Stochastic Form of the Growth of Wind Waves in a Single-Parameter Representation with Physical Implications

YOSHIAKI TOBA

Geophysical Institute, Tohoku University, Sendai 980, Japan

(Manuscript received 10 August 1976, in final form 18 January 1978)

ABSTRACT

It is shown that a simple relation, $E^* = 5.1 \times 10^{-2} \sigma_m^{*-3}$, for describing the conditions of growing wind waves, is supported by various available data, where $E^* = g^2E/u_*^4$ is dimensionless energy, $\sigma_m^* = u_*\sigma_m/g$ the dimensionless angular frequency at the maximum of the energy spectrum, g the acceleration of gravity and u_* the friction velocity of the air. This expression is an alternative form of the relation between dimensionless wave height and period, $H^* \propto T^{*3/2}$, which was previously proposed by the author (Toba, 1972) for energy-containing waves, and is extended to individual waves in the wind-wave field in a statistical sense. It is also shown, supported by various data, that the essential part of the one-dimensional energy spectra of growing wind waves should have the form $g_*u_*\sigma^{-4}$ for the high-frequency tail of the frequency spectrum, where g_* is g expanded to include the surface tension. This is the form previously proposed by the author (Toba, 1973b) as the one-dimensional spectral form consistent with the above power law relationship, instead of the $g^2\sigma^{-5}$ form proposed by Phillips (1958). By use of the power-law relationship for E^*, it is shown that the proportion of that part of momentum which is retained as wave momentum to the total momentum transferred from the wind to the sea can be expressed by a function of σ_m^*, which has essentially the same physical meaning as C/U, the ratio between the phase velocity of the energy containing wave and the wind speed. The value of the proportion decreases from about 6% in the form of an error function of C/U. A prediction equation for the growth of wind waves by a single-parameter representation is proposed, in which the rate of change of E^* is expressed by a formulation including the error function or by a simple stochastic form. The integration of the equation for the case of fetch-limited conditions is in excellent agreement with data compiled by Hasselmann *et al.* (1973). Reviewing results of recent wind-wave tunnel experiments, emphasis is given on the fact that wind waves are strongly nonlinear phenomena, especially for $C/U \ll 1$. A discussion is presented from this standpoint as to the physical basis for the existence of the simple power law relationship, the spectral form of $g_*u_*\sigma^{-4}$ and the stochastic form of the growth equation, and a systematic derivation of these relationships and equations is attempted.

1. Introduction

Quantitative study of the growth of wind waves was first initiated by Sverdrup and Munk (1947) through the introduction of the concept of the significant wave height. Since the 1950's (e.g., Pierson, 1952), irregular fluctuations of water level in wind waves have been treated by Fourier analysis techniques, assuming a superposition of free water waves. Since free waves may be generated at a still water surface, it has been postulated that the motion is irrotational, and that a pressure variation along the air-water interface is essential for the generation of wind waves. These assumptions became the foundation of modern theories, as represented by the mechanisms of wind-wave generation proposed by Phillips (1957) and Miles (1957), although the mechanisms have not yet been verified experimentally. The fact that the time scale of the generation and decay of the wind-wave field is large compared with wave period, has been the basis for the treatment of wind waves as an expansible into dispersive components and weakly nonlinear phenomenon (e.g., Hasselmann, 1968). Hasselmann *et al.* (1973), through a computation of nonlinear energy transfer using the empirical form of the spectrum obtained in an intensive observational project (JONSWAP), concluded that the energy flux across the peak of the spectra is due mainly to nonlinear resonant interactions among component waves, and plays the dominant role in the development of the wind-wave spectrum, although the mechanism of the momentum transfer from the air to the wind-wave field and the dissipation processes remain to be clarified.

