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Modelling Stock Returns with AR-GARCH Processes*

Elżbieta Ferenstein[1,2] and Mirosław Gąsowski[3]

Warsaw, Poland

Abstract

Financial returns are often modelled as autoregressive time series with random disturbances having conditional heteroscedastic variances, especially with GARCH type processes. GARCH processes have been intensely studying in financial and econometric literature as risk models of many financial time series. Analyzing two data sets of stock prices we try to fit AR(1) processes with GARCH or EGARCH errors to the log returns. Moreover, hyperbolic or generalized error distributions occur to be good models of white noise distributions.

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1 Introduction

Let S_t, $t = 0, 1, \ldots, T$, denote share prices observed at discrete moments. In the considered examples they are daily close prices of Elektrim and Okocim enterprise shares from the Warsaw Stock Exchange over a period 1994–2002. Graphs of the analyzed prices are given in Figures 1 and 3. Let R_t denote the log return at time t, so

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[1] *Address for correspondence:* Faculty of Mathematics and Information Science. Warsaw University of Technology. Pl. Politechniki 1, 00-661 Warsaw, Poland

[2] *Address for correspondence:* Polish-Japanese Institute of Information Technologies. Koszykowa 86, 02-008 Warsaw, Poland

[3] Bank Gospodarki Żywnościowej S.A. Kasprzaka 10/16, 01-211 Warsaw, Poland

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$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right), \qquad t = 1, 2, \dots T. \tag{1}$$

Let $X_t = R_t - \overline{R}$ be the mean-centred process, where $\overline{R}$ denotes the sample mean over the observation period. Within a class of autoregressive processes with white noises having conditional heteroscedastic variances we try to find reasonable models of $\{X_t\}$. Some well known processes from a broad class of GARCH processes are listed below. $\{X_t\}$ is called an autoregressive process of order k with an GARCH noise of order p, q, in short AR(k)-GARCH(p,q) process, if for $t = 0, \pm 1, \pm, 2, \dots$:

$$X_t = \varphi_1 X_{t-1} + \cdots + \varphi_k X_{t-k} + \varepsilon_t, \tag{2}$$

$$\varepsilon_t = \sigma_t v_t, \tag{3}$$

where $\{v_t\}$ is a strong white noise (iid (0,1)), and $\{\sigma_t\}$ satisfies the recurrence equation

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2. \tag{4}$$

Thus $\mathrm{Var}(\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \dots) = \sigma_t^2$, $E(\varepsilon_t) = 0$, $\mathrm{Cov}(\varepsilon_t, \varepsilon_s) = 0$, $t \neq s$. The residuals $\{\varepsilon_t\}$ satisfy (3) and (4) is an GARCH(p,q) process (Bolereslev 1986)). If in (4) all $\beta_i = 0$, then $\{\varepsilon_t\}$ is an ARCH(p) process introduced by Engle (1982). Above processes with heteroscedastic variances model such features of financial time series as risk variability, clustering data, leptokurtic property. Popular models of $\{v_t\}$ distributions are: normal, t-Student, GED (generalized error distribution), and hyperbolic. The latter three distributions have heavier tails than normal distributions (leptokurtic property). For instance, Eberlein and Keller (1995) have found that returns of some financial time series from German Stock may be considered as strong white noise from a hyperbolic distribution with $\mu = 0$, where the hyperbolic density is written as

$$f(x; \alpha, \beta, \delta, \mu) = \frac{\sqrt{\alpha^2 - \beta^2}}{2\alpha\delta K(\delta\sqrt{\alpha^2 - \beta^2})} \exp\left(-\alpha\sqrt{\delta^2 + (x-\mu)^2} + \beta(x-\mu)\right),$$

$\alpha > 0$, $|\beta| < \alpha$, K is a type I Bessel function, i.e.

$$K(t) = \frac{1}{2}\int_0^\infty \exp\left(-\frac{1}{2}t\left(x + \frac{1}{2}\right)\right) dx, \qquad t > 0.$$

Some disadvantage of GARCH processes in modelling financial returns is a symmetry of conditional variance of ε_t with respect to positive and negative values of $\varepsilon_{t-1}, \varepsilon_{t-2}, \dots$. In practice, one observes an leverage effect, i.e. asymmetric consequences of positive and negative innovations (the conditional variance tends to decrease if noise is positive-implying bigger returns). An EGARCH process (exponential GARCH) introduced by Nelson (1991) does not have this disadvantage. An EGARCH(p,q) process $\{\varepsilon_t\}$ satisfies (3) and below relations:

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 g(v_{t-1}) + \cdots + \alpha_p g(v_{t-p}) + \beta_1 \ln(\sigma_{t-1}^2) + \cdots + \beta_q \ln(\sigma_{t-q}^2),$$

where $g(v_t) = \theta v_t + \delta(|v_t| - E|v_t|)$.

