Estimates of the Local Rate of Vertical Diffusion from Dissipation Measurements

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ABSTRACT

Scaling of the turbulent energy equation suggests the balance of terms in the ocean is between turbulent production, dissipation and the loss to buoyancy. In this paper two models for the source of oceanic turbulence are considered; namely, production by the Reynolds stress working against a time variable mean shear, and the gravitational collapse of Kelvin-Helmholtz instabilities. Both of these shear instabilities are believed to be important in the ocean. Using values for the critical flux Richardson number and the measurements from studies of Kelvin-Helmholtz instabilities, the efficiency of turbulent mixing is shown to be comparable for the two models. Therefore, a general relationship between the dissipation rate and the buoyancy flux due to the small-scale turbulent velocity fluctuations is derived. The result is expressed as an upper bound on the value of the turbulent eddy coefficient for mass $K_\rho \leq 0.2\epsilon/N^2$. Values of K_ρ are calculated from recent oceanic measurements of energy dissipation. Isopycnal advection and doubly diffusive phenomena are not included in the model.

1. Introduction

This paper combines recent measurements of turbulent energy dissipation and the efficiency of turbulent mixing to estimate the local cross-isopycnal mass flux. Two different models of oceanic turbulence are used. In the first model the turbulence is maintained by the turbulent energy production of the Reynolds stress working against the mean shear (which may be varying slowly with time, i.e., a running mean) while the dissipation and the buoyancy flux act as sinks for the turbulent energy. In the second model the turbulence is associated with the gravitational collapse of Kelvin-Helmholtz instabilities. This turbulence decays with time. Some of the kinetic energy removed from the mean flow is dissipated while some is used for mixing. The first model is applied to the shear zone between the South Equatorial Current and the Atlantic Equatorial Undercurrent and may apply to some of the 5–40 m thick patches of turbulence found in other parts of the ocean. The second, or billow turbulence model, may apply to many of the active microstructure regions that have vertical scales on the order of 1 m. As discussed later, doubly diffusive convective regimes are not incorporated in these models.

2. Energetics—Derivations

The Navier-Stokes equation can be used to form two kinetic energy equations starting with the basic equation

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\alpha\frac{\partial p}{\partial x_i} - 2\epsilon_{ijk}\Omega_j u_k - g\delta_{i3} + \nu\frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (1)$$

where u_i is the velocity vector, p the pressure, α the specific volume, Ω_j the rotation vector, ϵ_{ijk} the three-dimensional permutation symbol, δ_{ij} the Kronecker delta, ν the coefficient of kinematic viscosity, x_3 is directed vertically upward and repeated indices indicate summation.

The velocity, specific volume and pressure can be written as a mean plus a fluctuation part

$$\left.\begin{array}{l} u_i = \bar{u}_i + u_i' \\ \alpha = \bar{\alpha} + \alpha' \\ p = \bar{p} + p' \end{array}\right\}. \quad (2)$$

We will discuss the definition of the mean state later.

Multiplying Eq. (1) by $\bar{u}_i$ and averaging yields

$$\frac{\partial}{\partial t}\frac{1}{2}\bar{u}_i\cdot\bar{u}_i + \bar{u}_j\frac{\partial}{\partial x_j}\frac{1}{2}\bar{u}_i^2 = -\bar{u}_i\alpha\frac{\partial p}{\partial x_i} - \bar{u}_3 g + \nu\bar{u}_i\frac{\partial^2\bar{u}_i}{\partial x_j\partial x_j} - \bar{u}_i\overline{u_j'\frac{\partial u_i'}{\partial x_j}}. \quad (3)$$

Usually, viscous effects on the mean motion are neglected. The velocities are normally considered

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nondivergent and the specific volume term written as the product of the means [since $\alpha'(\partial p'/\partial x_i) \ll \bar{\alpha}(\partial \bar{p}/\partial x_i)$], giving

$$\frac{\partial}{\partial t}\frac{1}{2}\bar{u}_i\cdot\bar{u}_i + \bar{u}_j\frac{\partial}{\partial x_j}\frac{1}{2}\bar{u}_i\cdot\bar{u}_i = -\bar{u}_i\frac{\partial p}{\partial x_i} - \bar{u}_3 g - \frac{\partial}{\partial x_j}(\overline{u_i u_j' u_i'}) + (\overline{u_i' u_j'})\frac{\partial \bar{u}_i}{\partial x_j}. \quad (4)$$

