THE $\Sigma - D$ RELATION FOR PLANETARY NEBULAE: PRELIMINARY ANALYSIS

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SUMMARY: An analysis of the relation between radio surface brightness and diameter, so-called $\Sigma - D$ relation, for planetary nebulae (PNe) is presented: i) the theoretical $\Sigma - D$ relation for the evolution of bremsstrahlung surface brightness is derived; ii) contrary to the results obtained earlier for the Galactic supernova remnant (SNR) samples, our results show that the updated sample of Galactic PNe does not severely suffer from volume selection effect - Malmquist bias (same as for the extragalactic SNR samples) and; iii) we conclude that the empirical $\Sigma - D$ relation for PNe derived in this paper is not useful for valid determination of distances for all observed PNe with unknown distances.

Key words. planetary nebulae: general – Radio continuum: ISM – Methods: analytical – Methods: statistical

1. INTRODUCTION

The relation between radio surface brightnesses and diameters of supernova remnants (SNRs), the so-called $\Sigma - D$ relation, has been subject of the extensive discussions in the last more than fourty years. Due to improvements of the observational techniques (radio-interferometers), the several hundreds planetary nebulae (PNe) were resolved in the last two decades at radio frequencies, but the $\Sigma - D$ relation for PNe was not discussed until now. By using radio data, some statistical methods were established in order to determine distances to PNe. The main method was related to the correlation between radius of PNe and brightness temperature – $R - T_b$ relation (Van de Steene and Zijlstra 1995, Zhang 1995, Phillips 2002). The different samples of Galactic PNe with known distances were defined in these papers. All the obtained empirical $R - T_b$ relations were used for determination of distances to PNe for which the independent distances (in order of $R - T_b$ dependence) were not obtained earlier.

The samples of Galactic PNe are better for statistical analysis than the samples of Galactic SNRs. The selection effects should be smaller in the case of PN samples. However, the selection effects surely influence the Galactic PN samples and the statistical determination of distances to Galactic PNe has to be highly uncertain.

The main objectives of this paper are the following:

i) to derive a simple form of the theoretical $\Sigma - D$ relation for PNe by analyzing the evolution of radio bremsstrahlung surface brightness,

ii) to discuss whether the updated sample of radio PNe is affected by the selection effects, and,

iii) to check whether the $\Sigma - D$ relation is valid for determination of distances to PNe.

2. ANALYSIS AND RESULTS

2.1. Theoretical $\Sigma - D$ relation for PNe

The thermal bremsstrahlung mechanism is responsible for radiation of HII regions at radio wavelengths. The bremsstrahlung volume emissivity ε_ν of a PN can be shown to be (Rohlfs and Wilson 1996):

$$\varepsilon_\nu [\text{ergs s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}] \propto n^2 T^{-1/2}, \qquad (1)$$

where n is the volume density and T is the thermodynamic temperature of interstellar medium (ISM).

The surface brightness can be expressed as:

$$\Sigma_\nu \propto \varepsilon_\nu D, \qquad (2)$$

where D is the diameter of PN. Combining Eqs. (1) and (2), we obtain:

$$\Sigma_\nu \propto n^2 T^{-1/2} D. \qquad (3)$$

Our next step is to express dependance of n and T on D. For a constant velocity mass flow the density distribution is $\varrho = \frac{\dot{\mathcal{M}}}{4\pi r^2 v}$, i.e. $n \propto D^{-x}$, where $x = 2$. Moreover, for the isothermal envelope with a power-law electron density distribution there is relationship between the shape of the density distribution and the power-law index of the radio continuum spectra (see Gruenwald and Aleman 2007, and references therein). Supposing that $n \propto D^{-2}$ and T=const. (HII regions are approximately isothermal at $T \sim 10^4$ K), we obtain the simplest form of the theoretical $\Sigma - D$ relation for PNe:

$$\Sigma_\nu \propto D^{-3}. \qquad (4)$$

This is a standard power-law form of the $\Sigma - D$ relation which can be written in general form as $\Sigma = AD^{-\beta}$, that is the same as in the case of SNRs.