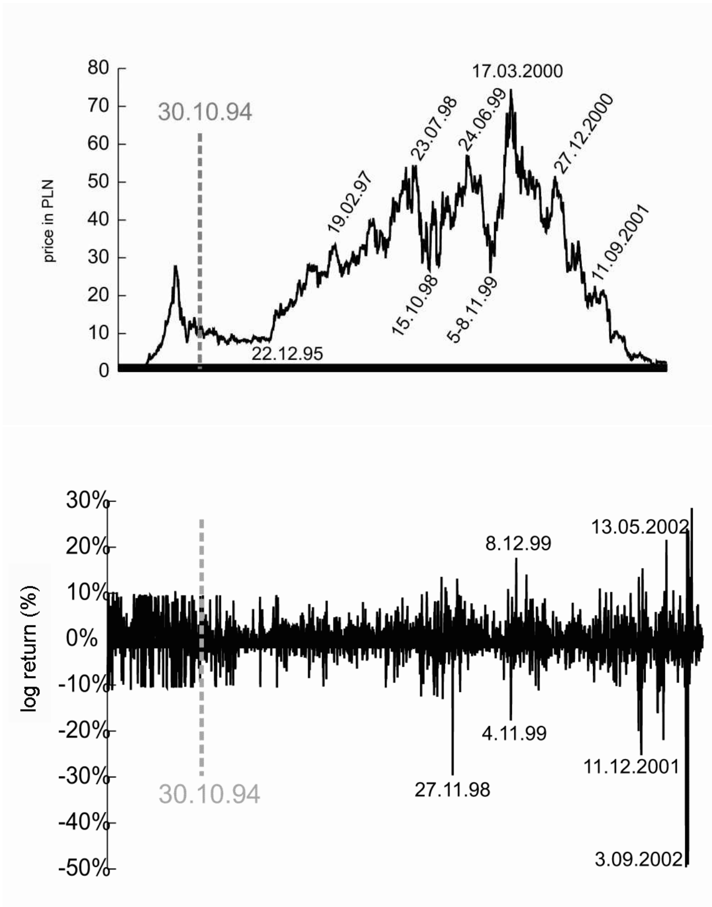

Figure 1: *Elektrim 26.03.92–09.12.02 (2398 observations).*

Another process modelling the leverage effect was introduced by Glosten *et al.* (1993). Its conditional variance is as follows

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2.$$

Sometimes, the simplest models of returns may be useful as proposed for instance, in the RiskMetrics process: $X_t = \sigma_t \nu_t$, $\{\nu_t\} \sim \text{iid}\, N(0,1)$, where σ_t^2 is a historical variance estimated as

$$\hat{\sigma}_t^2 = (1-\lambda) \cdot \sum_{j=0}^{n} \lambda^j X_{t-1-j}^2 = (1-\lambda) X_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2,$$

where the smoothing constant $\lambda = 0.94$ (0.96) for daily (monthly) returns, $n = \frac{\ln \gamma}{\ln \lambda}$, with γ such that $1 - \lambda^n = 1 - \gamma$.

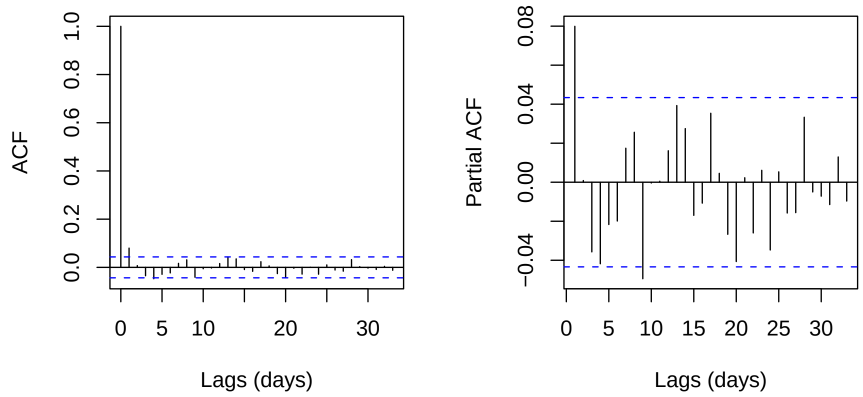

Figure 2: *Elektrim – sample ACF and PACF of log returns.*

2 Modelling empirical returns

Figures 1 and 3 present log returns of daily close prices of two enterprise shares– Elektrim and Okocim, from Warsaw Stock. If they were realizations of strong white noises their empirical autocorrelations

$$\widehat{\rho}(j) = \widehat{\gamma}(j)/\widehat{\gamma}(0), \quad j = 1, 2, \ldots, h, \quad h < T,$$

$\widehat{\gamma}(j) = T^{-1} \sum_{t=1}^{T-j} (R_{t+j} - \overline{R})(R_t - \overline{R})$, would have behaved for large T approximately as random samples from normal distribution with the mean 0 and variance T^{-1}. Hence, the null hypothesis that log returns are realizations of random samples should be rejected if more than 5% of sample autocorrelations fall out of $[-1.96/\sqrt{T}, 1.96/\sqrt{T}]$ or at least one is significantly far from this interval. Analysing graphs of sample autocorrelations (ACF) at Figures 2 and 4 we tend to reject the hypothesis. Small p-values of autocorrelation and portmanteau tests in Tables 1 and 2 justify the latter.