On the other hand, efforts to experimentally determine these processes occurring at the real air-water interface have received considerable emphasis (e.g., Kunishi, 1957, 1963; Shemdin, 1972; Wu, 1975; Banner and Phillips, 1974; Banner and Melville, 1976; Imasato and Ichikawa, 1977). Based

0022-3670/78/0494-0507$07.00

on findings by Wright and Keller (1971) that wind waves in the gravity-capillary region evolve from the wind-induced surface drift in water, Valenzuela (1976) revised the Miles mechanism by introducing shear flows on both sides of the air-water interface to describe the early stage of the generation of wind waves. Toba *et al.* (1975), Okuda *et al.* (1976, 1977) and Kawai (1977) demonstrated, by flow visualization studies of wind waves in wind-wave tunnels, that immediately (on the order of 1 s) after the initial generation of more regular waves by the wind they change into irregular wind waves, and the transition seems to occur through the following processes. A large nonuniformity in the distributions of the tangential stress of the wind and of the wind drift surface flow appears relative to each momentary crest of surface undulations. The tangential stress is concentrated at the crest and its windward face, and a convergence of the surface flow and a consequent downward thrust of water occurs near the crest, and somewhat to the leeward face, irrespective of whether or not the waves are breaking. Consequently, wind waves are always accompanied by forced convections, relative to the momentary wave profiles, producing a turbulent mode. Recent work by Rikiishi (1978) has demonstrated, through well-controlled experiments in a wind-wave tunnel, that the dispersion relation breaks down for component waves.

According to the above studies, it can be concluded that the growth of wind waves is a phenomenon in which strong nonlinearities are essential. Let us now consider a dimensionless parameter C/U, where C is the phase velocity of representative waves and U the wind speed at, say, the 10 m level. The value of the parameter determines whether the state of the wind-wave field is growing or is already in a developed state, i.e., essentially in a state of equilibrium. This parameter can be replaced by the ratio of C and the friction velocity of the air u_*, or by $T^* = gT/u_*$ where T is the representative wave period and g the acceleration of gravity. In any case, if C/U is large enough to achieve the equilibrium state (empirically determined to be $\sim$1.4 for a steady wind), waves near the frequency of the maximum spectral density will satisfy the condition of superposition of free component waves. If the wind becomes weak or ceases, C/U tends to infinity and the wind waves become swell which can be treated approximately as linear waves. However, in the case of wind waves of very small C/U, as is observed in wind-wave tunnels, nonlinearities are very large and it is almost impossible to treat them in a deterministic way. For wind waves of $C/U \approx 1.4$, the condition of strong nonlinearities will be the same for the higher frequency part.

The discussion in the present article is presented from the point of view that while wind waves are strongly nonlinear phenomena, regularities in the gross structure of growing wind waves are sought, in forms as simple as possible, by invoking an assumption of a kind of self-similar structure in the field of the wind waves and by dimensional self-consistency.

This line of approach is an extension of that of Kitaĭgorodskiĭ (1961), who introduced the similarity law in wind waves, Phillips (1958), who proposed the g^2-f^{-5} spectral form based on dimensional considerations, and Sverdrup and Munk (1947) who introduced the concept of significant waves and dimensionless parameters. With the intent of advancing the abovementioned point of view explicitly, the author (Toba, 1972, 1974a) proposed a 3/2 power law [expressed by Eq. (2.1) in the next section] for growing wind waves, and attempted to obtain a single-parameter growth equation of wind waves. Hasselmann *et al.* (1973) demonstrated the self-stabilizing nature of nonlinear interactions among component waves during the evolution of the spectrum and Hasselmann *et al.* (1976) proposed a rapid adjustment of the spectrum to a quasi-equilibrium level after the change of the wind, on the basis of source terms of nonlinear wave-wave interactions, and thus have presented *a posteriori* a basis for a single-parameter growth model. It is noteworthy that their treatment, based on the concept of component waves and weak nonlinear wave-wave interactions, leads to a similar single-parameter model. The author has independently been revising his own model and obtained (Toba, 1976) a somewhat different scheme of the single-parameter growth model from that proposed by Hasselmann *et al.* (1976).

