FE Analysis of Complex Discontinuous and Jointed Structural Systems (Part 1: Presentation of the Method - A State-of-the-Art Review)

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ABSTRACT

This study presents an efficient finite element analysis technique which shows great versatility in analysing complex discontinuous systems subjected to static, dynamic, or seismic loadings. The method incorporates discontinuities in the analysis of discontinuous structures by the use of interface elements designed to simulate the actual behaviour at the interfaces between contacting materials. Several case-problem studies that exhibit discontinuous behaviour have been performed in order to demonstrate the potential and applicability of the proposed method of analysis. One of these studies is reported in Part 2 of this paper, where a non-linear model for the analysis of unreinforced masonry walls is presented. Response results obtained, demonstrate that the overall response of a discontinuous system to external loading is significantly affected by behaviour at the interfaces between contacting materials. Through the inclusion of discontinuities with particular measurable properties, the proposed method of analysis conforms better to actual conditions than do other methods where a continuum is assumed.

KEYWORDS

Discontinuous structures, Seismic analysis, Joints, Interface element, Non-linear behaviour.

1 INTRODUCTION

1.1 The problem

In Civil Engineering practice, there is a variety of structures with interface discontinuities where the assumption of rigid interconnection between the contact surfaces is questionable. If, throughout the loading history, perfect bonding is maintained, then the presence of an interface offers no difficulties in the analysis of the system. However, if, at some point in the loading history, the bond breaks down and there is relative movement of the two mating surfaces, then special solution techniques must be employed.

The analysis of such discontinuous systems is compounded by the sliding and separation that may occur along the interfaces between adjacent blocks. In general, this occurs at shear levels that are significantly lower than the limiting shear of the block material; consequently, an analysis that assumes perfect bonding at the interface, would over-predict the shear transfer, and, depending on the specific application, this would lead to an over- or under-estimation of the response of the structure. Thus, the actual dynamic behaviour of the system can be determined only by a non-linear analysis technique that accounts for the effect of these discontinuities on the response of the system.

From a mathematical point of view, analytical solutions are possible only for a limited class of idealised interface problems. The complexities of the structures, of the material properties and of boundary conditions, have progressively led to the predominance of numerical models based on finite elements and finite differences. For cases in which discrete representation of discontinuities is required, the finite element approach provides the best modelling to date. This is achieved by using a mix of continuum elements and *joint* or *interface* elements.

1.2 Scope of the research

The objectives of this investigation are: (a) to formulate a realistic finite element model for the analysis of complex discontinuous systems subjected to static, dynamic, or seismic loadings; (b) to develop a finite element computer program that implements the above model; (c) to demonstrate the accuracy and capabilities of the program by using it for the analysis of specific discontinuous structures; and (d) to investigate the effect of interface discontinuities on the response of these structures.

In present-day practice there are many powerful finite element programs available commercially, capable of dealing with very large numbers of variables and formulations. The very complexity of such programs means that it is difficult to update them in order to introduce new developments in technology. An important feature of this work is the development of simple finite element computer programs, capable of analysing discontinuity problems in two and three-dimensions. These programs are constituted from various ‘building blocks’, in the form of FORTRAN subroutines, which can be changed or added to as desired to solve the problem in hand. Impact or seismic loading can be easily specified, and material non-linearity of the constituent materials can be based on elasto-plastic or elasto-viscoplastic models. Various numerical solution algorithms have been included in the programs so that the desired method can be selected to give the most economical analysis for each problem. The accuracy and performance of these algorithms were tested and verified against other known experimental and analytical solutions available in the literature.

In order to investigate the effect of interface discontinuities on the response of the structure, interface elements, designed to simulate the actual behaviour at the interfaces between adjacent blocks, have been included in the programs. The constitutive model of the interface element reflects the limiting value of the shear stress/normal stress ratio at the interface. Insertion of the interface element leads to the satisfaction of the compatibility condition at the interface and to the representation of the strain energy contributed by the discontinuous interface.

Owing to its length, the study is divided into two parts. In this part (Part 1), a comprehensive literature survey of the finite element modelling of the discontinuities is given, followed by the development of the three-dimensional interface element used for the analysis. A chronology of development of the main interface elements that exist in the literature, including their pertinent characteristics, is first given. The formulation of the stiffness matrix of the interface element is then presented, together with simplified constitutive relations to define its behaviour.

The accuracy and potential of the finite element program developed for the analysis is demonstrated in Part 2 of the study, by using it to analyse an original case-problem that exhibit discontinuous behaviour. The study is devoted to the analysis of a masonry wall assemblage subjected to static loadings and seismic base inputs. In particular, a non-linear three-dimensional finite element model for static and dynamic analysis of unreinforced masonry walls is presented, together with comparison of its performance against available experimental results. The model adopted treats bricks and mortar joints separately and allows for non-linear deformation characteristics and progressive local failure of both brick and mortar joints. The influence of mortar joints has been accounted for by using the interface element developed, to simulate the time-dependent sliding and separation along the interfaces.

1.3 Motives and research programme

Finite element analysis of the seismic response of complex discontinuous systems with material non-linearity presents a formidable problem. Extensive research shows that no work has apparently been done in this important area to date. The behaviour of masonry walls under earthquake forces is of primary importance for construction in earthquake-prone areas, since the majority of earthquake fatalities worldwide are caused by the failure of masonry structures. This has caused engineers to consider masonry as a structural material, to recognise its weaknesses and to consider ways to overcome these so that such damage can be significantly reduced in the future.