Table 1: *Elektrim.*

Test	Value of test statistic	p-value
Autokorrelation, lag 1	$\widehat{\rho}(1)\sqrt{T} = 3.610296$	0.0003
Ljung-Box	$Q_{LB}(1) = 13.0528$	0.0003
	$Q_{LB}(6) = 22.9662$	0.0008
	$Q_{LB}(12) = 29.5553$	0.0032
	$Q_{LB}(24) = 46.3575$	0.0040

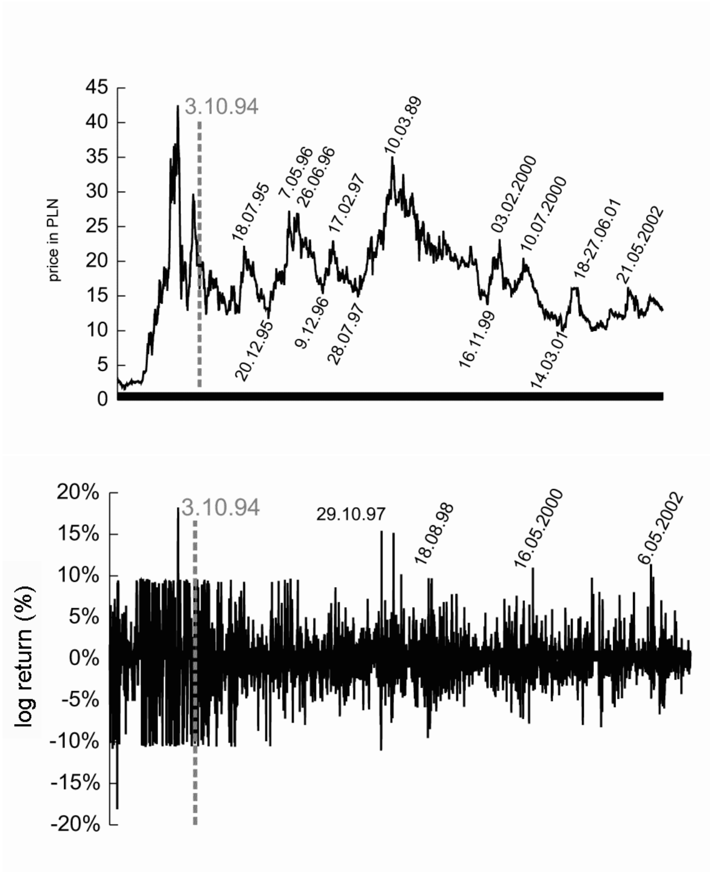

Figure 3: *Okocim 13.02.92–03.01.03 (2423 observations).*

Table 2: *Okocim.*

Test	Value of test statistic	p-value
Autocorrelation, lag 1	$\widehat{\rho}(1)\sqrt{T} = 2.474692$	0.0133
Ljung-Box	$Q_{LB}(1) = 6.1221$ $Q_{LB}(6) = 11.9245$ $Q_{LB}(12) = 20.9687$ $Q_{LB}(24) = 32.0897$	0.0133 0.0637 0.0508 0.1248

Ljung-Box statistics $Q_{LB}(h) = T(T+2)\sum_{j=1}^{h}\frac{\widehat{\rho}^{2}(j)}{T-j}$, based on strong white noise, have for long observation period T approximately the chi-square distribution wih h degrees of freedom. Graphs of partial autocorrelation functions (PACF) at Figures 2 and 4 have specific pikes for lags 1, and for lags bigger than 1 partial autocorrelations are close to 0. One may suspect then that AR(1) model can describe well the observed returns:

$$R_t - \mu = \Phi(R_{t-1} - \mu) + \varepsilon_t. \qquad (5)$$

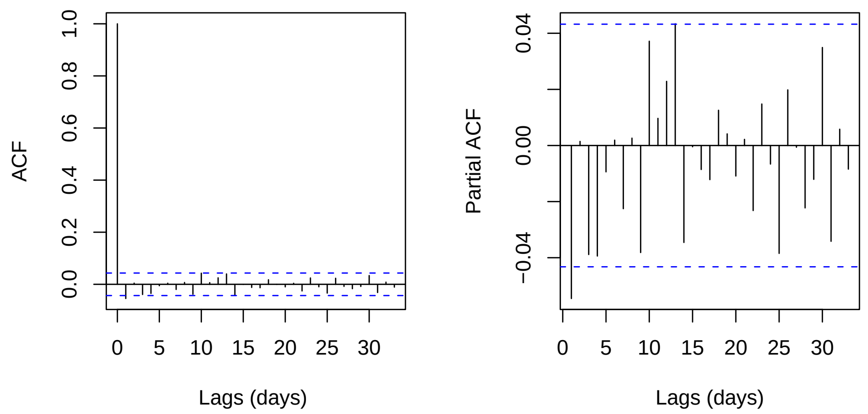

Figure 4: *Okocim – sample ACF and PACF of log returns.*

Assuming (5) and using quasi maximum likelihood method for the Elektrim data set we get the estimated model:

$$R_t + 0.0007429 = 0.07988(R_{t-1} + 0.0007429) + \widehat{\varepsilon}, \tag{6}$$

where estimates of model parameters in (6) maximize maximum likelihood function value obtained for $\{\varepsilon_t\} \sim$ iid $N(0, \sigma)$.

Based on p-values of statistics in Table 3, obtained for sample residuals $\{\widehat{\varepsilon}_t\}$, one cannot reject the hypothesis that residuals $\{\widehat{\varepsilon}_t\}$ form a strong white noise, and thus our log returns follow (5).

Table 3: *Elektrim.*

Test	Value of test statistic	p-value
Autocorrelation, lag 1	$\widehat{\rho}(1)\sqrt{T} = 0.008595$	0,993142
Ljung-Box	$Q_{LB}(1) = 0.0001$ $Q_{LB}(6) = 7.9749$ $Q_{LB}(12) = 15.0624$ $Q_{LB}(24) = 30.9048$	0.9931 0.2399 0.2380 0.1565

Analyzing the graph (Figure 5) of sample residuals $\{\widehat{\varepsilon}_t\}$ one can try to model residuals as a GARCH process since there is some clustering of close in time values. The Lagrange Multiplier test rejects the homoskedasticity hypothesis.

Processes listed in Table 4 have been chosen for farther analysis of residuals $\{\widehat{\varepsilon}_t\}$ behaviour. Particular models' orders minimize values of Akaike AIC$= -2\ln L + 2k$ or Schwarz SBC$= -2\ln L + k\ln n$ criteria, where k is a number of unknown parameters, n is a sample size, L denotes the maximum likelihood function. If more than one model have

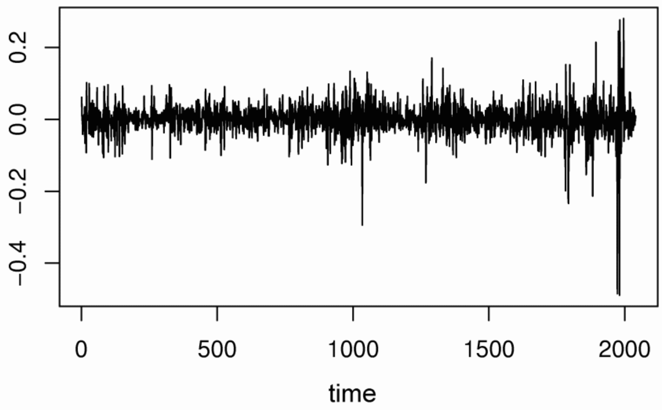

Figure 5: *Elektrim.*

Table 4:

Model	MSE	MAE	MPE	MMEO	MMEU
Homoscedastic	7.80208E-05	0.0023135	12396.3013	0.0122335	0.0310000
GARCH(2,2)	7.46337E-05	0.0021997	9243.13979	0.0126269	0.0265368
GARCH(2,2) with intercept	7.46492E-05	0.0022033	9147.99753	0.0126242	0.0265767
Stationary GARCH(1,3)	7.4127E-05	0.0021899	8985.63329	0.0126725	0.0264366
Stationary GARCH(1,3) with intercept	7.41663E-05	0.0021928	8902.94863	0.0126701	0.0264665
EGARCH(2,1)	7.496E-05	0.0021222	8339.03033	0.0124950	0.0261317
EGARCH(2,1) with intercept	7.49005E-05	0.0021256	8379.59961	0.0124678	0.0262241
Nonsymmetric GARCH(1,1)(1)	7.59514E-05	0.0022561	7785.23459	0.0125085	0.0269828
Nonsymmetric GARCH(2,2)(1)	7.5816E-05	0.0022114	7740.82165	0.0126797	0.0263438
Nonsymmetric GARCH(2,2)(2)	0.000606997	0.0206617	127872.13	0.0213179	0.1377665
GARCH(1,1) NTD	7.43914E-05	0.0022401	8989.68920	0.0126005	0.0266435
GARCH(2,2) NTD	7.50511E-05	0.0022210	9281.23602	0.0126727	0.0266436

comparable values of both criteria, then the model with lower number of parameters or lower *p*-values of significance parameters' tests is chosen.

