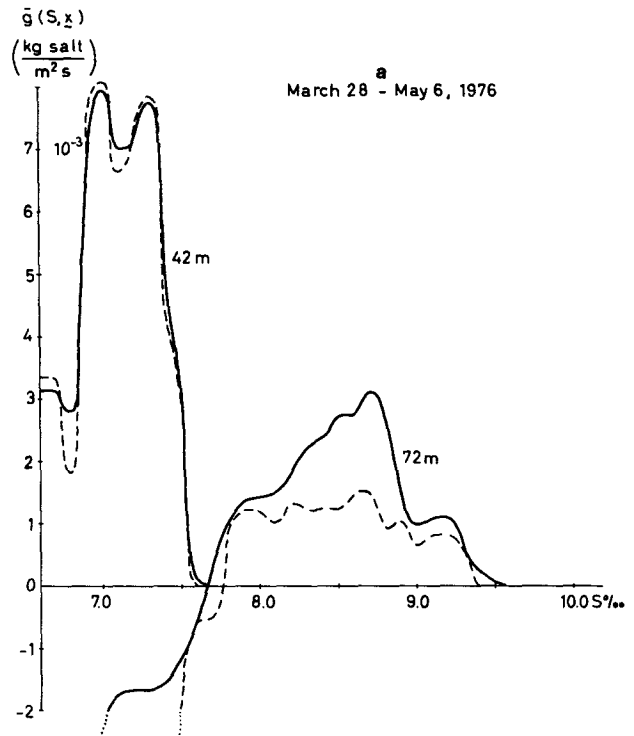


FIG. 4. Local time-averaged salt fluxes  $\bar{g}$  (a) and  $\bar{h}$  (b) (see text) from RCM data at 42 and 72 m in the coastal canyon over P, 28 March-6 May 1976. ( $+u \rightarrow 347^\circ$ ). The solid line represents fluxes calculated from instantaneous data, the dashed line from 14 h mean data.

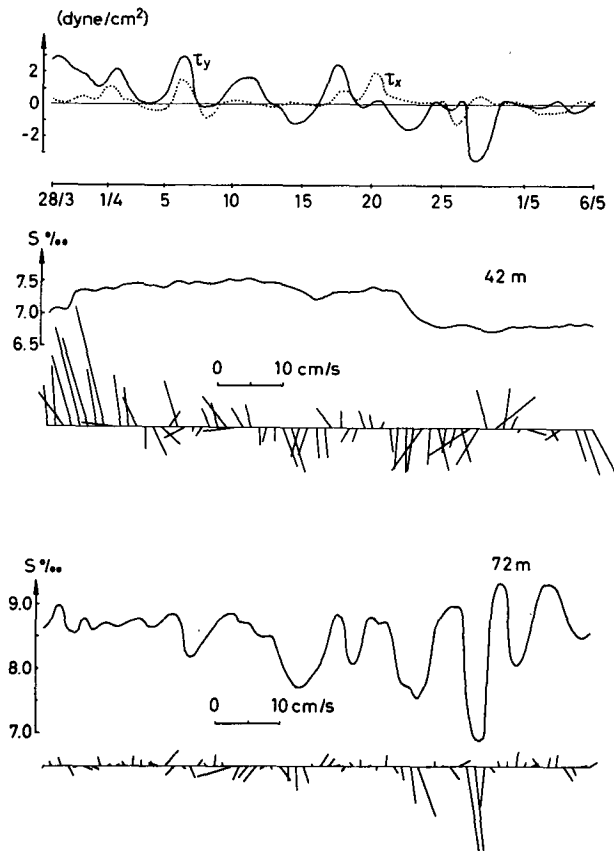


FIG. 5. 14 h mean values of salinity and velocity from the RCM data at 42 and 72 m in the coastal canyon over P. Also included are daily mean wind stress components calculated from 6 h measurements at Landsort's lighthouse.

salinity associated with offshore Ekman transport and vice-versa.