Reinforced by additional information from the Hasselmann *et al.* (1976) study, the first half of this article presents a more simple form of the single-parameter representation of growing wind waves along with a stochastic form of the growth equation. Later, a discussion and a possible interpretation of the physical basis of the scheme is presented.

2. The single-parameter representation for growing wind waves

We start with a description of the empirical basis of our single-parameter representation. The author (Toba, 1972, 1974a) has previously proposed a relation between the dimensionless significant wave height and period for growing wind waves which is a power law of the form

$$H^* = BT^{*3/2}, \quad B = 0.062, \tag{2.1}$$

where

$$H^* \equiv gH/u_*^2 \quad \text{and} \quad T^* \equiv gT/u_*,$$

and where H represents the significant wave height and T the significant wave period. Eq. (2.1) may be converted to a relation between the total energy E and the frequency f_m or the angular frequency σ_m of the spectral maximum, by

$$E^* = B_f f_m^{*-3}, \quad B_f = 2.1 \times 10^{-4}, \tag{2.2}$$

or

$$E^* = B_\sigma \sigma_m^{*-3}, \quad B_\sigma = 5.1 \times 10^{-2}, \tag{2.3}$$

where $E^* \equiv g^2E/u_*^4$, $f_m^* \equiv u_* f_m/g$, $\sigma_m^* \equiv u_*\sigma_m/g$. In the course of the conversion, the relations

$$f_m = (1.05T)^{-1} \tag{2.4}$$

after Mitsuyasu (1968) and Toba (1973b), and

$$E = \int_0^\infty \phi(\sigma)d\sigma = H^2/16 \tag{2.5}$$

after Longuet-Higgins (1952) for narrow-spectrum waves, were used, where $\phi(\sigma)$ is the spectral density. The absolute values of the numerical factors in (2.4) and (2.5) will not be universal, as suggested by Iwata *et al.* (1970, 1971) and also as described by Hasselmann *et al.* (1973) in terms of the variation of Phillips' constant α for individual cases. Consequently, much emphasis will be given in the present paper on the basic forms of equations rather than on values of the numerical factors in the equations.

Although (2.1) was derived by dimensional and macroscopic considerations, it is based on experimental data in a wind-wave tunnel obtained by Toba (1961, 1972), and on empirical formulas presented by Wilson (1965) and Mitsuyasu (1968, 1971). Consequently, (2.2) and (2.3) also are based on these results. Surprisingly, almost exactly the same expression may be obtained from the Hasselmann *et al.* (1973) JONSWAP formulas

$$\epsilon = 1.6 \times 10^{-7}\xi, \quad \xi \lesssim 10^4 \tag{2.6}$$

and

$$\nu = 3.5\xi^{-0.33}, \quad \xi \lesssim 10^4, \tag{2.7}$$

where

$$\left.\begin{aligned} \epsilon \equiv g^2E/U_{10}^4 \equiv C_D^2E^*, \quad \xi \equiv gF/U_{10}^2 \\ \nu \equiv U_{10}f_m/g \equiv f_m^*/(C_D)^{1/2} \end{aligned}\right\},$$

and where F is the fetch and C_D the drag coefficient defined by

$$u_*^2 = C_D U_{10}^2.$$

Eliminating the dimensionless fetch ξ from (2.6) and (2.7), we obtain

$$E^* = 2.0 \times 10^{-4} f_m^{*-3.03} \tag{2.8}$$

or

$$E^* = 5.3 \times 10^{-2} \sigma_m^{*-3.03}, \tag{2.9}$$

where the value of $C_D = 1.0 \times 10^{-3}$, which was used by Hasselmann *et al.* (1973) has been used. These may also be converted to

$$H^* = 0.061 T^{*3.03/2}. \tag{2.10}$$

These equations are considered to be identical with (2.1), (2.2) and (2.3).