The need for developing an accurate model of the seismic behaviour of masonry structures - masonry being probably the least understood of all construction materials - has motivated the initiation of the present research program, the steps of which are briefly described below. The work was first directed towards the development of a realistic three-dimensional model for the analysis of masonry walls subjected to static and dynamic loads. A finite element computer program was then written to implement the above model, incorporating the effects of discontinuities acting as planes of weakness, and non-linear inelastic behaviour of the constituent materials. Unfortunately, information on material properties of masonry under dynamic/seismic loading is complex, scarce and difficult to obtain. At present there appears to be very little knowledge about the dynamic properties of masonry, especially in the non-linear regime. The problem was further complicated by the unavailability of results, experimental or analytical, with which present results could be compared. For this reason, an experimental research program into the dynamic behaviour of three-dimensional masonry wall assemblages was originally intended to be performed at this stage. Unfortunately, due to shortage of funds, it was not possible to carry out experimental work as part of this project, necessary to verify and validate the results that would be obtained using the three-dimensional finite element computer program developed. Reliable results of tests on two-dimensional masonry walls were, on the other hand, available, and these were used to verify the accuracy of the present method of analysis. Thus, the full potential of the three-dimensional model developed could not be examined, since the external loading applied to the masonry wall panels had to be in-plane loading in order to compare results with available two-dimensional solutions.

Developing efficient techniques to handle all forms of non-linearities present in a discontinuous system is the necessary pre-requisite for understanding the earthquake behaviour of such systems. It was decided, therefore, to expand the finite element computer program developed, and to apply it for the analysis of other complex discontinuous and jointed structural systems. To make the program more versatile, various solution algorithms were added, and material non-linearity was based on elasto-plastic as well as elasto-viscoplastic models, with a choice of different yield criteria that can be adopted. A total Lagrangian formulation was used to allow for the geometrically non-linear behaviour of the structure.

In order to demonstrate the capabilities of the program, the proposed method has also been applied to the seismic response analysis of complex dam-reservoir systems (both gravity and arch), considering dam-water and dam-foundation interaction and including the effects of all the forms of significant non-linearities present in a realistic system. In this context, the dam is assumed to be constructed in blocks, with grouted or keyed joints between them. Even though these joints may provide full continuity under static loading, during dynamic response to an intense earthquake they tend to open. The influence of these joints to the overall response of the structure has been investigated by using a two-dimensional version of the interface element developed, to simulate the actual behaviour at the interfaces. Preliminary results are promising and there appears to be potential for simplifying the solution algorithm that would lead to significant computational economy. Results will be reported in due course.

Although applications to masonry structures and concrete gravity dams have only been described as important examples, several other types of problems, in which the effects of interface discontinuities play an important role in the accurate determination of the systems' response, can be dealt with by the method. Interface elements, with different characteristics and capabilities, may be used in conjunction with other two or three-dimensional linear or quadratic isoparametric elements, to efficiently analyse various types of contact problems and layered or jointed systems. The program may be eventually used as an effective tool for further research into the behaviour of complex discontinuous systems.

2 LITERATURE SURVEY

2.1 General

The application of the finite element method to the analysis of discontinuous systems has received a considerable interest in recent years. Examples of problems in which discontinuities play a prominent role in the physical behaviour of a system are numerous and include various types of contact problems and layered or jointed systems. The discontinuities in continuous systems are in fact interfaces between dissimilar materials and joints or fractures in the material.

A survey of the literature on finite element modelling of cracks and joints shows that two approaches are common for a representative analysis: the *discrete crack* and *smeared crack* approach and the use of *joint* or *interface* elements. Discrete or smeared crack models have only a limited ability to model sharp discontinuities, for which the use of joint elements is more appropriate. A brief description of the different approaches that exist in the literature is given below, together with some applications of interest. Particular attention, however, will be given to the use of joint elements, since the application of the method to the analysis of specific structures will be demonstrated.

2.2 Modelling of discontinuities

2.2.1 Discrete crack approach

The discrete crack approach requires monitoring the response and modifying the topology of the finite element mesh corresponding to the current crack configurations at each state of loading. Discrete crack models explicitly represent the crack as a separation of nodes. When the stress or strain at a node, or the average in adjacent elements, exceeds a given value, the node is redefined as two nodes and the elements on either side are allowed to separate. While this produces a realistic representation of the opening crack, a coarse discertization in the finite element model may result in misrepresentation of the propagating crack tip. A more serious problem is that, changing the formulation of the finite element model changes the number of equations to be solved and broadens the bandwidth of the stiffness matrix.

Skrikerud and Bachmann [1] developed a discrete crack procedure to account for the initiation, extension, closure and reopening of tensile cracks. They analysed Koyna dam, which experienced substantial cracking during the 1967 Koyna earthquake, although the impounded reservoir water was neglected in the analysis. Neglecting the water limits the applicability of the results because dam-water interaction significantly alters the dynamic response of gravity dams. The response results seemed to be dependent on mesh refinement and orientation, and indicated complete separation of the upper portion of the dam when subjected to an artificial ground motion with 0.50g peak acceleration.