Efficiency of model fitting may be evaluated via various measures of errors of squared residuals (estimators of conditional variance of log returns) forecasts, given in Table 4, where

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2)^2 \quad \text{(Mean Squared Error)}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2| \quad \text{(Mean Absolute Error)}$$

$$MPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2}{\widehat{\varepsilon}_t^2}\right| \quad \text{(Mean Absolute Percentage Error)}$$

$$MMEO = \frac{1}{T}\left(\sum_{t=1}^{T}(\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2)^+ + \sum_{t=1}^{T}\sqrt{(\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2)^-}\right)$$ (Mean Mixed Error of Over-predictions)

$$MMEU = \frac{1}{T}\left(\sum_{t=1}^{T}(\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2)^- + \sum_{t=1}^{T}\sqrt{(\widehat{\varepsilon}_t^2 - \widetilde{\varepsilon}_t^2)^+}\right),$$ (Mean Mixed Error of Under-predictions),

where $\widetilde{\varepsilon}_t^2 = E(\hat{\varepsilon}_t^2|\hat{\varepsilon}_{t-1}, \hat{\varepsilon}_{t-2}, \dots) = E(\sigma_t^2 v_t^2|\hat{\varepsilon}_{t-1}, \hat{\varepsilon}_{t-2}, \dots) = \sigma_t^2$,

$$a^+ = \max\{a, 0\}, \quad a^- = \max\{-a, 0\}.$$

Stationary GARCH(1,3) process and stationary GARCH(1,3) process with intercept are best models with respect to MSE measure:
$\hat{\varepsilon}_t = \sigma_t v_t$, where $\sigma_t = 0.000105 + 0.2413\hat{\varepsilon}_{t-1}^2 + 0.0766\sigma_{t-1} + 0.4302\sigma_{t-2} + 0.2026\sigma_{t-3}$, and
$\hat{\varepsilon}_t - 0.000878 = \sigma_t v_t$, where

$$\sigma_t = 0.000103 + 0.2414(\hat{\varepsilon}_{t-1} - 0.000878)^2 + 0.082\sigma_{t-1} + 0.4343\sigma_{t-2} + 0.1982\sigma_{t-3}.$$

Now, we will try to identify a distribution of the noise $\{v_t\}$ in the latter model. Table 5 presents results of testing the hypothesis that $\{v_t\}$ is the strong white noise. At the 5% significance level one cannot reject the hypothesis.

Table 5:

Test	Value of test statsistic	p-value
Ljung-Box	$Q_{LB}(1) = 3.3112$ $Q_{LB}(2) = 3.2188$ $Q_{LB}(6) = 5.9297$ $Q_{LB}(12) = 11.3775$	0.9156 0.1932 0.4311 0.4969
Turning point test	$Z = 655$	0.0521
Difference-sign test	$S = 1009$	0.9508

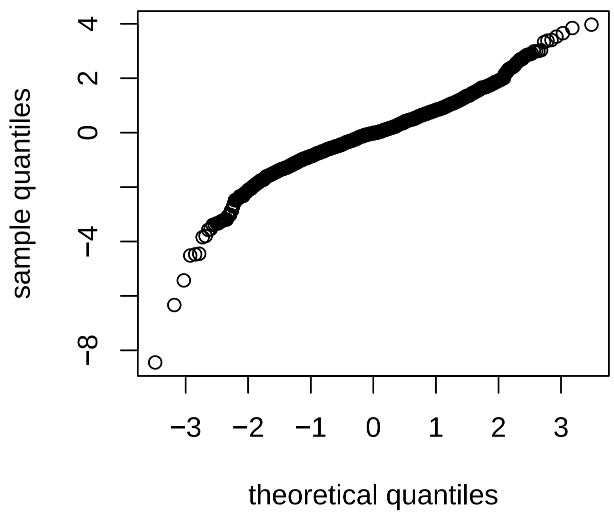

Figure 6: *Elektrim – quantile plot of residuals of log returns AR(1) model.*

Figure 6 presents the normal quantile plot of $\{v_t\}$ which does not resemble a straight line that suggests not normal distribution, with fat tails. Indeed, values of test statistics of Kolmogorov and Shapiro–Wilks tests are 0.0481 and 0.9565, and corresponding p-values are 0.0001595 and 0.22E-15, respectively. The t-Student, hyperbolic and general error distributions are often used as models of heavy tailed distributions. The density of GED(μ, σ, ν) distribution:

$$f(x;\mu,\sigma,\nu) = \frac{\nu}{\lambda 2^{(1+1/\nu)}\Gamma(1/\nu)} \exp\left(-\frac{1}{2}\left|\frac{x-\mu}{\lambda\sigma}\right|^{\nu}\right),$$

where $\lambda = \left(\frac{2^{-2/\nu}\Gamma(1/\nu)}{\Gamma(3/\nu)}\right)^{1/2}$ and μ, $\sigma > 0$, $\nu > 0$ are location, scale and shape parameters, respectively. In particular GED(0,1,2) = N(0,1). If $\nu < 2$ ($\nu > 2$), the density has thicker (thinner) tails than the normal one.

