Interaction of Ocean Waves with a Soft Bottom

S. V. HSIAO AND O. H. SHEMDIN

Jet Propulsion Laboratory, California Institute of Technology, Pasadena 91103

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ABSTRACT

Soft muddy bottoms have significant effects on properties of water waves which propagate over them. The wave dispersion equation is modified and wave energy is dissipated by the coupling between the waves in water and those induced in the mud layer. These effects are theoretically determined by assuming a viscoelastic mud layer. A boundary-value problem is solved for the water-mud system with sinusoidal waves. The theoretical dissipation rates are compared favorably with field measurements.

1. Introduction

Unusually high attenuation rates have been observed in the Mississippi River Delta area. Hurricane-generated waves were reported by Bea (1974) to have wave heights 68–78 ft in deep water only to be reduced in heights to 10–15 ft in water depths of 40–70 ft. Tubman and Suhayda (1976) measured wave heights and bottom accelerations at two stations in shallow water in the Mississippi River Delta area. The attenuation rates observed were much greater than normally attributed to bottom friction dissipation. The accelerations observed in the mud layer substantiated the dominant role of bottom motion in dissipating wave energy. Gade (1958, 1959) studied wave energy dissipation in shallow water [$h \leq (1/20)L$, where h is water depth and L is wavelength] by considering the bottom to be composed of viscous and viscoelastic materials in separate analyses. His solutions are restricted to shallow water and long waves, however.

Migniot (1968) performed laboratory measurements which demonstrated that wave-induced orbital motion does occur in mud layers and that soft layers can exhibit properties similar to those found in fluids. Carpenter *et al.* (1973) studied the mechanical properties of marine sediments taken from the Gulf of Mexico. They found a rather complex stress-strain relationship which is very difficult to apply analytically. Suhayda *et al.* (1976) measured wave height, pressure, current and bottom movement in East Bay, Louisiana. Their results show close relationships between bottom movement and surface waves. Tubman and Suhayda (1976) found that the wave energy loss due to mud motion was at least an order of magnitude greater than that due to percolation or friction. Mallard and Dalrymple (1977) studied the effects of an elastic bottom on water waves. Because they did not consider the viscous properties of mud, their results showed no energy dissipation. Dalrymple and Liu (1978) extended Gade's (1958) computation to intermediate water waves but the elastic property of mud was not considered. Rosenthal (1978) studied the wave energy dissipation due to the movement of a sandy bottom from field measurements. He found the dissipation rate to be insignificant in the sandy bottoms of the North Sea, offshore of Sylt. Yamamoto *et al.* (1978) and Madsen (1978) studied the response of a porous elastic bed to water waves. They indicated that the bed response is dependent both on the permeability and stiffness.

In this paper the interaction between water waves and a muddy bottom is studied by considering the bottom to be a viscoelastic material. A boundary-value problem for a two-layer flow system is solved for sinusoidal waves. The rates of wave energy dissipation are computed for different sediment and wave properties and then compared with field measurements.

2. The boundary-value problem

The muddy bottom is assumed to be homogeneous in this study. The stress-strain relationship for a muddy bottom is considered to have the property of a "Voigt body," i.e.,

$$\tau = G\epsilon + \mu\dot{\epsilon}, \tag{1}$$

where τ is shear stress, G shear modulus, μ dynamic viscosity, ϵ shear strain and $\dot{\epsilon}$ rate of shear strain. If the mud is incompressible and G, μ and mud

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density are all constants, the linear two-dimensional equations of motion may be expressed (Kolsky, 1963, p. 117) as

$$\frac{\partial^2 u}{\partial t^2} = J\nabla^2 u + \nu\nabla^2 \frac{\partial u}{\partial t} - \frac{1}{\rho_m}\frac{\partial^2 p}{\partial x \partial t}, \quad (2)$$

$$\frac{\partial^2 w}{\partial t^2} = J\nabla^2 w + \nu\nabla^2 \frac{\partial w}{\partial t} - \frac{1}{\rho_m}\frac{\partial^2 p}{\partial z \partial t}, \quad (3)$$

where u and w are the horizontal and vertical velocity components, respectively, t is time, x and z are the horizontal and vertical coordinates, respectively, ρ_m is the density of mud, $J = G/\rho_m$ and $\nu = \mu/\rho_m$.