2.2.2 Smeared crack approach

In the smeared crack approach [2, 3, 4, 5, 6] cracks and joints are modelled in an average sense by appropriately modifying the material properties at the integration points of regular finite elements. If the strain energy released by this softening is equal to the strain energy released by an opening discrete crack, then the global structural behaviour will be the same when the strain energy is redistributed. The criteria for cracking are similar to those applied in discrete models. Smeared cracks are convenient when the crack orientations are not known beforehand, because the formation of a crack involves no remeshing or new degrees of freedom. However, they have only limited ability to model sharp discontinuities and represent the topology or material behaviour in the vicinity of the crack. The method works best when the cracks to be modelled are themselves smeared out, as in reinforced concrete applications.

Pal [3] used a smeared crack method for tensile cracking in which the effects of cracking are redistributed over a finite element. The non-linear tensile and compressive behaviour of concrete, including strain rate effects, was represented by modifying an equivalent uniaxial stress-strain relationship. Analysis of the Koyna dam, again neglecting the water in the impounded reservoir, indicated that tensile cracks develop at both faces of the dam near the elevation of the discontinuity in the slope of the downstream face, although the cracks did not extend across the cross section. The response results should be viewed with caution because of the limitations of the smeared crack model, particularly with the coarse finite element mesh used in the analysis.

Mlakar [6] studied the earthquake response of three dams of different heights using a smeared crack model for the concrete. Although the study captured the important effects of cracking (for the short dam extensive cracking was shown near the heel of the dam, although, in the taller dams, cracking began near the heel followed by more cracks forming in the upper portions), no orientation of the cracks was provided.

A finite element procedure to model the non-linear earthquake response of concrete gravity dam systems was presented by El-Aidi and Hall [7, 8] using smearing techniques that include tensile cracking with subsequent opening, closing and sliding, as well as water cavitation in the reservoir. Additional applications of smeared crack models to the analysis of concrete gravity dams are discussed in Reference [5].

The *smeared crack* concept, based upon strain decomposition and first developed for use in concrete structures, has been extended to the analysis of masonry structures [9, 10]. The method is attractive if global analysis of large-scale masonry structures is required. It does not make a distinction between individual bricks and joints, but treats the masonry as an anisotropic composite such that joints and cracks are smeared out. An inherent limitation of the smeared crack approach is that discrete cracks are smeared out over an entire element and crack opening is modelled by the continuous displacement approximation functions of the conventional finite element approach. In view of this limitation, as well as other problems such as mesh-dependency due to tensile and compressive softening and difficulties of model calibration, smeared crack models should only be used with caution.

2.2.3 Interface smeared crack approach

Kuo [11] proposed an interface smeared crack model that combines the advantages of the discrete and smeared approaches described above. The model treats cracks discretely like joint elements, but, like smeared crack elements, it does not introduce additional degrees of freedom. Cracking is limited to element boundaries and, if the crack opening criterion is met at a boundary node, then the local element displacements are altered until stresses perpendicular to the interface are brought as close as possible to zero.

This ‘pushing back’ is performed at the local level, so a redefinition of the global problem is not necessary. The pushing back operation produces an unbalanced force, which must be taken up by other parts of the structure. The global stiffness is softened in the vicinity of the crack, so that the global behaviour will be accurate. Kuo's general scheme is based on the pushing back operation on stresses perpendicular to the interface, and it is only able to achieve convergence by applying a compensating factor unique to each problem. Furthermore, the method does not work in non-rectangular elements, or in interfaces at angles to the global coordinates.

Graves and Derucher [12] further developed Kuo's version into a usable analysis procedure and applied it to the seismic analysis of concrete gravity dams. The method pushes back displacements in elements bordering an open-crack interface to eliminate strains normal to the crack face. The amount of pushing back is found by dividing the normal strain by the appropriate strain interpolation function derivative. The method has certain limitations, such as failure to predict accurately the lengthening of periods caused by the softening of the structural stiffness because of cracking, thus producing non-conservative results. It is also considered impractical at greater excitation levels where cracks may open by large amounts due to rigid rocking of the adjacent blocks, and thus hundreds of pushing-back iterations would be required to reach a state of zero normal strain.

2.2.4 Other methods of approach

Vargas-Loli and Fenves [13] used the *crack band theory* [14] to model the tensile behaviour of concrete. The fracture of concrete is represented as a band of smeared cracks over a *crack band* of a certain width. Micro-cracking in the band is identified with the phenomenon of strain softening, which is represented by a stress-strain relationship that preserves the fracture energy of the material.

The authors incorporated the crack band model into a previously developed numerical procedure for computing the dynamic response of non-linear fluid-structure systems [15]. The earthquake response of the Pine Flat dam was computed using this procedure, showing that cracks tend to initiate near the stress concentrations in the monolith, mainly at the base and near the changes in the slope of the faces. Since dam-foundation interaction was not considered in the analysis, the stress concentration and cracking near the heel of the dam was mainly caused by the rigid foundation assumed in the analysis.

Another approach, the *method of constraints*, has been proposed by some investigators [16, 17, 18] to model discontinuities present in a system. According to this, interface discontinuities are represented by a sequence of double nodes, one on each side of the interface. The interconnection between the double nodes is controlled to simulate the physical behaviour of the interface, and the desired solution is obtained by modifying the global stiffness equations in a manner that all the interface conditions, such as compatibility and friction law, are satisfied. With this direct approach of simulating the interface behaviour, the need for constructing a slip element is eliminated.

This method has been used by Arya and Hegemier [19] to model masonry as a discontinuous system, where the discontinuities consist of the mortar joints. Limitations of the proposed method, together with some capabilities that have not been demonstrated by the authors, include the following: first, the assumption that interfaces do not intersect has been made. Secondly, the application of the method to dilatant interfaces has not been mentioned by the authors and no details are given of possible application to the post-peak behaviour of interfaces with strain softening. Joint elements, on the other hand, have successfully modelled complex geologies with intersecting joints, dilatant interfaces and post-peak behaviour of interfaces with strain softening [20, 21, 22], and their use will be discussed below.