Whatever the nature of the dynamics leading to these low-frequency (LF) velocity and salinity fields, it is of interest to study their contributions to the total "local" salt influxes. Values of  $\bar{g}$  and  $\bar{h}$  were recalculated using products of these 14 h mean values (Figs. 4a and 4b, dashed lines). The LF contributions appear to account for most of the total, although the high-frequency contributions are significant at 72 m. We will use these LF results in the following.

Since the width of the canyon ( $\sim 0.5$  km) is much less than the local internal radius of deformation ( $\sim 5$  km), the LF motions may be expected to occur as channel-type flows. We therefore assume that LF  $\bar{g}$  and  $\bar{h}$  are horizontally uniform on  $A$ .

Some information on the vertical variations of  $u$  and  $S$  during P can be obtained from the synoptic profiling data at station 4. In Fig. 6 the eight instantaneous  $S$  profiles are presented which fall into two distinct categories: linear structure (Fig. 6a, upwelling case?) and step structure (Fig. 6b, down-

welling case?). The highest salinity at 55 m is about 8‰ which is thus chosen as the lower bound for  $S$  in our subsequent calculations.

We also present  $U$  as a function of salinity for each of these two categories (Figs. 6c and 6d). The form of these profiles, although instantaneous and therefore expected to be "noisy", also tends to differ in the two cases.

For the "upwelling" case (Fig. 6c)  $u$  between 42 and 72 m might be approximated as a linear function of salinity. In the "downwelling" case (Fig. 6d), strong currents are found just at the upper edge of the salinity step which fall off rapidly to small constant values for  $S \geq 8.0$ ‰, our range of interest. In addition we note that the salinity at 72 m can be used to indicate an upwelling or downwelling situation. Guided by these results we assume the following values for the vertical dependencies of  $U$  and  $S$ :

- (i) for  $\bar{S}(72 \text{ m}) \geq 8.6$ ‰, linear interpolation (with extrapolation) between  $\bar{U}, \bar{S}(42 \text{ m})$  and  $\bar{U}, \bar{S}(72 \text{ m})$ .
- (ii) for  $\bar{S}(72 \text{ m}) < 8.6$ ‰, linear interpolation (with extrapolation) between  $\bar{U}, \bar{S}(72 \text{ m})$  and  $U_0, S_0(82 \text{ m})$  taken to be 0 and 9.5‰, respectively.

Values of  $\bar{g}_i, \bar{h}_i(Z_i, S \geq 8.0$ ‰) have been calculated using (11) and (12) from the 14 h mean RCM data interpolated in this way at 5 m vertical intervals. Finally, the cross-isohaline transports have been estimated using

$$\bar{A}d, \bar{F}(S) \approx \sum_{i=1}^p \bar{g}_i, \bar{h}_i(S, Z_i) A_i(Z_i), \quad (13)$$

[where  $A_i$  is the partial area calculated from section AA' (Fig. 2) for each depth interval] and are presented in Figs. 7a and 7b (solid lines).

From these results and the initial and final salinity distributions reconstructed from RCM and profiling data, we find  $\tau_0(S) \leq 0.2\tau$  over P and our range of  $S$ . The quasi-steady assumption on which these calculations are based appears justified. The start and end of P were in fact chosen such that  $S$  distributions were as similar as possible, minimizing  $\Delta J$  and thus  $\tau_0$ .

To test the sensitivity of our results, we tried two other possible choices for the vertical dependencies of  $U$  and  $S$ . First, we assumed linear interpolation (and extrapolation) of instantaneous ( $\Delta t = 15$  min)  $U$  and  $S$  between 42 and 72 m for all  $S$  (Figs. 7a and 7b, dashed line). Second, we assumed for  $\bar{S}(72 \text{ m}) \geq 8.6$ ‰ linear interpolation (with extrapolation) between  $S_0(42 \text{ m})$  and  $\bar{S}(72 \text{ m})$ , where  $S_0(42 \text{ m}) = 7.4$ ‰ and for  $\bar{S}(72 \text{ m}) < 8.6$ ‰ linear interpolation (with extrapolation) between  $\bar{S}(72 \text{ m})$  and  $S_0(82 \text{ m})$  with  $S_0(82 \text{ m}) = 9.5$ ‰. At all  $S$ ,  $\bar{u} = \bar{u}(72 \text{ m})$ . In this case (Figs. 7a and 7b, dotted line) only RCM data at 72 m and profiling data were used, not RCM data at 42 m, above the "rim" of the