A corresponding formula has been given in Hasselmann *et al.* (1976) as their Eq. (3.6):

$$\epsilon = 5.3 \times 10^{-6}\nu^{-10/3}. \tag{2.11}$$

Our formulas (2.2) or (2.3) may be converted to the same form of the expression and yield

$$\epsilon = 7.1 \times 10^{-6}\nu^{-3}. \tag{2.12}$$

In Fig. 1 is shown a comparison of (2.11) and (2.12) together with a composite data set presented in Fig. 10 from Hasselmann *et al.* (1976). Although the superiority of (2.12) over (2.11) is not conclusive from the figure, it seems that (2.12) corresponds slightly more closely to the data.

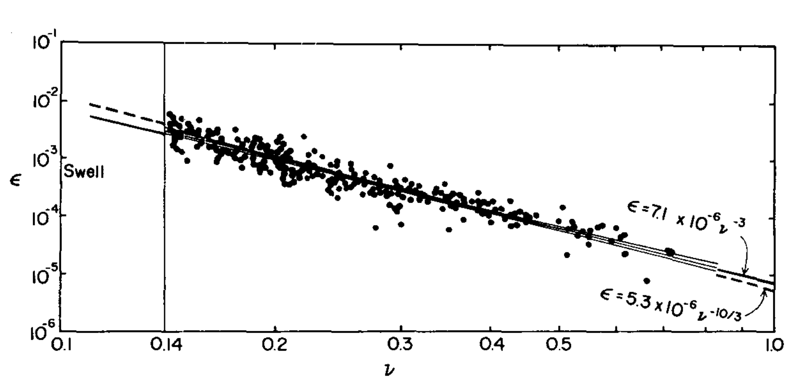

FIG. 1. Reproduction of Hasselmann *et al.* (1976) Fig. 10-J plus positions of straight lines of Eqs. (2.11) and (2.12). The positions are shown only at the right and left ends. The thin straight line flanked by hyperbolas represents the regression line for data points and the standard deviation envelope. The line of (2.12) is almost inside the standard deviation envelope.

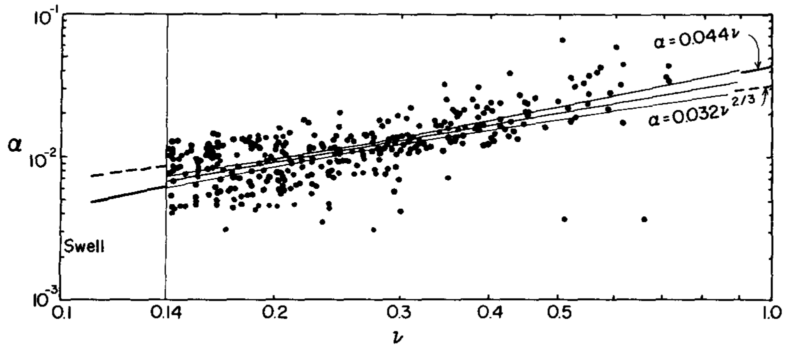

FIG. 2. As in Fig. 1 except for Hasselmann *et al.* (1976) Fig. 9-J plus positions of straight lines of Eqs. (2.19) and (2.20). The line of (2.19) is inside the standard deviation envelope.