To obtain a periodic solution of Eqs. (2) and (3) in response to a monochromatic wave, a streamfunction,

$$\psi = s(z)e^{i(kx-\omega t)}, \quad (4)$$

is assumed, where k is wavenumber and ω wave frequency. The streamfunction is defined such that $u = \partial\psi/\partial z$ and $w = -\partial\psi/\partial x$. Substituting for ψ in Eqs. (2) and (3) and eliminating p, the following fourth-order equation is obtained:

$$\frac{d^4 s}{dz^4} + \left(-2k^2 + \frac{\omega^2}{J - i\nu\omega}\right)\frac{d^2 s}{dz^2} + \left(k^4 - \frac{\omega^2 k^2}{J - i\nu\omega}\right)s = 0. \quad (5)$$

The solution of Eq. (5) is given by

$$s = Ae^{kz} + Be^{-kz} + Ce^{mz} + De^{-mz}, \quad (6)$$

where

$$m = k\left[1 - \frac{\omega^2}{k^2(J - i\omega\nu)}\right]^{1/2}, \quad (7)$$

and A, B, C and D are constants which are determined from the solution of the water-wave equation and boundary conditions.

Assuming incompressible and irrotational water motion, the governing equation in the water layer is the Laplace equation

$$\nabla^2\phi = 0, \quad (8)$$

where ϕ is the velocity potential function defined by $u = \partial\phi/\partial x$ and $w = \partial\phi/\partial z$. The boundary conditions are considered to be (see Fig. 1):

$$p = 0 \quad \text{at} \quad z = h \quad (9a)$$

$$w = \partial\eta/\partial t \quad \text{at} \quad z = h \quad (9b)$$

$$p_{\text{water}} = p_{\text{mud}} \quad \text{at} \quad z = 0 \quad (9c)$$

$$(\partial u/\partial z + \partial w/\partial x)_{\text{mud}} = 0 \quad \text{at} \quad z = 0 \quad (9d)$$

$$w_{\text{water}} = w_{\text{mud}} \quad \text{at} \quad z = 0 \quad (9e)$$

$$u = 0 \quad \text{at} \quad z = -H \quad (9f)$$

$$w = 0 \quad \text{at} \quad z = -H. \quad (9g)$$

Here η is the water surface displacement and a the amplitude of water waves defined as

$$\eta = a \exp[i(kx - \omega t)].$$

The solution of Eq. (8) becomes

$$\phi = -\frac{iag}{\omega}[\cosh k(z-h) + (\omega^2/gk)\sinh k(z-h)]e^{i(kx-\omega t)}, \quad (10)$$

which specifies the constants [see Eq. (6)]

$$A = \frac{\lambda}{2\Lambda}\left[(k^2 + m^2)e^{kH}\cosh mH - \frac{k}{m}(k^2 + m^2)e^{kH}\sinh mH - 2K^2\right], \quad (11)$$