It is possible that x in density distribution is slightly higher, $x \gtrsim 2$, and that the temperature is not strictly constant throughout the nebula. We can expect to see temperature gradients in PNe arising from radiation hardening. More energetic photons will travel further and when they are absorbed by the PN they will impart greater kinetic energy to the ions thereby producing a higher temperature. Using the numerical model results given by Evans and Dopita (1985), we calculate the dependence between $\log T_e$ and $\log D$ and find the low slope (≈ 0.1). Therefore, this only slightly changes the slope of the theoretical $\Sigma - D$ relation. The value $\beta = 3$ is then a theoretical lower limit, and the $\Sigma - D$ relation could only be steeper, as one can see from Eq. (3).

2.2. The empirical $\Sigma - D$ relation for PNe

The most important prerequisite for deriving a proper empirical $\Sigma - D$ relation is defining of a representative sample of PNe. The distances to the calibrators have to be determined by accurate methods, e.g. trigonometric or spectroscopic parallaxes of

central stars in PNe, or by a method that uses the expansion of nebulae. On the other hand, all samples suffer from the severe selection effects that arise from limitation in sensitivity and resolution, but the most severe selection effect for the Galactic samples of PNe is Malmquist bias; i.e. intrinsically bright PNe are favored because they are sampled from a larger spatial volume compared to any given flux limited survey. The result is a bias against low surface brightness nebulae such as highly evolved old PNe. In this paper we use the updated sample of PNe at the distances less than 0.7 kpc collected by Phillips (2002). The influence of Malmquist bias in this sample is limited because of the limitation in distances to PNe. In addition, we assume that the distances are accurately determined for this sample of relatively close PNe. The empirical $\Sigma - D$ relation at 5 GHz for 44 calibrators with distances less than 0.7 kpc (Phillips 2002) has the form:

$$\Sigma_{5\text{GHz}} = 2.33^{+0.88}_{-0.64} \cdot 10^{-22} D^{-2.07 \pm 0.19}. \qquad (5)$$

The parameters A and β are calculated by least-squares fitting procedure with correlation coefficient -0.86. The corresponding $\Sigma_\nu - D$ diagram is shown in Fig. 1.
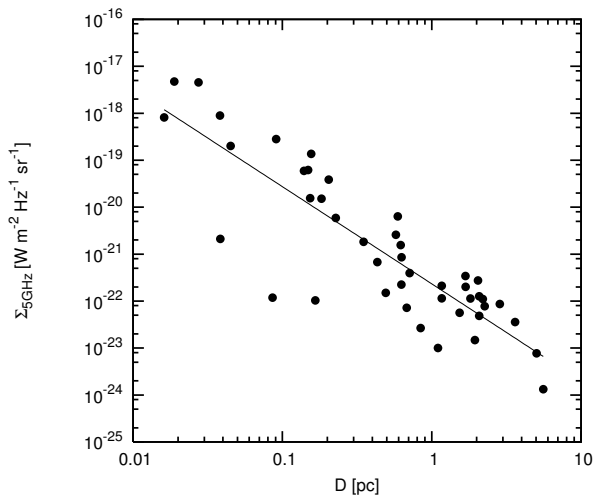

Fig. 1. *The $\Sigma - D$ diagram at 5 GHz for 44 Galactic PNe with distances less than 0.7 kpc.*

The form of Eq. (5) is very close to the so-called trivial $\Sigma - D$ form with $\beta = 2$ (for details see Arbutina et al. 2004). The additional test in order to estimate the validity of Eq. (5) pertains to the possible dependence between the luminosity and diameter of PNe. The $L_\nu - D$ diagram is shown in Fig. 2. The scatter in $L_\nu - D$ plane shows that the correlation between L_ν and D is poor (correlation coefficient = -0.06) and therefore the physical dependence between L and D could not be confirmed by this statistical procedure.
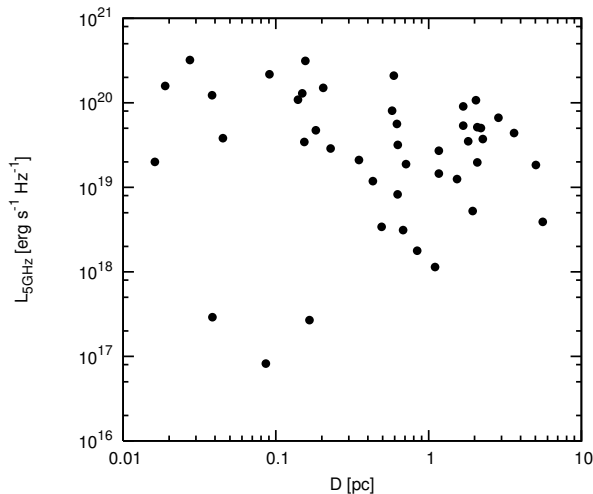
Fig. 2. *The $L - D$ plot at 5 GHz for 44 Galactic PNe with distances less than 0.7 kpc.*