A form of the one-dimensional energy spectra of wind waves which is consistent with the above −3 power relation, is $gu_*\sigma^{-4}$, instead of $g^2\sigma^{-5}$ which was proposed by Phillips (1958) for the high-frequency side of the spectral maximum (cf. Toba, 1973a). The essential part of the spectra may be described by

$$\phi(\sigma) = \alpha_s g u_* \sigma^{-4}, \qquad (2.13)$$

although some correction factors may be needed to express the entire spectrum {e.g., a Pierson-Moskowitz (1964) type factor $\exp[-\text{constant}\,(\sigma_m/\sigma)^4]$, together with a Hasselmann *et al.* (1973, 1976) type peak-enhancement function}. Also, if we include the gravity-capillary range, g in (2.13) should be replaced by g_* as proposed by Toba (1973b, 1974a), where

$$g_* = g(1 + Sk^2/\rho_w g), \qquad (2.14)$$

where S is the surface tension, k the wavenumber and ρ_w the density of water. Kawai *et al.* (1977) presented experimental support for the (2.13) spectral form, from spectra obtained for growing wind waves, under ideal onshore wind conditions obtained at the Shirahama Oceanographic Tower Station, Kyoto University. The mean and standard deviation of m, in the form of $\phi(\sigma) = \alpha'\sigma^{-m}$, were determined by the method of least squares and yielded $m = 4.15 \pm 0.25$. Under the assumption of $m = 4$, the mean and standard deviation of α_s in (2.13) were determined to be

$$\alpha_s = 0.062 \pm 0.010. \qquad (2.15)$$

Mitsuyasu (1977) also supported (2.13) with (2.14) for gravity-capillary wave range by a detailed field observation.

Further support is obtained in the data presented by Hasselmann *et al.* (1976). They show in their Fig. 3 that

$$\lambda \equiv \epsilon\nu^4/\alpha \equiv Ef_m^4/\alpha g^2 = 1.6 \times 10^{-4}, \qquad (2.16)$$

where α is the Phillips' constant for their g^2f^{-5} spectral form

$$\phi_f(f) = \alpha g^2(2\pi)^{-4}f^{-5} \exp\left\{-\frac{5}{4}\left(\frac{f_m}{f}\right)^4 + \ln\gamma^{\exp[-(f-f_m)^2/2c^2f_m^2]}\right\}, \qquad (2.17)$$

where

$$c = \begin{cases} c_a, & f \leq f_m \\ c_b, & f \geq f_m. \end{cases}$$

From (2.16) we get

$$\alpha = 6.25 \times 10^3 \epsilon\nu^4. \qquad (2.18)$$

Substituting (2.12), we obtain

$$\alpha = 0.044\nu = 0.044 u_* f_m/(C_D)^{1/2} g. \qquad (2.19)$$

Although Hasselmann *et al.* (1976) proposed

$$\alpha = 0.032\nu^{2/3}, \qquad (2.20)$$

again it appears that (2.19) corresponds slightly more closely to the composite data set in their Fig. 9 as is shown in Fig. 2.

If we assume similarity in the main part of the spectrum, then the high-frequency part of the spectra lies on the same line as the peak moves as the wind waves develop for a constant wind. Since this assumption is supported by field data by Kawai *et al.* (1977), it is considered from the shape of (2.19) that the g^2f^{-5} form of the spectrum is reasonably replaced by gu_*f^{-4}. Thus the level of the spectrum is proportional to u_* and independent of the fetch, as supported by the results of Toba (1973b) and Kawai *et al.* (1977).

The next section will start from the premise that the relation expressed by (2.3) [or Eqs. (2.2) or (2.12)] holds, and that the main high-frequency part of the spectrum has the form of (2.13).

3. Partition of wind stress to waves and current and the stochastic form of the growth of wind waves

Beginning with relation (2.3), by which the process of the growth of wind waves may be treated simply by a single dimensionless parameter, the parameter σ_m^* is used initially and then converted to E^*. The reason will be developed in due course.