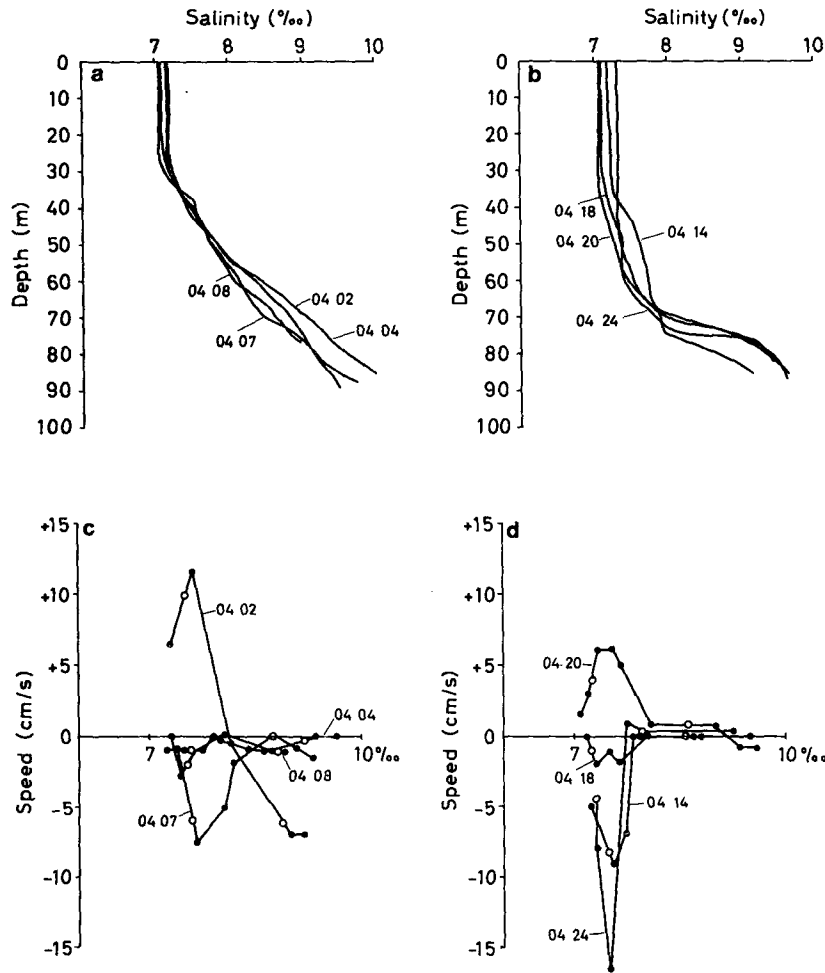


FIG. 6. Synoptic profiles of salinity (a,b) and onshore velocity (c,d) at station 4 in the coastal canyon over P. (+u → 347°). The open circles in Figs. 6c and 6d represent the values at 42 and 72 m for each profile.

canyon. For  $S < 8.5\text{‰}$  the results diverge greatly but for  $S \geq 8.5\text{‰}$  they only vary by at most a factor of 3.

c. Mean cross-isohaline fluxes

We can now estimate mean cross-isohaline advective and diffusive fluxes by dividing the mean transports by the mean areas occupied by each isohaline surface over P. Since this step may be expected to introduce added uncertainty (see Section 2) we will limit ourselves to calculating the fluxes at  $S = 8.5\text{‰}$ . From the profiling sections, the depth to the  $8.5\text{‰}$  isohaline was never found to differ by more than 2 m between stations 3 and 4 (Fig. 2). From the RCM and profiling data we estimate  $\bar{Z}(8.5\text{‰}) = 69$  m. Then from our knowledge of the detailed depth distribution in the canyon from a bathymetric survey we obtain  $\bar{C}(8.5\text{‰}) = 6.5 \times 10^5 \text{ m}^2$ .