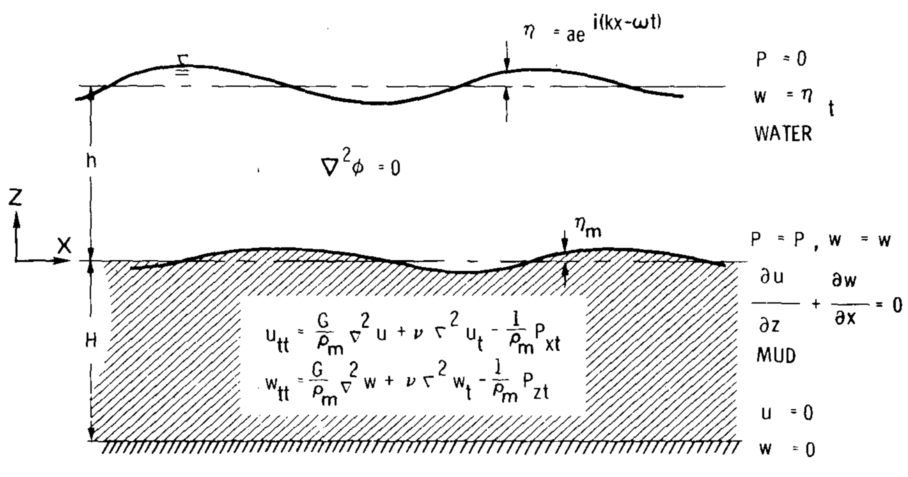

FIG. 1. Schematic definition of bottom motion boundary-value problem.

$$B = \frac{-\lambda}{2\Lambda}\left[(k^2 + m^2)e^{-kH}\cosh mH + \frac{k}{m}(k^2 + m^2)e^{-kH}\sinh mH - 2k^2\right], \quad (12)$$

$$C = \frac{\lambda}{2\Lambda}\frac{k}{m}[k^2 e^{mH}\cosh kH - kme^{mH}\sinh kH - (k^2 + m^2)], \quad (13)$$

$$D = \frac{-\lambda}{2\Lambda}\frac{k}{m}[k^2 e^{-mH}\cosh kH + kme^{-mH}\sinh kH - (k^2 + m^2)], \quad (14)$$

where

$$\Lambda = (k^2 + m^2)\cosh kH \cosh mH - \frac{k}{m}(k^2 + m^2)\sinh kH \sinh mH - 2k^2, \quad (15)$$

$$\lambda = r\frac{ag}{\omega}[\cosh kH - (\omega^2/gk)\sinh kH], \quad (16)$$

and r is the ratio of water density to mud density. The dispersion relation becomes

$$k = \frac{\omega^2}{g}\frac{1 + \tanh kh\Omega}{\tanh kh + \Omega}, \quad (17)$$

where

$$\Omega = r\frac{(m^2 - k^2)[\sinh kH \cosh mH - (k/m)\cosh kH \sinh mH]}{(m^2 + k^2)[\cosh kH \cosh mH - (k/m)\sinh kH \sinh mH] - 2k^2}. \quad (18)$$

Eq. (17) is reduced to the rigid-bottom dispersion equation

$$\omega^2 = gk\tanh kh, \quad (19)$$

under the following conditions:

(i) $H = 0$, or no mud layer.
(ii) $G \to \infty$ or $\nu \to \infty$, or rigid mud layer.
(iii) $r \to 0$, or very large mud density.

The wavenumber k is a complex number. Let

$$k = k_r + ik_i,$$

where k_r is the real part and k_i the imaginary part of the wavenumber. The water surface displacement becomes

$$\eta = ae^{-k_i x}e^{i(k_r x - \omega t)},$$

where the wavelength $= 2\pi/k_r$ and k_i is the attenuation coefficient.

3. Computation method

Eq. (17) is complex and does not have an apparent analytical solution. We have resorted to numerical methods to obtain a solution. The technique used is the one introduced by Hamming (1973, Section 5.2). This is an extended bisection method and is applied here to find the complex zeros of the function

$$F = k - \frac{\omega^2}{g}\frac{1 + \tanh kh\Omega}{\tanh kh + \Omega}.$$

The two-dimensional domain of k investigated is $0.5\,k_\infty \leqslant k_r \leqslant 1.5\,k_0$ and $0 \leqslant k_i \leqslant 0.3\,k_0$, where k_0 is the wavenumber of a firm bottom computed from Eq. (19), and $k_\infty = \omega^2/g$ is the deep-water wavenumber.