The left-hand side represents the total derivative of the kinetic energy of the mean motion $E_m = \frac{1}{2}\bar{u}_i^2$. The first term on the other side is work done by the pressure gradient, then the gravitational terms is subtracted to account for the hydrostatic pressure gradient. The last two terms are the divergence of the mean advection of a turbulent correlation and the Reynolds stress times the mean shear. It is this last term which also appears in the equation for the turbulent kinetic energy—providing the transfer between the two forms of kinetic energy. The divergence term serves to redistribute the energy in space.

Multiplying Eq. (1) by u_i', averaging and assuming the turbulent flow to be nondivergent allows us to write the viscous term as the sum of the divergences of the velocity times the stress and the dissipation ϵ. Thus

$$\frac{\partial}{\partial t}\frac{1}{2}\overline{u_i'\cdot u_i'} + \bar{u}_j\frac{\partial}{\partial x_j}\frac{1}{2}\overline{u_i'\cdot u_i'} = -\overline{u_i'\alpha'}\frac{\partial \bar{p}}{\partial x_i} - \overline{(\bar{\alpha}+\alpha')u_i'\frac{\partial p'}{\partial x}} + \nu\frac{\partial}{\partial x_j}(u_i'\sigma_{ij}') - \epsilon - (\overline{u_i'u_j'})\frac{\partial \bar{u}_i}{\partial x_j}, \quad (5)$$

where

$$\sigma_{ij}' = \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right),$$

$$\epsilon = \nu\sigma_{ij}'\sigma_{ji}'/2. \quad (6)$$

From (5) we see that the turbulent kinetic energy is modified by 1) the work done by the buoyancy flux $[-\overline{u_i'\alpha'}\partial\bar{p}/\partial x_i \cong (g/\bar{\rho})\overline{u_3'\rho'}]$; 2) the correlation of the turbulent velocity times the turbulent pressure gradient and the specific volume; 3) the divergence of the viscous diffusion of turbulent kinetic energy; 4) the dissipation; and 5) the Reynolds stress times the mean shear. Thus it is via the turbulent kinetic energy that stratification and dissipation come to affect the mean circulation.

Now we need to scale Eq. (5) to determine the dominant terms. This exercise forces us to define the averaging process. The concept of a mean and a fluctuating part requires a separation between what is the mean motion and the fluctuating part. One wants the mean motion to be larger in scale and to change more slowly in time than the fluctuating part. Since we are interested in the effects of the small-scale turbulence we separate the flow as follows. The fluctuating part will include those variations on scales which contribute significantly to the mean-square shear, i.e. the dissipation scales. Most internal waves, inertial currents, etc., will be considered part of the "mean" motion. The problem of separating internal waves from turbulence may be serious but is not amenable to solution here. Usually they will have different time scales. Thus it is assumed that the dissipation is due to random motions which cause mixing and not wavelike motions which do not.

Scaling the equations to determine the dominant terms can be done for some atmospheric flows. Monin and Yaglom (1975) indicate that for a parallel shear flow in the absence of a destabilizing heat flux, the pressure velocity correlation and the turbulent diffusion terms are generally neglected. Both of these terms can be written as divergences and therefore affect only the distribution of turbulent kinetic energy in space, but not the total quantity. Thus for a steady-state situation the balance is between the turbulent production, the dissipation and the work against buoyancy. We will use this balance for the oceanic situation. Applied to the ocean this balance can be written as

$$\overline{u_i'u_j'}\frac{\partial \bar{u}_i}{\partial x_j} = -\epsilon - \frac{\overline{u_3'\rho'}g}{\bar{\rho}}. \quad (7)$$