Based on the results from the first vertical interpretation scheme values of  $\bar{A}_d$ ,  $\bar{F}$ ,  $\bar{A}_d/\bar{C}$ ,  $\bar{F}/\bar{C}$  and  $\bar{K}_s$  at  $S = 8.5\text{‰}$  are presented in Table 1. The mean turbulent salt diffusion coefficient  $\bar{K}_s$  is defined by

$$\bar{K}_s(\overline{\partial S/\partial z}) \equiv \bar{F}/\bar{C}$$

with  $\overline{\partial S/\partial z}$  at  $8.5\text{‰} = 7 \times 10^{-2}\text{‰}(\text{m})^{-1}$ .

Several field programs to study the mean inflow of salty water as a function of  $S$  into the Baltic through the Bornholm Strait and the Stolpe Furrow have been carried out by investigators from the Oceanographic Institute in Gothenburg over the past few years (Petrén and Walin, 1976; Rydberg, 1978). Such data permit calculation of mean advective and diffusive salt fluxes over the whole Baltic [ $\tau_0(S) \leq$  one year for the Baltic]. Rydberg found that the volume flux of salty water ( $S > 8\text{‰}$ ) almost doubled between the Bornholm Strait and the Stolpe Furrow (Fig. 2) due to entrainment of surface water

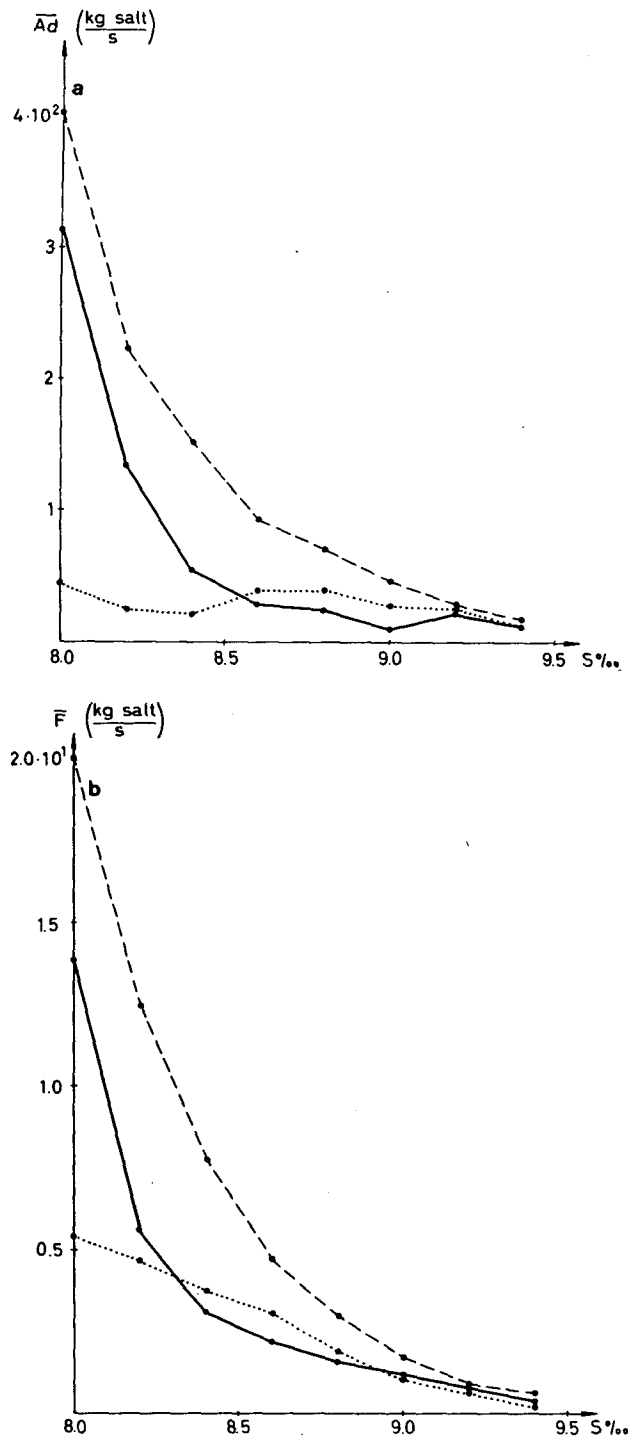


FIG. 7. The mean cross-isohaline advective (a) and diffusive (b) salt transports as a function of salinity in the coastal canyon over P. The different lines represent results of different vertical interpolation schemes described in the text.

into this gravity current. By the end of the Stolpe Furrow the inflowing dense water has reached its equilibrium depth and a transition from the "inflow", downward entrainment regime to the

"spreading", internal mixing-upward entrainment regime takes place.