2.2.5 Use of joint elements

All of the *crack* models reviewed above have only limited ability to model sharp discontinuities present in many structural systems. Joint elements are more appropriate for modelling opening and closing of discrete cracks and joints and have been used in numerous applications where interface discontinuities play an important role in the physical behaviour of the system. A chronology of development of the main joint elements, including their pertinent characteristics, is given in Section 3.2 of this study.

Applications include various types of contact problems and layered and jointed systems. Discontinuities in the form of faults, joints, bedding planes, or interfaces with underground support systems are present to some extent in virtually all rock environments of interest to the engineer. The presence of joints in rock has long been recognised as an important factor influencing the mechanical behaviour of the media, and it has been accounted for in non-linear analysis by considering isolated discrete joints by a number of investigators [23, 24, 25, 26].

The stability of soil, rock slopes and underground excavations or foundations is a major problem where the influence of joints may be completely dominant. The effect of discontinuities or joints on the response of circular tunnels founded in layered geological media was investigated by Lee & Zaman [27] by using interface elements to represent the deformation behaviour of the joints.

Other engineering problems with interface discontinuities involve soil-structure interaction systems where large relative movements often occur between the structure and the soil, such as debonding-rebonding and slippage, as well as local yield of the soil in the neighbourhood of the structure. Interface elements can be effectively used to allow for such movements and for the transfer of shear stresses across the interfaces.

The load-deformation behaviour of laterally loaded structures (piles) has been studied by Desai & Appel [28] using interface elements capable of allowing relative displacements around the pile, as they occur in reality under lateral loading. Toki et al. introduced the joint element into the dynamic analysis of soil-structure systems to simulate time-dependent sliding and separation along the interface of soil and structure [29, 30] and extended the proposed method to encompass a more realistic three-dimensional soil-structure system [31].

Finite element studies of the interface behaviour in reinforced embankments on soft grounds have been performed [32] using interface elements that allow slip to occur on the soil-reinforcement interface according to a Mohr-Coulomb strength criterion. The effect of differential displacement due to heterogeneity at the interfaces of rockfill dams with an earth core has been studied by Sharma et al. [33] using an isoparametric and numerically integrated curved joint element with variable thickness. Numerous other problems involving soil-structure interaction can be identified. These include problems whose foundations are subjected to inclined loadings; soil-footing, soil-piling and soil-culvert systems; soil-retaining wall behaviour, etc.

Yet, another important class of discontinuity problems concerns the inelastic non-linear behaviour of masonry, where the discontinuities are block or brick-mortar interfaces and block or brick-grout interfaces. A method that accounts for the non-linear behaviour of masonry, considering masonry as a two-phase material, was first developed and applied to solid masonry by Page [34], and to grouted and hollow concrete masonry by Hegemier et al. [35]. They used a finite element model considering masonry as a continuum of isotropic elastic bricks acting in conjunction with mortar joints possessing specialised and restricted properties. The joints were modelled as linkage elements with non-linear deformation characteristics, drawing an analogy with the behaviour of jointed rock.

Goodman's et al. [23] technique, used to analyse problems of this nature in rock mechanics, was used in a similar way by A.D. Tzamtzis [36, 37], for the analysis of other types of discontinuous structures, including masonry. The ‘microscopic’ model proposed by the author for the analysis of masonry walls, offers a more realistic alternative to an analysis based on isotropic elastic behaviour, since it has the ability to reproduce non-linear behaviour caused by material characteristics and local joint failure. However, application of this model to the analysis of large masonry structures is difficult due to the large number of elements needed to separately model the component materials and their interfaces.

Sub-structuring and mesh-refinement techniques were also used in the analysis of masonry walls subjected to in-plane concentrated loads, so large wall panels can be modelled without the need for excessive computer storage requirements [38]. Since masonry walls are regular assemblages of identical structural units, the sub-structuring concept can be easily employed to considerably reduce the cost of analysis compared with the conventional way of modelling which requires a large number of nodes and elements if the bricks and joints are to be modelled separately.

An additional category of localised non-linearity is encountered in structures with joints that may open or close during loading. An important example of these is a concrete dam built as a system of independent concrete monoliths separated by joints. Apparently, only recently there has been an attempt to account for joint opening in the dynamic analysis of dams. The joint behaviour in arch dams was first described by Clough [39]. Joint elements with increased sophistication [40, 41, 42], borrowed from those used in rock mechanics [23], have been used to represent the gradual opening and closing of vertical contraction joints and predetermined horizontal cracking planes present in an arch dam.

Row and Schricker [41] first performed a dynamic analysis of a three-dimensional arch dam that accounts for joint opening. Joint opening effects were modelled by zero-length gap elements which have finite stiffness in compression and zero stiffness in tension. Results indicate that gap opening occurred in all the five contraction joints assumed for the Xiang Hong Dain Dam, when subjected to the El Centro 1940 N-S record, amplified in intensity by a factor of 3.0 to produce a peak ground acceleration of 0.96g.