In some cases two complex zeros are found in the domain investigated. In examining the two solu-

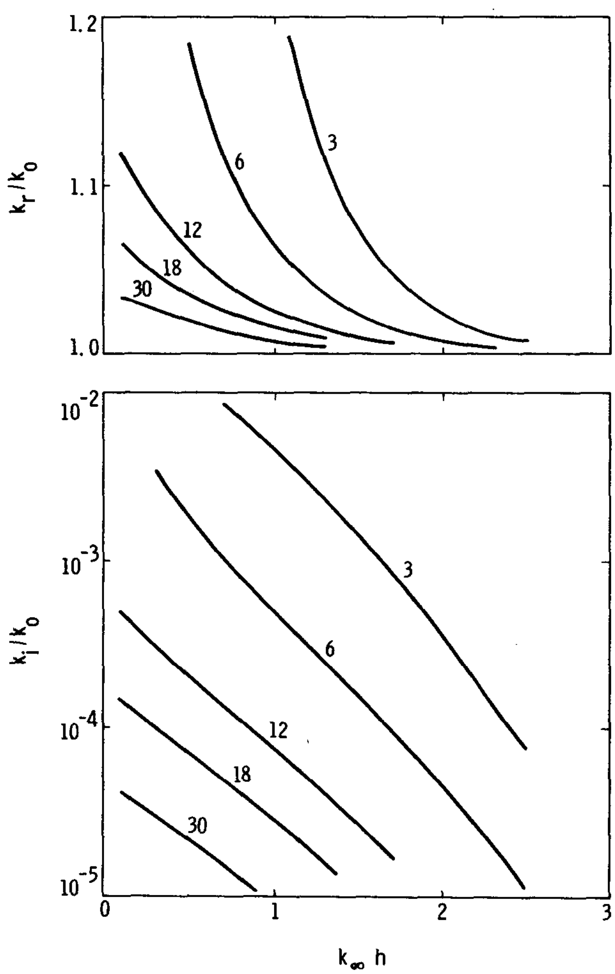

FIG. 2. Nondimensional real and imaginary parts of wavenumber. $H/h = 1.0$, $\nu/\omega h^2 = 0.003$. Numbers on the curves refer to $J/\omega^2 h^2$.

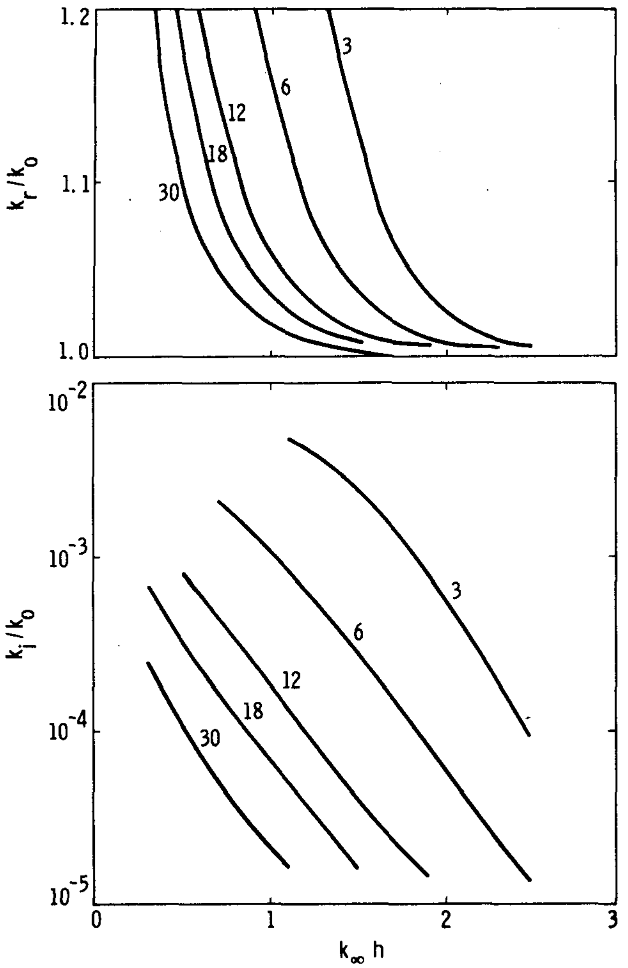

FIG. 3. As in Fig. 2 except for $H/h = 3.16$, $\nu/\omega h^2 = 0.003$.