For our sample residuals $\{v_t\}$, maximum likelihood estimators for normal, t-Student, GED, hyperbolic distributions have following values, respectively:

- $\widehat{\mu} = -0.03203267$, $\widehat{\sigma} = 1.002$
- $\widehat{\nu} = 11$
- $\widehat{\mu} = -0.01862165$, $\widehat{\sigma} = 0.991337337$, $\widehat{\nu} = 1.137376404$
- $\widehat{\alpha} = 1.612786198$, $\widehat{\beta} = -0.05965513$, $\widehat{\delta} = 0.481525055$, $\widehat{\mu} = 0.025483705$.

One may measure discrepancy between the fitted and empirical distributions using the functions:

$$h_K(x) = |F_n(x) - F_0(x)|$$

or

$$h_{AD}(x) = h_K(x)/\sqrt{\mathrm{F}_0(x)\cdot(1-\mathrm{F}_0(x))},\ \ x \in (-\infty, \infty),$$

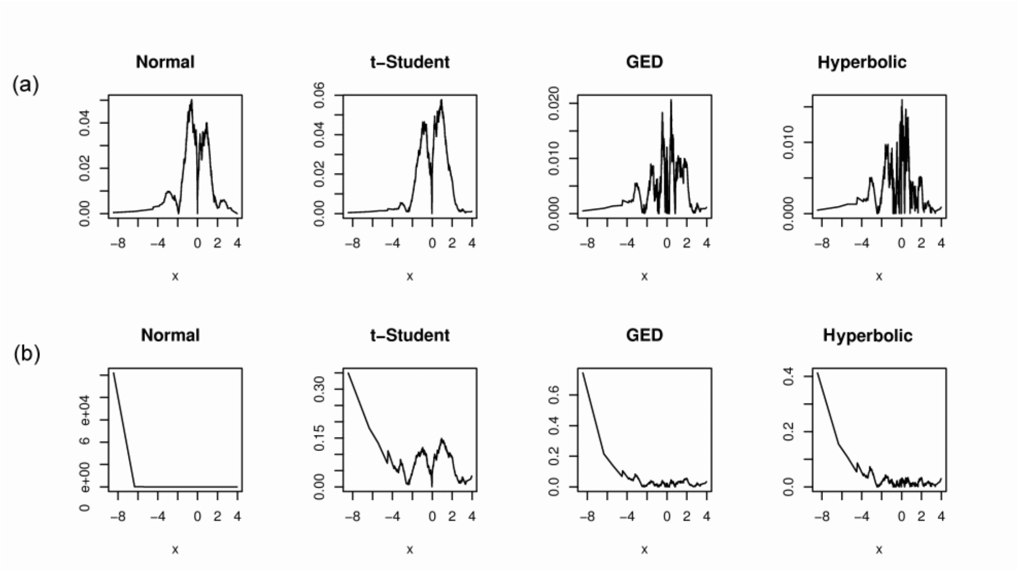

Figure 7: *Elektrim: (a) Kolmogorow statistics $h_K(x)$ for log return residuals (b) Anderson-Darling statistics $h_{AD}(x)$ for log return residuals.*

where F_n and F_0 are empirical and fitted distribution functions, respectively. The function statistic h_{AD} measures discrepancy in tails of the distributions better than h_K. From graphs at Figure 7 we state that GED and t-Student distribution functions are closer to the empirical one, especially at the tails.

Table 6 presents values of Anderson-Darling $AD = \sqrt{n}\sup\{h_{AD}(x), x \in (-\infty,\infty)\}$ and Kolmogorow–Smirnow $K = \sqrt{n}\sup\{h_K(x), x\in(-\infty,\infty)\}$ statistics with the corresponding p-values. Only GED and hyperbolic distributions are not rejected.

Table 6: *Elektrim.*

Distribution	Value of AD	Value of K	p-value
Normal	46.03884×10^6	2.269845	0.00001
t-Student	15.807146	2.607259	0.00001
GED	33.60154	0.932649	0.3530
Hyperbolic	18.630039	0.722223	0.6880

Figures 8 and 9 show the histogram with the fitted densities and empirical densities on a logarithmic scale.