First we consider the duration-limited case, in which we define the proportion G, of that part of momentum which is retained as the wave momentum M to the total momentum transferred from the wind to the sea, as

$$G \equiv \frac{1}{\tau}\frac{dM}{dt}, \qquad (3.1)$$

where τ is the wind stress and t the wind duration. From the definition $(1 - G)$ is the proportion of the wind stress which is accumulated as the momentum of the drift current. The momentum ΔM of the component waves between σ and $\sigma + \Delta\sigma$ may be expressed as

$$\Delta M = \Delta(E/C) = \rho_w g A_\sigma{}^2/2C = \rho_w \sigma A_\sigma{}^2/2, \quad (3.2)$$

where C is the phase velocity, ρ_w the density of water and A_σ the wave amplitude of the component waves. Using (2.13)

$$A_\sigma{}^2/2 = \phi(\sigma)\Delta\sigma = \alpha_s g u_* \sigma^{-4}\Delta\sigma,$$

and consequently (3.2) becomes

$$\Delta M = \alpha_s \rho_w g u_* \sigma^{-3}\Delta\sigma. \qquad (3.3)$$

Finally we obtain

$$M = \sum \Delta M = \delta_M \int_{\sigma_m}^{\infty} \alpha_s \rho_w g u_* \sigma^{-3} d\sigma = \delta_M \alpha_s \rho_w g u_* \sigma_m{}^{-2}/2. \quad (3.4)$$

The factor δ_M has entered since the lower limit of the above integral has been taken as σ_m instead of zero, and is a constant as long as the spectra exhibit similarity.

Under the condition where u_* is constant, (3.1) is then expressed by use of (3.4) as

$$G = \frac{1}{\tau}\frac{dM}{dt} = \frac{1}{R_\sigma}\frac{d(\sigma_m^{*-2})}{dt^*},$$

$$R_\sigma = \frac{2\rho_a}{\delta_M \alpha_s \rho_w}, \qquad (3.5)$$

where $t^* \equiv gt/u_*$ and ρ_a is the density of the air. If the values in (2.15) of $\alpha_s = 0.062$, $\rho_a = 1.21 \times 10^{-3}$ (for 15°C), $\rho_w = 1.03$ (for 15°C, 35‰) and $\delta_M = 4/3$ are used (the value of δ_M for the case where the spectral form is symmetrical, in a logarithmic diagram, with respect to the peak frequency can be easily derived), the value of R_σ becomes

$$R_\sigma = 0.028. \qquad (3.6)$$

For the fetch-limited case, if we approximate the velocity of transport of the wave energy by the group velocity at the peak frequency $C_{gm} = g/2\sigma_m$, the duration t may be converted to the fetch F by

$$dF = C_{gm} dt,$$

and it follows that

$$dt^* = 2\sigma_m^* dF^*, \qquad (3.7)$$

where

$$F^* \equiv gF/u_*{}^2.$$

Since the dispersion relation for component waves may not always hold, as mentioned in the Introduction, and since this situation may also be true for component waves in the directional spectrum as demonstrated by Rikiishi (1978), a better formulation of the above is not expected in the present situation. Consequently, we use (3.7) and obtain

$$G = \frac{1}{2R_\sigma \sigma_m^*}\frac{d(\sigma_m^{*-2})}{dF^*}. \qquad (3.8)$$

Eqs. (3.5) and (3.8) may also be converted to

$$G = \frac{1}{R_T}\frac{d(T^{*2})}{dt^*} \qquad (3.9)$$

and

$$G = \frac{T^*}{4\pi R_T'}\frac{d(T^{*2})}{dF^*}, \qquad (3.10)$$

where by use of (2.4)

$$R_T = (2\pi/1.05)^2 R_\sigma = 36R_\sigma, \quad R_T' = (2\pi)^2/(1.05)^3 R_\sigma = 34R_\sigma, \qquad (3.11)$$

or for (3.6),

$$R_T \approx R_T' \approx 1.0. \qquad (3.12)$$

It should be noted that the equivalent form of these equations may be derived by use of (2.1) for the significant waves; namely, using $C = gT/2\pi$ and $\overline{A_\sigma{}^2}/2 = H^2/16$,