The ratio of the buoyancy flux to the turbulent production is defined to be the flux Richardson number R_f. The value must be less than 1 for steady state. In fact, there are good arguments (Stewart, 1959) that the value is considerably less than 1 for maintained turbulence in a shear flow. Essentially, the physical argument is that for a shear $\partial\bar{u}_1/\partial x_3$ the turbulent production goes into $u_1'^2$ and from there is distributed by pressure velocity correlations to $u_2'^2$ and $u_3'^2$. There is viscous dissipation in each component but a loss to buoyancy only via the vertical component. Thus the loss to buoyancy is probably much less than the dissipation. The possibility of a maximum value for R_f above which the turbulence cannot be maintained in steady state is very appealing. Such a value constitutes a critical flux Richardson number.

Britter's (1974) measurements suggest the critical value for R_f is 0.18–0.2. Businger (1973) discusses atmospheric measurements. Ellison's (1957) theoretical prediction is ~0.15. At higher values of the flux Richardson number too much energy is going into the buoyancy flux and the turbulence will be suppressed. If the time scale for the variation of the mean field is large compared to $u'^2\epsilon^{-1}$ then the turbulence will be close to equilibrium. Thus we can apply the steady-state value for R_f critical to a slowly varying mean state.

If we define the eddy coefficient for density K_ρ by

$$K_\rho = \frac{g\overline{u_3'\rho'}\bar{\rho}^{-1}}{N^2}, \quad (8)$$

where N is the Väisälä frequency. Then K_ρ can be related to ϵ, N and R_f as

$$K_\rho = \frac{R_f\epsilon}{(1 - R_f)N^2}, \quad (9)$$

If $R_f \leq R_f$ crit = 0.15, then

$$K_\rho \leq \frac{0.15\epsilon}{0.85N^2} < 0.2\frac{\epsilon}{N^2}. \quad (10)$$

Eq. (10) can now be used to relate dissipation measurements to the buoyancy flux in a steady shear zone such as the Atlantic Equatorial Undercurrent. It may also be applicable to thick (10 m or more) patches of turbulence found in various regions of the ocean.

There are patches of temperature and velocity microstructure in the ocean that have scales of 1 m. These thin features could well be the result of Kelvin-Helmholtz instabilities. Gregg (1977) shows oceanic temperature, salinity and density profiles that look like the laboratory profiles of Thorpe (1973) and Koop (1976). There are two sources of information on the mixing efficiency of Kelvin-Helmholtz (KH) billows. Thorpe has done measurements on KH billows. His 1975 review paper comments that 10% of the energy removed from the mean field can go into mixing and up to 16% might be radiated away via internal waves. Koop (1976) reports on extensive measurements of the ratio of the increase in potential energy to the decrease in kinetic energy as a function of gradient Richardson number. Koop defines $\mathrm{Ri} = g(\Delta\rho/\rho)h_0/(\Delta u)^2$, where h_0 is the initial scale of the velocity shear. His experiments were done in a flume tank with shear and a density gradient. The most efficient mixing occurred at small gradient Richardson numbers with a value of $\Delta PE/\Delta KE = 0.25$. As the value of R approached 0.20 the ratio $\Delta PE/\Delta KE$ approached 0.01. From these data we can get a range of values for the multiplication factor $\gamma = (\Delta PE/\Delta KE)/(1 - \Delta PE/\Delta KE)$, which converts the already dimensionally correct ratio ϵ/N^2, into an eddy coefficient for mass transport. The different values of γ are listed in Table 1.

Thus we see that the factor $R_f/(1 - R_f)$, derived from the concept of a critical flux Richardson number, is half as large as the maximum value of γ derived from Koop's forced shear flow and 30–60% larger than the values deduced from Thorpe's work. As the gradient Richardson number increases, and we would have to think of this as the local value increasing everywhere, the factor γ becomes smaller. Thus using Eq. (10) is a reasonable way to estimate an upper bound for diffusion from measurements of the local dissipation rate for the two sources of turbulence considered.

TABLE 1. Relative efficiency of mixing by Kelvin-Helmholtz billows and the critical flux Richardson number.