In columns 1 of tables 7, 8, 9 (for Elektrim data) there are listed best (according to AIC or SBC criteria) AR(1) – xGARCH models of log returns found in two ways:

- two step procedure described so far
- one step maximum likelihood parameter estimation

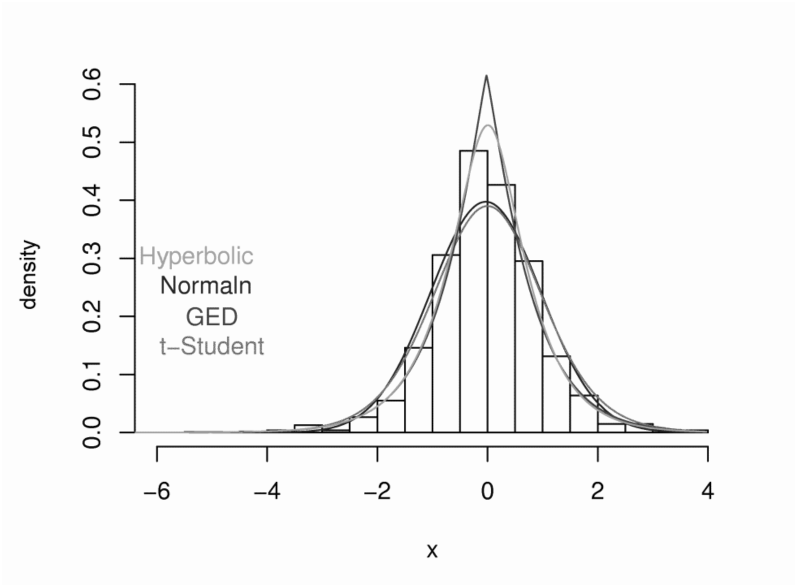

Figure 8: *Elektrim – Histogram of noise* $\{v_t\}$ *in GARCH model of log return residuals.*

Table 7: *Percentage of the observed log returns out of 99% prediction limits.*

No	Noise Log Return Model	Stand. Normal	Normal	GED	Hyper- bolic	t-Student
1	iid standardized return	1.7638	1.7148	0.8329		0.8329
2	1-step est. AR(1)-iid noise	1.7647	1.7647	0.8824		0.8824
3	1-step est. AR(1)-GARCH(1,1)	0.0000	0.6373	0.7843	1.4706	1.4216
4	1-step est AR(1)-GARCH(2,1)	2.3629	2.3529	1.2255	1.3235	1.2255
5	1-step est. AR(1)- station.GARCH(1,2)	2.4510	2.3529	1.2745	1.5196	1.3235
6	1-step est. AR(1)-EGARCH(1,2)	2.3529	2.3529	1.2745	1.2745	1.2745
7	2-step est. AR(1)- station.GARCH(1,3) with intercept	2.4510	2.4510	1.3235	1.4706	1.3235
8	2-step est. AR(1)- station.GARCH(1,3)	2.4510	2.4510	1.3235	1.5196	1.3235
9	2-step est. AR(1)-EGARCH(2,1) with intercept	2.3039	2.3039	1.3725	1.2255	1.2745
10	2-step est. AR(1)-EGARCH(2,1)	2.3039	2.3039	1.3725	1.2745	1.2745
11	GARCH(1,1)	2.3518	2.4008	1.2739	1.5189	1.2739
12	Classic Risk Metrics	1.4063	2.0090	0.9041	0.8036	1.2054

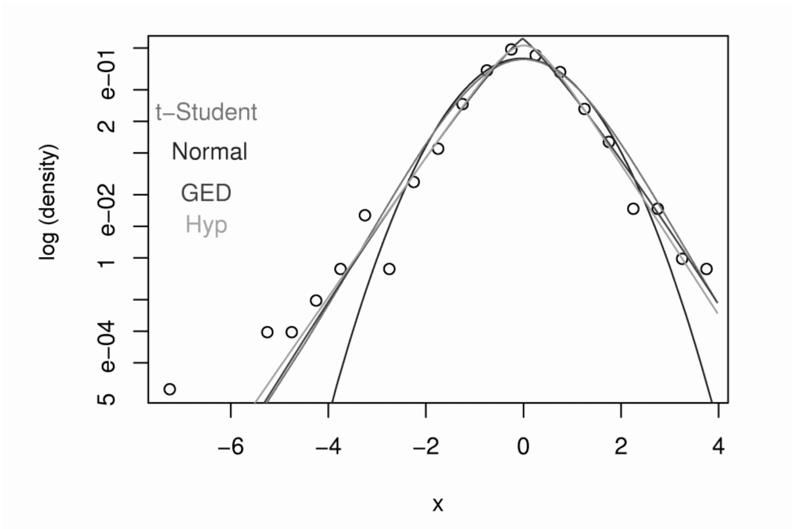

Figure 9: *Observerd relative frequences and fitted densities from Figure 8 on logarithmic scale.*