$$M = E/C = \rho_w g \overline{A_\sigma{}^2}/2C$$

may be reduced to

$$M = \pi \rho_w H^2/8T,$$

and by use of (2.1) we obtain

$$G = \frac{1}{R_T''}\frac{d(T^{*2})}{dt^*}, \qquad (3.13)$$

$$G = \frac{T^*}{4\pi R_T''}\frac{d(T^{*2})}{dF^*}, \qquad (3.14)$$

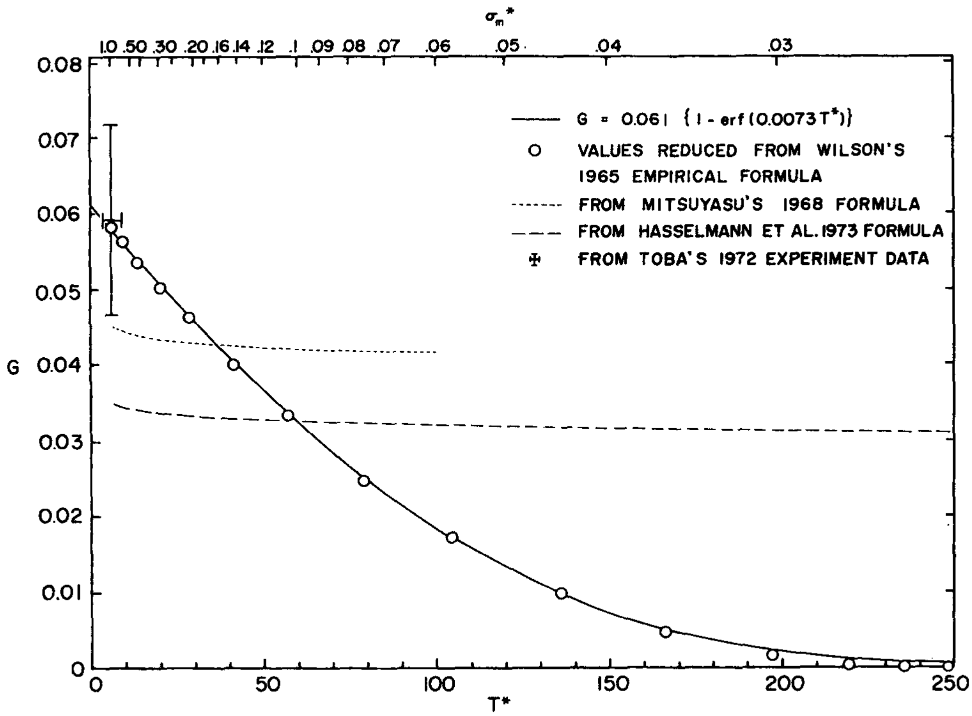

FIG. 3. Variation of G, the proportion of that part of momentum which is retained as the wave momentum to the total momentum transfered from the wind to the sea, with dimensionless wave period T^* or angular frequency σ^*. Values derived from (3.10) by substituting (3.16) are shown by the open circles; values derived from (3.8) by substituting (3.24) and (2.7) are given by the dashed line and broken line, respectively; and the average value and the standard deviation obtained by applying the author's wind-wave tunnel data to (3.10) are shown by the cross mark. The full line represents (3.18) with (3.20) and (3.23).

where

$$R_T'' = 8\rho_a/\pi\rho_w B^2, \quad B = 0.062. \qquad (3.15)$$

The same values of ρ_a and ρ_w as before give $R_T'' = 0.78$, which is 20% different from (3.12). However, this discrepancy will not affect the fetch-limited prediction equation since the value of G will be determined empirically, as will now be developed.