$\Delta PE/\Delta KE$	$\gamma = (\Delta PE/\Delta KE)/(1 - \Delta PE/\Delta KE)$	Source
0.25	0.33	Koop (1976), Ri = 0.043
0.15	0.18	Critical flux Ri
0.1	0.14	Thorpe (1975) accounting for the 16% radiated by internal waves
0.1	0.11	Thorpe (1975), no radiation
0.01	0.01	Koop (1976), Ri = 0.2

3. Applications

The first set of dissipation data we will examine is from the equatorial Atlantic (Crawford and Osborn, 1979a). The Atlantic Equatorial Undercurrent is a rather special oceanic situation for there is a large mean shear above the core. The South Equatorial Current is flowing at 0.3 m s^{-1} to the west and at ~100 m the Undercurrent is flowing almost 1 m s^{-1} to the east. The shear over 10 m intervals reached values well in excess of 10^{-2} s^{-1} with values as large as 6×10^{-2} reported by Bruce and Katz (1976). This region of high shear coincides with the region of maximum dissipation. Crawford and Osborn (1979b) examine the energetics of the current and scale the turbulent energy equation to justify using ϵ as an estimate of the local production. Their dissipation estimates are combined with other measurements of the mean flow regime to allow an integrated version of Eq. (3) to be compared with measurements of the mean properties for the South Equatorial Current and the Equatorial Undercurrent above the velocity maximum. The comparison is quite favorable. Since that work was done the results of some temperature microstructure measurements taken on the same cruise have become available (Osborn and Bilodeau, 1980).

Following Crawford and Osborn (1979a) we can identify three major regions of the water column: 1) the shear region above the Equatorial Undercurrent, 2) the core region of the Equatorial Undercurrent, and 3) the shear region below the Equatorial Undercurrent. Table 2 shows the results for K_ρ calculated from (10) using the dissipation data from Crawford and Osborn (1979a) and density data from Katz (personal communication). Values of K_T were calculated from the results in Osborn and Bilodeau (1980) using the model of Osborn and Cox (1972).

TABLE 2. Dissipation and eddy coefficient estimates from data collected in the equatorial Atlantic. Average values for K_T and K_ρ were derived by averaging individual estimates of K_ρ and K_T not from average ϵ, N^2, etc., and then applying Eq. (10). Dissipation estimates are derived from an instrument called the "Camel", while the temperature gradient variance comes from measurements with instruments called "Pumpkins". Numbers following the instrument indicate drop number.

	Atlantic Equatorial Undercurrent average values			
	High shear above core	Core region	Shear below core	
ϵ	3×10^{-3} cm^2 s^3	3×10^{-5}	$10^{-4} \rightarrow 10^{-3}$	
K_ρ	4 cm^2 s^{-1}	0.015	$0.3 \rightarrow 3$	
K_T	$(2 \pm 1) \times 1.9$ cm^2 s^{-1}	$(2 \pm 1) \times 0.01$	$(2 \pm 1) \times 0.04 \rightarrow 0.2$	
			Camel 18, Pumpkin 19	
Depth	20–42 m	42–60		
$\bar{\epsilon}$	7×10^{-3} cm^2 s^3	5×10^{-5}*		0°02′N 24°00′W
K_ρ	2.6 cm^2 s^{-1}	0.013		
K_T	$(2 \pm 1) \times 3.2$ cm^2 s	$(2 \pm 1) \times 0.04$		
			Camel 23, Camel 24, Pumpkin 24	
Depth	20–50 m	50–106	106–148	
ϵ	2.5×10^{-3} cm^2 s^{-3}	2.5×10^{-5}	1×10^{-4}	0°18′S 28°01′W
K_ρ	3.5	0.01	0.32	
K_T	$(2 \pm 1) \times 0.8$	$(2 \pm 1) \times 0.005$	$(2 \pm 1) \times 0.043$	
			Camel 28, Camel 29, Pumpkin 30	
Depth	20–60 m	60–110	28 29 11–130 115–152	
ϵ	3.4×10^{-3}	3×10^{-5}	$(1 - 2) \times 10^{-3}$	0°02′N 32°59′W
K_ρ	6.4	0.014	2.4 3.2	
K_T	$(2 \pm 1) \times 1.6$	$(2 \pm 1) \times 0.006$	$(2 \pm 1) \times 0.2$	

* Estimated.