$\bar{A}d$ ,  $\bar{F}$ ,  $\bar{A}d/\bar{C}$ ,  $\bar{F}/\bar{C}$  and  $\bar{K}_s$  at  $S = 8.5\text{‰}$  for the Baltic inside of the Stolpe Furrow are also presented in Table 1 based on Rydberg's results with  $\bar{C}(S = 8.5\text{‰}) = 8.3 \times 10^{10} \text{ m}^2$  and  $\partial S/\partial z(S = 8.5\text{‰}) = 0.1\text{‰ m}^{-1}$ . A coastal zone of  $\sim 10 \text{ km}$  width (cf. Fig. 3) around the Baltic with similar flux intensities as those calculated for the coastal canyon would account for most of the total upward advective and diffusive fluxes in the Baltic.

#### 4. Discussion

In Section 2 we developed a method for calculating cross-equiscalar surface advective and diffusive transports in semi-enclosed regions applicable to data collected at fixed points in geopotential space. This was then applied in Section 3 to estimate cross-isohaline salt transports in a coastal canyon in the Baltic from recording current meter and profiling data. Our intention here is to review the steps taken and analyze the assumptions made there with the object of discussing the requirements for the method's successful application.

As expressed in Eqs. (11) and (12) knowledge of the functions  $\bar{g}$  and  $\bar{h}$  defined over a sufficiently long time at all points at the opening of a semi-enclosed region is enough to calculate the mean cross-isohaline advective and diffusive transports there. In every natural situation the key question is then at how many points must  $\bar{g}$  and  $\bar{h}$  be known such that vertical and horizontal interpolation based on them should yield accurate approximations. Certainly, this choice must be guided in each case by the hydrographic situation—as well as it is known—and by physical reasoning.

In our study area a dominant feature is the permanent halocline. A minimum vertical separation of about 10 m would have been needed to resolve it. About two additional RCM's would have been required (had they been available) to enable direct vertical interpolation of  $\bar{g}$  and  $\bar{h}$ . Likewise, at least one additional RCM to the side of the first mooring could have been used to interpolate horizontally.

Lacking sufficient resolution to calculate  $\bar{A}d$  and  $\bar{F}$  directly in this manner, we sought additional information on the dominant current and stratification situations in the canyon. We found that low-frequency (LF) motions contributed most to  $\bar{g}$  and  $\bar{h}$  and that coastal upwelling-downwelling seemed to account for most of these motions. Linear vertical profiles of LF  $U$  and  $S$  were chosen guided by some measurements for each of these situations. LF  $\bar{g}$  and  $\bar{h}$  were constructed at intermediate depths from these profiles and LF  $\bar{g}$  and  $\bar{h}$  at 42 and 72 m. Finally  $\bar{A}d$  and  $\bar{F}$  were calculated under the addi-



TABLE 1. A comparison of mean cross-isohaline advective and diffusive transports and fluxes and turbulent exchange coefficients between the values in our coastal canyon and the means over the whole Baltic across the 8.5‰ isohaline.

	$\bar{A}d$ ( $10^2$ kg salt $s^{-1}$ )	$\bar{F}$ ( $10^2$ kg salt $s^{-1}$ )	$\bar{A}d/\bar{C}$ ( $10^{-6}$ kg salt $m^{-2} s^{-1}$ )	$\bar{F}/\bar{C}$ ( $10^{-6}$ kg salt $s^{-1}$ )	$\bar{K}_s$ ( $cm^2 s^{-1}$ )
Baltic	2890	980	3.4	1.1	0.1
Canyon	0.4	0.03	70	5.0	0.8

tional assumption of horizontal homogeneity in the narrow canyon. Two other *linear* vertical interpolation schemes yielded comparable results for  $S \geq 8.5\%$ .