tions, it is found that one solution gives k_r/k_0 values that do not approach 1.0 at large $k_\infty h$ values. This implies that such waves do not have the expected deep-water wave properties. Such solutions were deleted because of lack of interest. The two solutions are analogous to the two modes of interfacial waves (Lamb, 1945, art. 231). The computational results are presented in the following section.

4. Results

Four sets of curves are computed from the solution of Eq. (17) and are discussed in this Section. Fig. 2 shows the real and imaginary parts of k/k_0 versus $k_\infty h$ for $H/h = 1.0$, $\nu/\omega h^2 = 0.003$, and five J/ω^2h^2 values. The mud effect is shown to decrease with increasing water depth. Also shown in the figure is the stiffness effect of the bottom specified by J/ω^2h^2 values. The bottom-motion effect on waves decreases with increasing stiffness (larger J/ω^2h^2 values). The case of rigid bottom is approached when $J/\omega^2h^2 \to \infty$.

Figs. 3 and 4 are similar to Fig. 2 except that thicker mud layers are considered, $H/h = 3.16$ in Fig. 3 and $H/h \to \infty$ in Fig. 4. Comparing Figs. 2 and 3, k_r/k_0 and k_i/k_0 are both found to be larger for $H/h = 3.16$ than for $H/h = 1.0$, indicating higher dissipation rates in the thicker mud layer. Small differences are observed, however, when comparing Figs. 3 and 4. The latter suggests that an upper limit exists for the mud thickness beyond which the solution is equivalent to an infinitely thick mud layer.

The influence of mud viscosity on wave motion is illustrated in Fig. 5. The dependence of k_r/k_0 and k_i/k_0 on $k_\infty h$ is calculated for $H/h \to \infty$, $J/\omega^2h^2 = 6$, and six $\nu/\omega h^2$ values. It is found that k_r/k_0 increases with decreasing $\nu/\omega h^2$ up to a value of 0.03 beyond which no further change occurs. The wave attenuation rates k_i/k_0 are also shown in Fig. 5; they indicate an increase first and then a decrease with the increasing of $\nu/\omega h^2$. This is attributed to the dissipation rate dependence on both mud viscosity and extent of mud motion. The dissipation rate increases with mud viscosity, but an excessively viscous mud retards motion and causes a decrease in the total