The factor (2 ± 1) is used to convert components of the mean square gradient to the total variance of the gradient. The three stations included in Table 2 were located close to the core of the undercurrent as seen in Bruce and Katz (1976). The values of K_T and K_ρ can be compared with the expectation that the eddy coefficients for different scalar variables should be the same (Munk, 1966). The high shear region above the core of the undercurrent probably represents one of the best places to expect the assumptions leading to Eq. (10) to be valid.

The results are encouraging in the upper shear layer. Both K_ρ and K_T are large and of the same order of magnitude. The results are consistent with the turbulence in the *upper shear layer* being maintained in the presence of stable stratification. Through the core of the current ϵ decreases and N increases so that K_ρ is 100 times smaller than in the water above. The values for K_T are also smaller in the range of 10 times the molecular value for heat. Thus, in this region, K_ρ and K_T are comparable but small. In the shear region below the core values for ϵ vary by a factor of 10 between the pair of profiles, 23 and 24, and the other pair, 28 and 29. Here, the values of K_ρ are an order of magnitude larger than the values of K_T. Both estimates of the eddy coefficient show the same variation as the values of the dissipation. One possible interpretation is that the flux Richardson number is considerably smaller (by a factor of 10) in the lower shear region.

Another set of data is available from the waters adjacent to the island of Santa Maria in the Azores (Osborn, 1978). The data show thick regions (10–40 m) of relatively high dissipation. These features are most noticeable in the seasonal thermocline adjacent to the island but are also seen at depth away from the island. The Ozmilov scale $(\epsilon/N^3)^{1/2}$ is on the order of 1 m in these turbulent regions. Thus the scale of the constituent eddies is smaller than the full vertical scale of the active turbulence.

The model of shear production balancing dissipation and the buoyancy flux may still be applicable. Osborn (1978) suggests that the thick dissipation patches may be regions where the turbulence is supported by the Reynolds stress working against the local mean shear. This mean shear would be time variable, largely due to inertial and internal waves, and hence the turbulence would grow and decay with time. Stewart (1959) discusses the equations for the individual components $\overline{u'^2}$, $\overline{v'^2}$ and

$\overline{w'^2}$ as well as the Reynolds stress term $\overline{u'w'}$. Increases in the shear can lead to increased turbulent energy production which leads to an increase in the Reynolds stress. Thus the turbulence could increase until the shear started to decrease. The picture is one of a time varying turbulence level in response to a time varying mean shear. There would always be a low level of background turbulence perhaps decaying in time but ready to grow with an increase in the local mean shear. This mechanism could produce turbulent patches, such as reported by Gregg (1977, Fig. 15a), with vertical scales of tens of meters even with static stability on scales of 1 m and larger.

The dissipation data below the upper layer, which is in direct contact with the atmosphere, can be separated into three oceanic regimes:

1) The region well away from the island (~80 km west) and below the surface mixed layer. These data encompass the region 200–750 m depth, which is the maximum operating depth of the instrument.

2) The seasonal thermocline adjacent to the island (within 16 km) corresponding to the depth interval between 100 and 150 m.

3) The region below the seasonal thermocline adjacent to the island, ~150–750 m.

Values of K_ρ for the three regions in the Azores below the wind-mixed upper layer are shown in Table 3. The first two values are quite large compared to what one has come to expect from temperature microstructure using the Osborn-Cox model (Gargett, 1976; Gregg, 1977). The lower value below the seasonal thermocline adjacent to the island compared to the same depth offshort is a reflection of the relative ϵ values and is quite apparent in the averaged values of ϵ.