The effect of crack formation at the concrete-rock interface was examined by O'Connor [43] by placing an isoparametric curved surface interface element along the concrete-rock contact surface of the arch dam. In the most complete work to date, a discrete joint model represented by non-linear springs was developed by Dowling and Hall [44]. The spring-stiffness coefficients were obtained from a separate two-dimensional analysis of a typical arch, although compressive and sliding non-linearities were not included in the study. The earthquake analysis of Pacoima dam demonstrated that contraction joint opening, particularly in the upper portions of the dam, can occur even under a moderate earthquake ground motion.

Fenves et al. [45] used non-linear joint elements to model the opening and closing of contraction joints, and combined them with linear shell, solid, and fluid elements to model an arch dam system. The contraction joints were modelled directly, in contrast to using generalised non-linear springs to represent the joint behaviour. The substructure procedure used for the analysis; i.e. the cantilever sections (as defined by the contraction joints in the model) and the foundation are linear substructures, and the set of joint elements constitutes a single non-linear substructure. Results indicate that if joint opening is not included in the earthquake analysis, unrealistically large tensile stresses develop in the arch direction. Allowing joint opening relieves the tensile stresses, and the maximum arch stresses are reduced by 50-60%. The maximum cantilever stresses are increased however, as loads are transferred from the arches to the cantilevers. The joint opening behaviour is dependent on the presence of keys in the contraction joints, and additional study is necessary to determine the effect of sliding.

The effect of contraction joints on the earthquake response of an arch dam has only recently being investigated and, therefore, there are very few publications on the subject [45]. A summary of the joint elements used for arch dam analysis is given in Reference [46]. This paper concentrates on the opening/closing performance of a family of isoparametric joint elements with zero thickness and discusses remedies to the notorious oscillatory tendency in the family. Similar studies to examine these effects on the earthquake response of gravity dams do not appear in the literature, apart from the work reported by A.D. Tzamtzis [47].

3 METHOD OF ANALYSIS – USE OF A 3D INTERFACE ELEMENT

3.1 General

For the efficient non-linear analysis of discontinuous systems, it is necessary to consider relative slip, debonding and cycles of closing and opening of the interfaces, since these can significantly affect the overall behaviour of the structure. To account for this behaviour, a special interface element is been proposed, together with some simplified constitutive laws to define its behaviour. Insertion of the interface element between the contact surfaces of a discontinuous system leads to the satisfaction of the compatibility condition at the interface and to the representation of the energy contributed by the discontinuous interface.

3.2 Joint or interface elements - A chronology of development

Various types of *joint* or *interface* elements have been developed to date by many investigators to represent joint behaviour. A general chronology of development of the main joint elements that exist in the literature, including their pertinent characteristics, is given below.

The method of using special joint elements and adding their stiffness to the global stiffness of the structure was first used in rock mechanics by Goodman et al. [23] to represent the behaviour of jointed rock masses. The authors represented the joint element as a simple one-dimensional tube, with eight degrees of freedom, offering resistance to compressive and shear forces acting normally and parallel to its axis. The normal and shear resistances were expressed as products of the relative normal and axial displacements between the two faces of the element and the unit stiffnesses of the joint in the two directions. Later, Mahtab and Goodman [25] extended the jointed rock model to three-dimensions, and developed a two-dimensional (plane) joint element of zero thickness. Thus, although each of the nodes of the joint element had three degrees of freedom, the displacement components were functions of the planar coordinates only.

Goodman and Dubois [48] first introduced dilatancy in the joint element stiffness matrices using a perturbational approach. Another study paying attention to stress dependant constitutive laws for the joints, allowing for joint dilatancy and criteria for crack initiation in the blocks, was presented by de Rouvray and Goodman [49]. Reduction from peak shear strength to its residual value was dealt with using iterations on the joint shear stiffness rather than on the load vector, as in Goodman and Dubois [48]. This appears to give better stability and faster convergence of the solution.

Later, Ghaboussi et al. [26] formulated a linear interface element covering a wide range of joint properties, including dilatancy. The joint element developed avoids some theoretical difficulties of other problems by defining the displacement degrees-of-freedom at the nodes of the element to be the relative displacements between opposing sides of the slip surface. A similar joint element has also been developed by the authors for axisymmetric problems, the stiffness matrix of which is also given in the study.

When the adjacent blocks are modelled with quadratic solid elements for higher accuracy of solution, quadratic joint elements must be used to provide displacement compatibility required at the contact between blocks. An isoparametric and numerically integrated curved joint element, with variable thickness, has been developed by Sharma et al. [33] to study the effect of differential displacements due to heterogeneity at the interfaces of rockfill dams with an earth core. A general-purpose program, with sequential construction and non-linear material behaviour, has been developed by the authors incorporating mixed-graded linear and parabolic elements in conjunction with linear and parabolic joint elements.

Buragohain and Shah [50] have also developed curved isoparametric line and axisymmetric interface elements for use in situations involving straight or curved contact surfaces. They also developed a quadratic isoparametric surface element of arbitrary curvature [51] to tackle problems where the contact surfaces are arbitrarily curved.

The introduction of a rotation stiffness in the joint element is a desirable feature, considering the combinations of slip and rotation taking place in assemblies of rock blocks, and this has been done by Goodman and John [52]. The joint element considered was a zero thickness element, accounting for dilation and strain softening behaviour.

Van Dillen and Ewing [53] developed a three-dimensional joint element, operational within a large general purpose computer program. The constitutive relations for the joint element were posed in terms of plasticity theory where dilatation was considered to be plastic strain in the normal direction and slip was taken to be plastic shear strain.