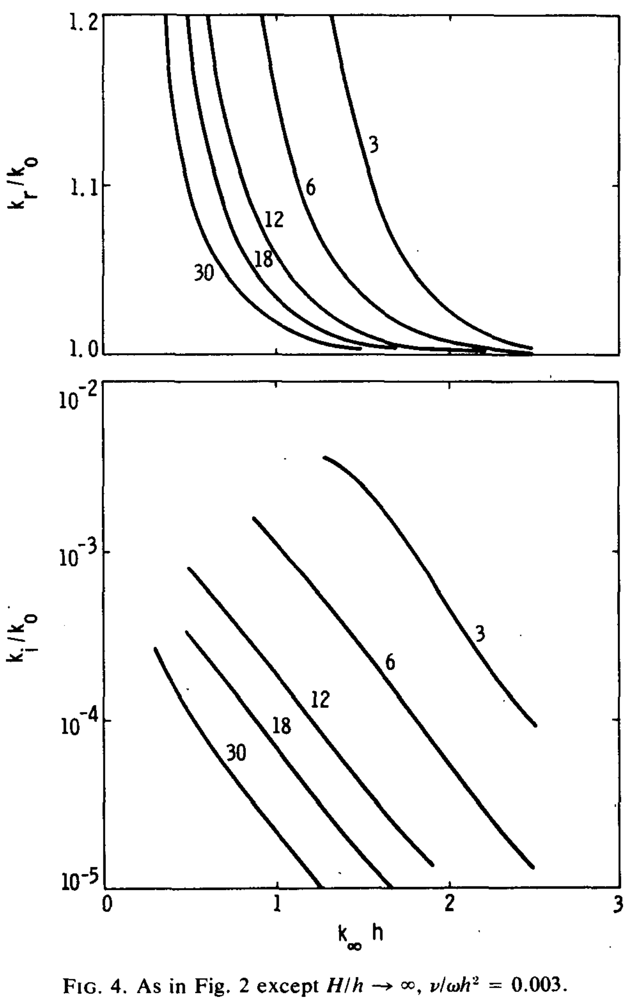

FIG. 4. As in Fig. 2 except $H/h \to \infty$, $\nu/\omega h^2 = 0.003$.

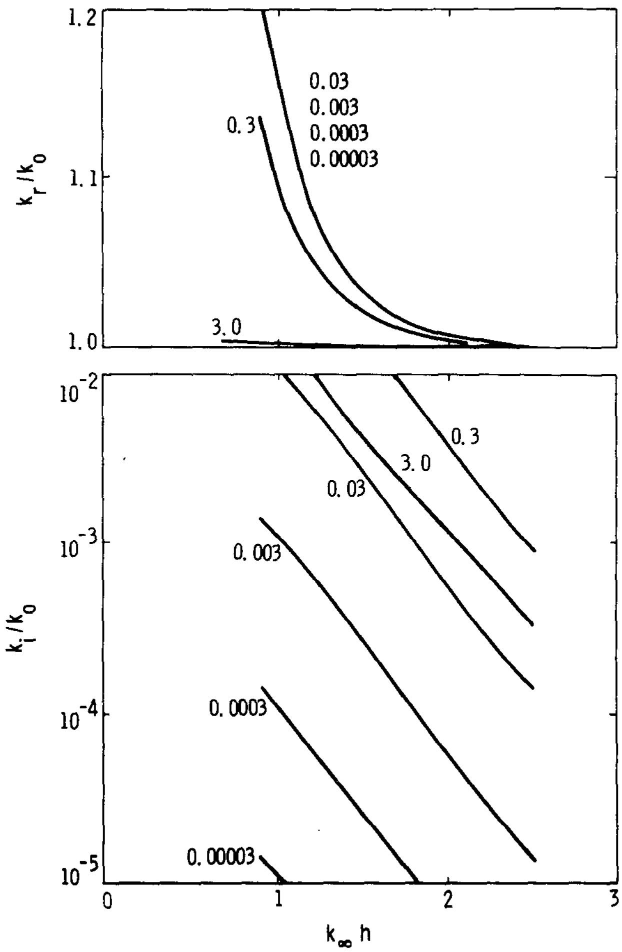

FIG. 5. As in Fig. 2 except $H/h \to \infty$, $J/\omega^2h^2 = 6$; numbers refer to $\nu/\omega h^2$.

rate of energy dissipation. In the limit $\nu/\omega^2 \to \infty$, the bottom becomes firm and no wave energy is dissipated.

5. Comparison with field measurements

The wave attenuation measurements, reported by Tubman and Suhayda (1976) in East Bay, Louisiana, are used to compare with the theoretically predicted attenuation rates. The wave height measurements were obtained in 19.2 and 5.3 m water depths. In that region the bottom consisted of soft mud deposited recently by the Mississippi River. Their measured wave heights are shown in Fig. 7.