How good might we expect such linear interpolations to be? These appear to be most critical in the case of velocity. At any one time complex velocity distributions might exist due, for instance, to internal waves. Inasmuch as these motions are reversible, however, they should not contribute to  $\bar{g}$  or  $\bar{h}$ . (If the amplitudes of such motions were considerably greater than those from LF motions, one might still get a significant contribution from them due to instrument inaccuracies. This appeared not to be so here.)

On the other hand, one can imagine LF-irreversible processes which could violate our simple linear assumptions. For instance, internal waves could propagate into the canyon in the halocline, increase in amplitude due to decreasing depth and width in the canyon, and finally break where the halocline is in contact with the bottom. The resulting turbulence would mix water from the upper and lower edges of the halocline giving rise to a net inflow there as well as a net outflow near the center of the halocline. Thus the results in Section 3 should be viewed with a healthy dose of scepticism. I feel that the upwelling—downwelling signal was the dominant one and was approximated reasonably well. The reader may reach other conclusions based on his experience and physical intuition. I have tried to state clearly the assumptions on which the results depend. In either case the example in Section 3 has shown how the conservation formalism developed in Section 2 might be applied.

Certainly, it would be preferable to calculate  $\bar{A}d$  and  $\bar{F}$  directly from a sufficiently dense set of  $\bar{g}$  and  $\bar{h}$ . This would imply an extensive instrumental effort in most cases (Even more so when the mean *fluxes* are sought. Then instantaneous areas occupied by equiscalar surfaces in a semi-enclosed region are also needed.) However, the rewards of

obtaining separate estimates of advective and diffusive transports may make such an effort worthwhile. Instrumentation now exists which could do the job quite well, such as the cyclosonde (Van Leer *et al.*, 1974), for example. This method may prove useful for problems ranging from fjord circulation to oceanic coastal upwelling.

*Acknowledgments.* I wish to thank the staffs of the Oceanographic Institute, Gothenburg, and Askö Laboratory of the Zoological Institute, Stockholm, for their help with the field work. The assistance of Kristina Hansson, Agneta Hilding, Ulf Jonasson and Agneta Malm with data analysis and manuscript preparation is gratefully acknowledged. Discussions with G. Walin on "salt space" have been fruitful. This work has been supported by the Swedish Research Council under Contracts B-2969 and G-3752.

#### REFERENCES

- Csanady, G. T., 1975: Hydrodynamics of large lakes. *Annual Reviews of Fluid Mechanics*, Vol. 7, Annual Reviews, Inc., 357–386.
- Haamer, J., 1974: Current measurements with gelatine pendulums. *Vatten*, 1, 49–52.
- Petrén, O., and G. Walin, 1976: Some observations of the deep flow in the Bornholm Strait during the period June 1973–December 1974. *Tellus*, 28, 74–87.
- Rooth, C., and G. Östlund, 1972: Penetration of tritium into the Atlantic thermocline. *Deep-Sea Res.*, 19, 481–492.
- Rydberg, L., 1978: Deep water flow and oxygen consumption within the Baltic. Rep. No. 27, Oceanographic Institute, University of Gothenburg, 12 pp.
- Turner, J. S., 1973: *Buoyancy Effects in Fluids*. Cambridge University Press, 367 pp.
- Van Leer, J., W. Düing, R. Erath, E. Kennelly and A. Speidel, 1974: The Cyclosonde: An unattended vertical profiler for scalar and vector quantities in the upper ocean. *Deep-Sea Res.*, 21, 385–400.
- Veronis, G., 1977: Use of tracers in circulation studies. *The Sea*, Vol. 6, Wiley, 169–188.
- Walín, G., 1977: A theoretical framework for the description of estuaries. *Tellus*, 29, 128–136.
- Welander, P., 1971: The thermocline problem. *Phil. Trans. Roy. Soc. London*, A270, 69–73.