A model to simulate the non-linear properties of joint elements and a method of non-linear analysis based on an incremental procedure was presented by Ge Xiurun [54]. The models suggested for the analysis consisted of four parts: shear displacement, normal deformation, and shear strength models, as well as a relationship between unit shear stiffness and normal stress of the joint element.

Two new developments and refinements in the modelling of geological discontinuities have been presented by Heuze and Barbour [22]. An axisymmetric joint element has been formulated to operate along all directions, unlike that in a previous formulation by Ghaboussi et al. [26], together with a new model to account for the dilatant effects of rock joints.

A more sophisticated joint/interface element applicable to two and three-dimensional finite element analysis was presented by Beer [55], based on assumptions similar to those of References [26] & [53], but a general isoparametric formulation was used instead and the element was of zero thickness, particularly suited to the modelling of rock joints and fractures.

Several other joint/interface elements have been developed since; each of them applicable to particular structures and with different characteristics. It is believed though, that the interface elements mentioned above demonstrate the main capabilities and applicability of the family to the analysis of discontinuous structures.

3.3 Interface element for three-dimensional analysis

The formulation of the stiffness matrix of a three-dimensional interface element is presented in this section. The geometry of the element is shown in Figure 1.

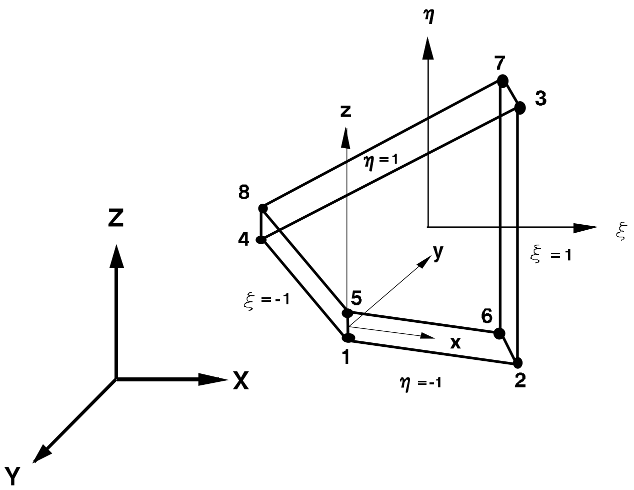

Figure 1. Three-dimensional interface element

Its purpose is to permit large relative movements to occur between adjacent blocks, and the transfer of shear stresses across the interfaces. The element has been used before for the analysis of jointed rock slopes [25] and other discontinuous structures [36, 37, 48].

With reference to Figure 1, the displacements at any point within the element can be expressed as

$$\{u_i\} = [B_i]\{q_i\} \tag{1}$$

where $\{u_i\}$ is the vector of displacement components, and $\{q_i\}$ the vector of nodal displacements.

Matrix $[B_i]$ contains the interpolation functions of the element and is given by

$$[B_i] = \begin{bmatrix} h_i & 0 & 0 \\ 0 & h_i & 0 \\ 0 & 0 & h_i \end{bmatrix} \tag{2}$$

where

$$h_i = \frac{1}{4}(1 \pm \xi_i)(1 \pm \eta_i) \tag{3}$$

The relative displacements between the top and the bottom of the element can then be computed as

$$\{u_i\} = \left[[B_i]\{q_i\}_{top} - [B_i]\{q_i\}_{bottom}\right] = [N]\{q\} \tag{4}$$

Thus,

$$[N] = [-B_1 \quad -B_2 \quad -B_3 \quad -B_4 \quad B_1 \quad B_2 \quad B_3 \quad B_4] \tag{5}$$

and

$$\{q\}^T = \{u_1 \;\; v_1 \;\; w_1 \ldots u_8 \;\; v_8 \;\; w_8\} \tag{6}$$

On minimising the potential energy of the element, we obtain the stiffness matrix, $[K^/]$, of the interface element in the local coordinates. Thus,

$$[K^/] = \iint [N]^T [k][N] \, dx \, dy \tag{7}$$

where [k] is the material property matrix containing unit shear and normal stiffnesses. This area integral can be easily computed if we make a change of the variables by writing:

$$dx \, dy = \mathrm{Det}[J] \, d\xi \, d\eta \tag{8}$$

where [J] is the Jacobian matrix.

Equation (7) can now be integrated numerically, using the Gauss quadrature formula for example. In general, for a n-point Gauss integration scheme, we can write

$$[K^{/}] = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[N(\xi_i,\eta_j)\right]^T [k]\left[N(\xi_i,\eta_j)\right] \det[J]\, w_i w_j \qquad (9)$$

in which w_i and w_j are the weighting coefficients.

Finally, the joint stiffness $[K^{/}]$ is transformed into its global counterpart [K] through

$$[K] = [H]^T [K^{/}][H] \qquad (10)$$

in which [H] denotes the 3x3 transformation matrix between local and global coordinate axes.

3.4 Interface behaviour

The interface problem is complex and inherently non-linear. The development of improved models of material behaviour is made possible by the increased sophistication of numerical methods of stress analysis. Adequate determination of constitutive laws to define the behaviour at the interfaces is important, and a number of publications have given details of non-linear elastic and elasto-plastic constitutive laws that can be used. In the simple procedure adopted here, only the shear and normal stiffness of the interface element are defined, and therefore the material property matrix in Eq. (7) takes the form

$$[k] = \begin{bmatrix} k_{sx} & 0 & 0 \\ 0 & k_{sy} & 0 \\ 0 & 0 & k_{nz} \end{bmatrix} \qquad (11)$$

in which k_{sx} and k_{sy} denote unit shear stiffnesses along x and y directions respectively; and k_{nz} denotes unit normal stiffness along the z direction. This definition of [k] assumes that the shear and normal modes are uncoupled, and ignores the effects of shear dilatancy which was considered to be negligible in this case because of the type of joint [45]. In general, joints can be *dilatant*, if shearing produces expansion or contraction of the joint, or *non-dilatant*, if there is no coupling between the direct and the shearing aspects. Because of the lack of coupling, the elasticity matrix of a non-dilatant joint is diagonal.