Table 8: *Percentage of the observed log returns out of 95% prediction limits.*

No	Noise Log return model	Stand. Normal	Normal	GED	Hyper-bolic	t-Student
1	iid standardized return	4.9976	4.9976	4.5076		3.2337
2	1-step est. AR(1)-iid noise	5.0490	5.0490	4.7549		3.2353
3	1-step est. AR(1)-GARCH(1,1)	1.8627	5.0000	5.0000	4.5098	3.8725
4	1-step est. AR(1)-GARCH(2,1)	4.6569	4.8529	4.2157	4.3137	3.7255
5	1-step est. AR(1)- station. GARCH(1,2)	5.0000	4.8039	4.2157	4.3137	3.6275
6	1-step est. AR(1)-EGARCH(1,2)	5.1471	5.1961	4.3137	4.6078	3.8725
7	2-step est. AR(1)- station.GARCH(1,3) with intercept	4.8529	4.8529	4.0686	4.4118	3.7254
8	2-step est. AR(1)- station.GARCH(1,3)	4.9020	4.8039	4.1667	4.2647	3.7255
9	2-step est. AR(1)-EGARCH(2,1) with intercept	5.0000	5.1471	4.3627	4.5098	3.7745
10	2-step est. AR(1)-EGARCH(2,1)	5.1471	5.1471	4.4118	4.5098	3.7745
11	GARCH(1,1)	5.0955	5.1445	4.2626	4.6056	3.7237
12	Classic RiskMetrics	4.9292	5.6755	5.0226	5.2235	4.6710

Table 9: *Percentage of the observed log returns out of 90% prediction limits.*

No	Noise Log return model	Stand. Normal	Normal	GED	Hyper- bolic	t-Student
1	iid standardized return	6.9084	6.9574	8.0843		5.6345
2	1-step est. AR(1)-iid noise	6.9608	6.9118	8.1373		5.6373
3	1-step est. AR(1)-GARCH(1,1)	4.9610	10.3431	10.441	8.8235	6.5686
4	1-step est. AR(1)-GARCH(2,1)	8.3333	8.3824	8.6275	9.2157	8.1275
5	1-step est. AR(1)- station.GARCH(1,2)	8.2843	8.2353	8.4314	9.0686	6.3725
6	1-step est. AR(1)-EGARCH(1,2)	8.2353	8.1863	8.6275	8.9706	6.6176
7	2-step est. AR(1)- station.GARCH(1,3) with intercept	8.4804	8.4804	8.6275	9.2157	6.3235
8	2-step est. AR(1)- station.GARCH(1,3)	8.5294	8.4804	8.5784	9.0686	6.3725
9	2-step est. AR(1)-EGARCH(2,1) with intercept	7.9902	8.0882	8.2843	8.9786	6.3725
10	2-step est. AR(1)-EGARCH(2,1)	8.0392	8.0392	8.2843	8.9216	6.3725
11	GARCH(1,1)	8.0843	8.3213	8.4762	9.0152	6.3204
12	Classic RiskMetrics	8.6891	9.6434	9.8945	9.9950	8.3877

References

Blaesid, P. and Sorensen, M. (1992). 'Hyp'—a computer program for analyzing data by means of the hyperbolic distribution, *Research Report*, 248, University Aarhus, Dept. Theor. Statist.

Eberlein, E. and Keller, U. (1995). Hyperbolic distributions in finance, *Bernoulli*, 1, 281-299.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, *Econometrica*, 50, 987-1008.

Bollerslev, T. (1986). Generalised autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31, 307-327.

Gąsowski, M. (2002). *Modelling financial returns with nonlinear time series from ARCH family processes*, M. Sc. Thesis, Warsaw University of Technology, Warsaw.

Glosten, L. R. *et al.* (1993). Relationship between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, 48, 1779-1801.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, 59, 347-370.

Resum

Les rendibilitats financeres es modelen sovint com a sèries temporals auto-regressives amb pertorbacions aleatòries amb variàncies condicionals heterocedàstiques, especialment amb processos de tipus GARCH. Els processos GARCH han estat intensament estudiats en la literatura financera i economètrica com a models de risc de moltes sèries financeres. Analitzant dos conjunts de dades de preus d'actius tractem d'ajustar processos AR(1) amb errors GARCH o EGARCH a les log-rendibilitats. A més, distribucions hiperbòliques o d'errors generalitzats resulten ser bons models de distribucions de soroll blanc.

MSC: Primary 62M10, 91B84; secondary 62M20

Paraules clau: processos auto-regressius, models GARCH i EGARCH, variància condicional heterocedàstica, log-rendibilitats financeres