The model was applied to data collected with the same instrument system during the Fine and Microstructure Experiment (FAME). Data were taken at several sites in the western North Atlantic, near Bermuda, in the Gulf Stream as well at an open ocean site (Gargett and Osborn, 1979a). Table 4 shows the values of K_ρ derived from 200 m averages of ϵ and N^2, provided by Gargett (personal communication). There is an ambiguity in the correct way to average. Estimated values of ϵ corresponding to the noise level of the undercurrent ($<10^{-5}$ cm^2 s^{-3}) can actually represent water with much lower dissipation rates. This effect is most pronounced at the deeper level for the open ocean site where the estimate for K_ρ would be about three times less if values of ϵ corresponding to the noise level are set to zero before averaging the data over 200 m intervals.

TABLE 3. Dissipation and diffusion estimates from the Azores.

Depth (m)	ϵ (cm^2 s^{-3})	K_ρ (cm^2 s^{-1})	Location
1. 200–700	8.6×10^{-5}	2	Below the seasonal thermocline 80 km from the Island
2. 100–150	2.7×10^{-4}	5	Seasonal thermocline adjacent to the Island
3. 150–750	2×10^{-5}	0.3	Below the seasonal thermocline adcent to the Island

TABLE 4. Estimates of eddy coefficients from the FAME data presented for different locations in space.

Location	Seasonal thermocline (cm^2 s^{-1})	Main thermocline (cm^2 s^{-1})
Open ocean site	0.3	0.3
Near Bermuda	0.12	0.3
Gulf Stream	0.07	0.5

4. Error bands

The dissipation values are based on measurements of the shear variance. The uncertainty in the measurements of shear variance are reported at ±50% (Osborn, 1978) and ±45% (Crawford, 1976). There is an uncertainty for spatial averaging (Crawford, 1976) and the uncertainty in converting the mean signal shear to dissipation using the isotropic relation between the shear variance and ϵ. The finite noise level of the measuring system places a lower bound on the measurement and makes the average value ambiguous when a substantial part of the water column is at or below the noise level. Calculations of K_ρ and K_T are also statistically limited by the amount of data available. It is unclear how much data are required to form a stable estimate for either ϵ or $\chi = 2\kappa\overline{(\nabla\theta')^2}$.

For the above reasons the values calculated in this paper should be considered on an order-of magnitude basis. The large values shown in Table 3 indicate a difference in the water column between the near island seasonal thermocline and the region below. The water below the seasonal thermocline away from the island is different from that sampled adjacent to the island. Values of K_ρ that differ by a factor of 2 should be considered equivalent values.

5. Discussion

The eddy coefficient calculated by (10) parameterizes the diffusion due to the small-scale turbulence. It obviously cannot include events such as the MEDOC 69 convective penetration through the thermocline. Nor does the eddy coefficient include the vertical turbulent diffusion due to large-scale advection along sloping isopycnal surfaces. The model used to derive K_ρ estimates the buoyancy flux associated with the local small-scale turbulent fluctuations that are responsible for the dissipation. Hence, the buoyancy flux and therefore the derived eddy coefficient is associated with the small-scale turbulence and represents the local cross-isopycnal diffusion. Whenever one uses a model for deriving an eddy coefficient, it is crucial to keep in mind the assumptions associated in the model to allow the correct interpretation of the value derived. For example, Osborn and Cox (1972) derived a method to estimate the vertical eddy coefficient for heat from measurements of the small-scale temperature gradient variance. In deriving that model, terms which included the advection of temperature variance were neglected. Hence that model is inappropriate in frontal regions where there are large advective contributions to the $\overline{(\theta')^2}$ balance.

Values for K_T of 0.14 cm^2 s^{-1} correspond to 100 times the molecular coefficient. Thermal microstructure measurements in the Pacific (Gregg, 1977; Gargett, 1976) suggest that this ratio of eddy diffusivity to molecular diffusivity is not unreasonable. Gregg's values from FAME (personal communication) indicate a Cox number on the order of 100 from data collected near Bermuda. A larger value for K_ρ such as suggested by two of the Azores regimes seems too large. Perhaps the time-dependent effects are important in this case. The limited sample does not allow us to estimate the temporally averaged value and so we may be getting the apparently large values due to specific mixing events occurring during the sampling period. Hence these values of K_ρ in the Azores represent an excessively large value to the upper bound of K. The value of R_f might have been considerably below the value of 0.15 used in the derivation of Eq. (10). Dissipation that occurs in well-mixed portions of the water column is not associated with its proportionate share of mass transport. Thus the estimate of $\overline{\rho'w'}$ and hence K_ρ, from ϵ is an upper bound.