Cores of similar bottom material from the Gulf of Mexico were analyzed by Carpenter *et al.* (1973). In their tests, the shear stress and the shear strain were measured while the rate of shear strain was kept constant. If one assumes that the scale of mud motion is equal or less than that of water motion, then the shear strain and the rate of shear strain are equal or less than ka and ωka [$O(10^{-1})$ or less for linear waves], respectively. Using their data for

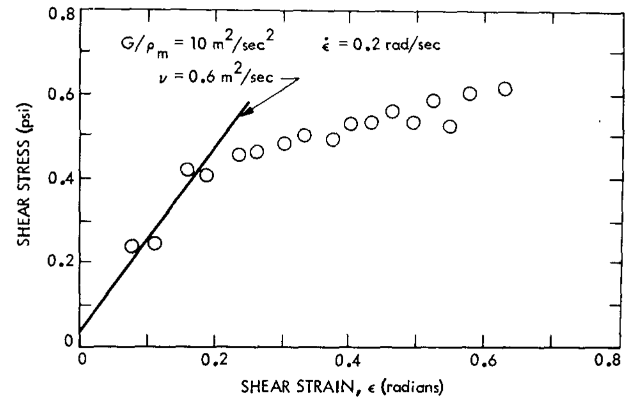

FIG. 6. Stress-strain relationship of mud sample taken from the Gulf of Mexico (Carpenter *et al.*, 1973).

$\dot{\epsilon} = 0.2$ rad s^{-1}, shown in Fig. 6, we find the values of $\nu = 0.6$ m^2 s^{-1}, and $J = 10$ m^2 s^{-2} are representative in the range of $\epsilon = 0 - 0.2$ rad.

The computed wave height attenuation is shown in Fig. 7 based on $\nu = 0.6$ m^2 s^{-1}, $J = 10$ m^2 s^{-2} and $H = 3.5$ m. An independent computation of wave height attenuation based on bottom friction was reported by Tubman and Suhayda (1976) and is also shown in the same figure. It is shown that the observed wave energy dissipation is predicted by the viscoelastic mud model. The dissipation rates are also computed by using Gade's (1958) and Dalrymple and Liu's (1978) models. They are both one order of magnitude higher. Alternatively, the bottom friction mechanism, using a reasonable friction coefficient (of order 0.01), cannot explain the high dissipation rate observed; a high friction coefficient cannot be justified in this area because of the very fine bottom sediment found there. In areas where coarse sand is found, sand ripples can form under certain wave conditions. The latter gives rise

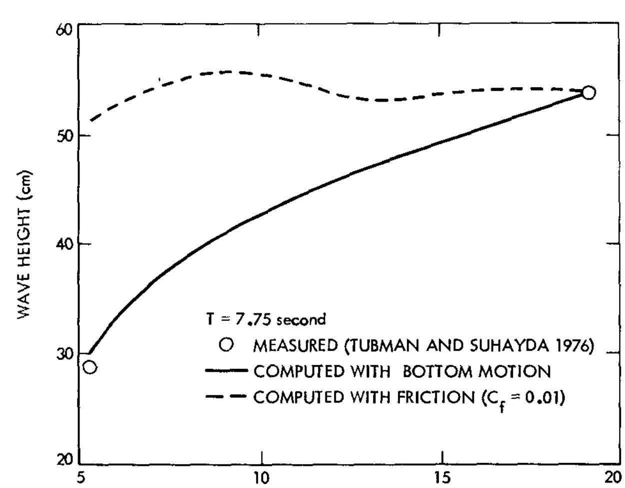

FIG. 7. Wave attenuation over soft mud in shallow water.

to high friction coefficients and high dissipation rates (see Hsiao and Shemdin, 1978). The dissipation due to percolation through the bottom is negligible because of the low coefficient of permeability of fine sediments. The poro-elastic model of Yamamoto *et al.* (1978) is also examined. It is found that for very fine sediments the wave energy dissipation due to this poro-elastic mechanism is negligible.

6. Summary and conclusion

The viscoelastic mud model presented in this paper demonstrates that a soft muddy bottom can have a significant effect on damping of surface water waves. The water-mud interaction decreases the wavelength of surface waves and produces a high dissipation rate. Using representative mud property parameters the proposed model predicts wave energy decay rates consistent with field observations.

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