The behaviour of the joint material is both complex and non-linear. Figure (2) shows the idealised constitutive relations of the joint element in the normal and tangential direction to the joint that can be adopted for the sake of simplicity. The joint behaviour can be characterised as elastic-perfectly plastic, and incapable of withstanding any tensile stress in the direction normal to the bed joint. In relating stress to deformation in the direction normal to the joint, two distinct stages are defined (Figure 2): (1) Separation, which occurs when the normal strain is less than or equal to zero; the joint cannot now sustain any tensile stress in the normal direction. During separation, both normal and shear stiffnesses of the interface element are set equal to zero; consequently shear or direct stress cannot be transmitted across the joint. (2) Contact, which is restored when the normal strain returns to the value at which separation occurred.

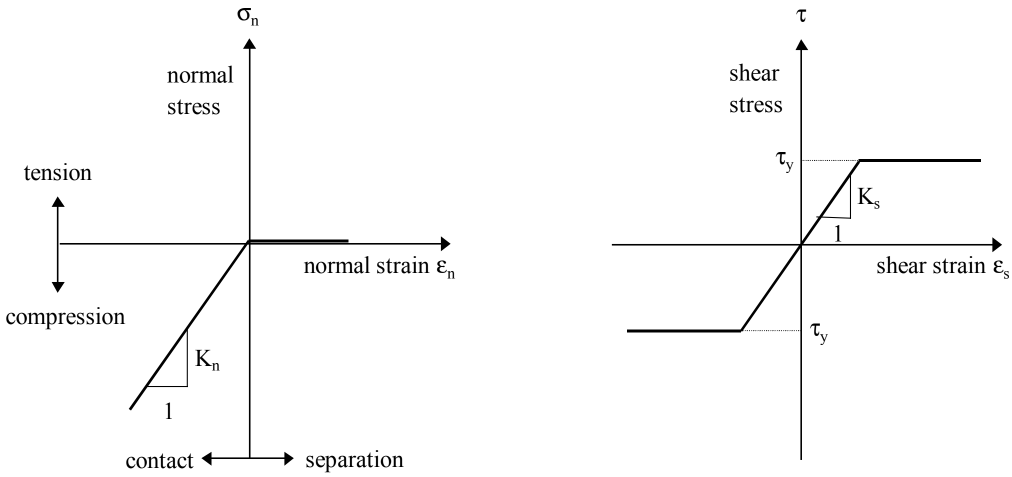

Figure 2. Constitutive relations of the joint element

The tangential stress-strain relationship is assumed to be elastic-perfectly plastic, based on the Mohr-Coulomb yield criterion:

$$\tau_y = c + \sigma_n \tan\phi \tag{12}$$

in which c and ϕ denote cohesion and the friction angle respectively. Within the range of $\tau<\tau_y$, sliding does not occur and the behaviour is elastic. If shear stress reaches its yield value, τ_y, slippage is assumed to occur. During slippage, joint stiffness in the normal direction is assumed to remain constant, but a reduced (or residual) shear stiffness is allocated to the joint. The non-linear behaviour of the joints can be treated by assigning the joint properties corresponding to the level of stress obtained from the last load step in a step-by-step loading procedure.

3.5 Factors influencing joint parameters

In order to predict the 'potential behaviour' of the joints under loading, three distinct joint parameters must be introduced in the analysis [23]: (a) The unit stiffness across the joint, k_n; (b) the unit stiffness along the joint, k_s; and (c) the shear strength along the joint (defined by c and ϕ). In some cases it is desirable to introduce off-diagonal stiffness terms (Eq. 11) to account for dilation during shearing in the stiffness analysis.

Many factors may influence these joint parameters, and these should be taken into account before assigning the relevant properties to the joints. In particular, the value of k_n will depend on the contact area ratio between the two joint surfaces and the relevant properties of the joint filling material, if present. The tensile strength of joints is usually considered to be negligible. The value of k_s will depend on the roughness of the joint surfaces, which can be determined by the distribution, amplitude, and inclination of the asperities; as well as the relevant properties of the joint filling material, if present. The value of shear strength will depend on the friction along the joint, the cohesion due to interlocking, and the strength of the filling material, if present. Thus, the filling material present in the joint may have a decisive effect on all the three parameters mentioned above. Moisture in a joint may also influence all three parameters, indirectly, through the influence on the filling material properties, or directly, by altering the frictional strength of an unfilled joint. In general, therefore, the factors influencing the values of the joint parameters are difficult to quantify for a given joint. Even if relevant data for each factor could be measured, a decision on the relative influence of each factor has to be made in order to obtain the parameters required. Therefore, the direct measurement of the individual joint parameters is necessary in order to obtain realistic values [23].

The joint stiffness concept is relatively new, and no values are to be found in reports and publications about joint parameters, apart from some data in the results of direct shear tests performed on individual specimens with joints. These values, however, can only be used for the analysis of the particular structure under consideration, since the wide range of possible joint conditions indicates the likelihood of extremely different response to the applied load for joints of different classifications and characteristics.