3. DISCUSSION

The theoretical $\Sigma_\nu - D$ relation (Eq. (4)) for PNe, derived in this paper, describes a trend of decreasing radio surface brightness with increasing diameter of an object. The radiation mechanism used in this simple derivation is thermal bremsstrahlung. This is the basic process of production of the radio radiation in HII regions. The theoretically derived slope ($\beta = 3$) is steeper than the slope from the empirical relation given by Eq. (5). This discrepancy can be explained by the low quality of the sample of Galactic PNe or by the assumptions used in derivation of theoretical relation. Due to small variation in power-law density distribution with $x \gtrsim 2$ (Gruenwald and Aleman 2007, and references therein) and approximately constant temperature of expanding envelope of PNe, theoretical slope can be slightly steeper than in Eq. (4). Therefore, we conclude that the theoretical relation has the correct form, but our empirical relation is under influence of biases that could make the slope shallower. On the other hand, there are some attempts to show that evolution of PNe are not linear in log-log scales (e.g. Phillips 2004). These different dependences cannot be derived from the thermal bremsstrahlung radiation formula (Eq. (1)).

A very interesting feature regarding the empirical relation for Galactic PNe (Eq. (5)) is that the slope is approximately equal to the slope of trivial $\Sigma - D$ relation. Therefore, we conclude that Malmquist bias is not so severe as in cases of Galac-

tic SNR samples. This slope ($\beta \approx 2$) was obtained for the extragalactic samples of SNRs (except M82 sample) where Malmquist bias is small, because all the SNRs are at the approximately same distance (see Urošević 2002, Urošević et al. 2005).

The large scatter in $L_\nu - D$ plane (Fig. 2) suggests that the slope in Eq. (5) does not have real and valid physical interpretation. It is a kind of luminosity-diameter scattering artefact which produces the trivial $\Sigma \propto D^{-2}$ form. Therefore, the relation defined by Eq. (5) is not precise enough for determination of valid distances to Galactic PNe. This is due to the different biases: the limitations in sensitivity and resolution of radio surveys, the source confusion, Malmquist bias (in mild form), mixture of different types of PNe in the same sample, and insufficient precision in determining the distances to the 44 calibrators.

4. SUMMARY

The main results of this paper may be summarized as follows:

i) The theoretical $\Sigma_\nu - D$ relation for the radio evolution of thermal bremsstrahlung surface brightness of PNe in form of $\Sigma_\nu \propto D^{-3}$ is derived.

ii) Our results show that the updated sample of Galactic PNe does not severely suffer from volume selection effect - Malmquist bias (same as in cases of the extragalactic SNR samples). This is opposite to results obtained earlier for the Galactic SNR samples.

iii) Due to analysis of the $L_\nu - D$ dependence, we conclude that the $\Sigma_\nu - D$ relation for Galactic PNe is not useful for reliable determination of distances for all observed PNe with unknown distances.

The above observation leads to the more general comment that PNe may have very different initial conditions leading to independent evolutionary paths. These paths could follow the same theoretical $\Sigma - D$ curve but with varying intercepts, leading to the scatter such as the one found in this paper.

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$\Sigma - D$ РЕЛАЦИЈА ЗА ПЛАНЕТАРНЕ МАГЛИНЕ: ПРЕЛИМИНАРНА АНАЛИЗА

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Приказана је анализа тзв. $\Sigma - D$ релације између површинског сјаја на радио-фреквенцијама и дијаметра планетарних маглина (ПМ): i) изведена је теоријска $\Sigma - D$ релација за еволуцију површинског сјаја створеног закочним зрачењем; ii) супротно резултатима добијеним раније за узорке сачињене од Галактичких остатака супернових, наши резултати показују да најновије формирани узорак Галактичких ПМ не трпи велики утицај због запреминског селекционог ефекта, тзв. Малмквистовог селекционог ефекта (исто важи за вангалактичке узораке остатака супернових); и iii) закључујемо да емпиријска $\Sigma - D$ релација за ПМ изведена у овом раду није употребљива за поуздана одређивања даљина до свих посматраних ПМ са непознатим даљинама.