The derivation of a similar eddy coefficient for the buoyancy flux [Eq. (10)] has been done by Ozmidov (1965) on the basis of a critical mixing distance. In fact, the functional dependence on ϵ and N^2 is determined by dimensional reasoning as soon as they are made the only available descriptors of the flux. The derivation in this paper differs from Ozmidov's in that a basis is provided for evaluating the constant of proportionality between K_ρ and ϵ/N^2.

The model of oceanic turbulence is also different. In Ozmidov's model of turbulence there are mixing layers up to some vertical thickness determined by the stratification. Eq. (10) is derived without requiring that the stratification determine the vertical scale of the turbulent patches. Rather, the vertical scale of the turbulent patches is felt to be determined, in some cases, by the vertical scale of the "local mean shear" and in other cases by the billow size.

Olbers (1976) uses the internal wave spectrum and ideas about the energy transfer to estimate the energy that is available to turbulence. Using a value of 3 for $\epsilon(K_\rho N^2)^{-1}$ he derives the buoyancy flux due to the energy lost from internal waves. His value of K_ρ = 0.3 cm^2 s^{-1} would correspond in this paper to a value of 0.16 cm^2 s^{-1} when the differences in the assumed efficiency of mixing are taken into account.

Lilly *et al.* (1974) use a value of $R_f = 0.25$ to convert estimates of dissipation rates in the atmosphere to diffusion fluxes for buoyancy. Weinstock (1978) presents an analytical derivation for K_z in the atmosphere. However, the associated volume value for R_f would be ~0.44 which seems too large if one is inclined to accept Stewart's (1959) reasoning.

The models for turbulent diffusion discussed in this paper use mechanical energy extracted from the flow field to increase the potential energy of the system by raising the center of mass. In contrast, convection driven by the difference in diffusion rates (doubly diffusive convection) removes potential energy from the vertical distribution of one constituent of the density field (heat or salt) and uses that energy to raise the potential energy due to the other constituent. The efficiency in converting potential energy in one scalar field to potential energy of the other scalar field varies from 15% for the more stable portions of the diffusive regime (Turner, 1973) to 56% for salt fingers (Schmitt, 1977). Quantitative evaluation of the role of doubly diffusive processes in oceanic transport of scalar properties cannot be done with present information. While thermohaline and the mechanical sources may both contribute to the energy dissipation and the buoyancy flux, more insight is needed to separate out their relative contributions to the density flux.

6. Concluding remarks

This paper relates measurements of energy dissipation to the local rate of cross-isopycnal turbulent mixing. Two different shear instability mechanisms are used to estimate the efficiency of turbulent mixing. Doubly diffusive phenomena are not incorporated into this discussion since they represent vertical convection driven by gravitational potential energy, whereas this paper is concerned with vertical transport due to turbulence whose energy is derived from the local kinetic energy in the ocean.

A value for the buoyancy flux of 15% of the turbulent production is used as an upper bound. This value is consistent with the two different sources of the turbulent fluctuations. Large values in excess of 1 $cm^2\ s^{-1}$ for the vertical eddy coefficient are found in the shear region of the Atlantic Equatorial Undercurrent as well as in parts of the seasonal and main thermoclines adjacent to the Azores. Values well below 1 $cm^2\ s^{-1}$ are found in some of the Azores data and in the dissipation measurements from FAME. Much more data are needed to separate out temporal and spatial variation in the dissipation rates. Work is also needed to determine the statistics of the distributions of ϵ and χ. These results would show how much data are needed to reliably estimate average values. More detailed measurements in the vertical are necessary to determine the relationship, if any, between the distributions of the local values of N^2 and ϵ.

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