4 CONCLUSIONS

This study presents an efficient finite element analysis technique which shows great versatility in analysing complex discontinuous systems subjected to static, dynamic, or seismic loadings. The method incorporates discontinuities in the analysis of discontinuous structures by use of interface elements designed to simulate the actual behaviour at the interfaces between contacting materials. The capabilities and applicability of the method have been demonstrated in Part 2 of this study, by using it to analyse an original case-problem; an unreinforced masonry wall assemblage subjected to static loadings and seismic base inputs. Numerical results obtained, verify the accuracy of the method against available experimental and theoretical results.

The proposed procedure shows good potential for application to a number of complex systems. Although applications to masonry structures and concrete gravity dams have only been performed by the authors, several other types of problems and jointed systems, in which the effects of interface discontinuities play a prominent role in the accurate determination of the systems' response, can be dealt with by the method. A general-purpose finite element program has been developed for the analysis, incorporating the effects of discontinuities acting as planes of weakness, and non-linear inelastic behaviour of the constituent materials. Various solution algorithms have been developed and incorporated in the program so that the desired method can be selected to give the most economical analysis for each problem.

An examination of results, obtained from the individual case studies examined, demonstrates that the overall response of a discontinuous system to external loading is significantly affected by behaviour at the interfaces between the contacting materials. These discontinuities constitute planes of weakness and exhibit non-linear inelastic behaviour, such as interface slippage and separation. Consequently, an analysis that assumes perfect bonding at the interface, over-predicts the shear transfer between the contact surfaces and, depending on the specific application, this leads to an over- or under-estimation of the system response. Through the inclusion of discontinuities with particular measurable properties, the method of analysis discussed conforms considerably better to actual conditions than do other methods where a continuum is assumed.

The provision of joint elements at the interfaces permits freedom of movement at the nodes and results in improved distribution of stresses between the contact surfaces. The interface element presented adequately simulates the joint behavioural features and its use in the finite element analysis of discontinuous systems may lead to a more rational design of these structures. It is important to note that before the method can be adopted in design-type codes, it must be verified and validated against other significant experimental data.

5 SCOPE FOR FURTHER RESEARCH

Finite element analysis of the seismic response of complex discontinuous systems with material non-linearity presents a formidable problem. Extensive investigation shows that no work has apparently been done in this important area to date. The present study provides a basic step towards the better understanding and solution of this very complex and important class of problems in which the effects of interface discontinuities play a prominent role in the behaviour of the system. Due to shortage of funds, it was not possible to carry out experimental work as part of this project necessary to verify and validate the results obtained using the proposed method of analysis. It is clear, however, that in order to understand thoroughly the structural behaviour of each individual discontinuous system to the various loading conditions, a considerable amount of investigative work, both experimental and theoretical, still remains to be done.

At the present time it is true to say that numerical solution capabilities are in advance of the knowledge of fundamental material behaviour. This is particularly true for dynamic problems, where there is a substantial lack of information on material response under transient conditions, especially in the non-linear regime. Experimental work is thus very much in need, and this should be especially directed towards the better understanding of interface behaviour and the parameters that influence it. The joint stiffness concept is relatively new, and no values are to be found in reports and publications about joint parameters, apart from some data in the results of direct shear tests performed on individual specimens of joints. There are many types of joints, and detailed quantitative data on the mechanical behaviour of all types have yet to be obtained both in linear and non-linear regimes. Additional tests should be carried out in order to obtain reliable constitutive and strength properties of multi-axially loaded jointed systems, and to explore the possible range of joint behaviour under dynamic loading conditions. A meaningful representation of joints in finite element analysis must closely model the actual prototype characteristics, since the wide range of possible joint conditions indicates the likelihood of extremely different response to applied load for jointed systems with different characteristics.

Non-linear finite element methods offer the possibility of conducting ‘numerical experiments’ to provide insight into material behaviour which could not be obtained by experiment alone. The application of the proposed method to problems in applied mechanics are numerous, and it can be used as an effective tool for further research into the behaviour of complex discontinuous systems. Interface elements with different characteristics and capabilities may be used in conjunction with other two or three-dimensional linear or quadratic isoparametric elements to efficiently analyse various types of contact problems and layered or jointed systems. Detailed objectives for further investigation cannot be proposed for each of the variety of structures encountered in Engineering, but recommendations for further research on the particular type of structures examined in Part 2 of this study (i.e., masonry structures) are given, together with a comprehensive literature survey on the subject.

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APPENDIX I. NOTATION

The following notation has been used in this paper:

[B]	=	Matrix of shape functions of interface element
[H]	=	A transformation matrix
[J]	=	The Jacobian matrix
[K]	=	Element stiffness matrix in global coordinates
$[K^{/}]$	=	Element stiffness matrix in local coordinates
[M]	=	Mass matrix
h_i	=	Interpolation (or shape) function
[k]	=	Element material property matrix
k_{sx}, k_{sy}	=	Unit shear stiffness along the x and y directions
k_{nz}	=	Unit normal stiffness along the z direction
$\{q_i\}$	=	Vector of nodal displacements
$\{u_i\}$	=	Vector of displacement components
u, v, w	=	Displacement components parallel to x, y, z axes
w_i, w_j	=	Weighting coefficients for Gaussian integration
η, ξ	=	Natural coordinates of the interface element
τ	=	Shear stress