The empirical formula of Wilson (1965),

$$C_D^{1/2}T^*/2\pi = 1.37\{1 - [1 + 0.008(C_DF^*)^{1/3}]^{-5}\}, \qquad (3.16)$$

can be substituted into (3.10), together with a value of

$$C_D \approx 1.2 \times 10^{-3}, \qquad (3.17)$$

yielding G-T^* plots as shown by the open circles in Fig. 3, which are well represented by the following form including the error function as shown by the full line in Fig. 3:

$$G = G_0[1 - \mathrm{erf}(b_T T^*)], \qquad (3.18)$$

or

$$G = G_0[1 - \mathrm{erf}(b_\sigma \sigma_m^{*-1})], \qquad (3.19)$$

where

$$\mathrm{erf}(x) = \frac{2}{(\pi)^{1/2}}\int_0^x \exp(-\zeta^2)d\zeta.$$

Here G_0 represents the maximum value of G, *viz.*, the value of G when wind waves are underdeveloped. The value of G decreases with increasing σ_m^{*-1}, or T^* consistent with the form of the error function, or as a simple stochastic form. The value of b_T (or b_σ) has been determined by the condition that the value of G reduces to 1% of G_0 at $C/U = 1.37$ or

$$T^* = 1.37 \times 2\pi/C_D^{1/2} = 248(=T_1^*),$$

which corresponds to fully developed conditions from (3.16). This value corresponds approximately to the limiting frequency of $\nu = 0.14$ by Hasselmann *et al.* (1976). To be more precise, $C/U = 1.37$ corresponds to $\nu = 0.11$ and $\nu = 0.14$ corresponds to $C/U = 1.1$. The b_T and b_σ then become

$$b_T = 1.82/248 = 7.3 \times 10^{-3}, \qquad (3.20)$$

$$b_\sigma = (2\pi/1.05)b_T = 4.4 \times 10^{-2}. \qquad (3.21)$$

As to the value of G_0, it should be noted that, since (3.10) contains R_T, the substitution of (3.16) gives an empirical value of the product $R_T G$. Consequently, we obtain only an empirical value of $G_0 R_T$ instead of the value of G_0 itself. Using an average value of the drag coefficient of $C_D \approx 1.2 \times 10^{-3}$, we obtain

$$G_0 R_T = 0.062. \qquad (3.22)$$

This value of C_D has been adopted since Toba's (1972) wind-wave tunnel data also gives a value of $G_0 R_T = 0.062$. Moreover, Kondo (1975) has recently reported a value of 1.2×10^{-3} as representative of C_D. If we adopt the value of $R_T = 1.0$ given by (3.12), we obtain

$$G_0 = 0.062. \qquad (3.23)$$

However, since the final form of the prediction equation contains G_0 in the form only of the product $G_0 R$, the estimate of G_0 itself does not affect the essential story.

On the other hand, if we use Mitsuyasu's (1968) empirical formula,

$$f_m^* = 1.00 F^{*-0.330} \; (F^* = 10^2 \sim 10^6), \qquad (3.24)$$

with (3.8), we obtain

$$G = 0.044 \sigma_m^{*\,0.03} \qquad (3.25)$$

or

$$G = 0.046 T^{*-0.03}. \qquad (3.26)$$

Likewise, if we use the Hasselmann *et al.* (1973), JONSWAP empirically derived formula (2.7) for $\xi \leq 10^4$ with (3.8), we obtain

$$G = 0.035 \sigma_m^{*\,0.03} \qquad (3.27)$$

or

$$G = 0.037 T^{*-0.03}, \qquad (3.28)$$

where the value of $R_\sigma = 0.028$ has been used. Eqs. (3.26) and (3.28) are entered in Fig. 3 by a dotted line and a broken line, which indicate that G has an almost constant value of approximately 4 and 3%, respectively. However, if we consider the existence of an equilibrium nondimensional peak frequency, it is reasonable that the value of G has the form of a decreasing function of T^*, such as in Eq. (3.18). It is concluded that although the empirical formulas (3.16), (3.24) and (2.7) themselves are quite similar, especially for smaller dimensionless fetches, large differences exist between (3.18), (3.26) and (3.28) since they have been obtained through differentiation procedures.

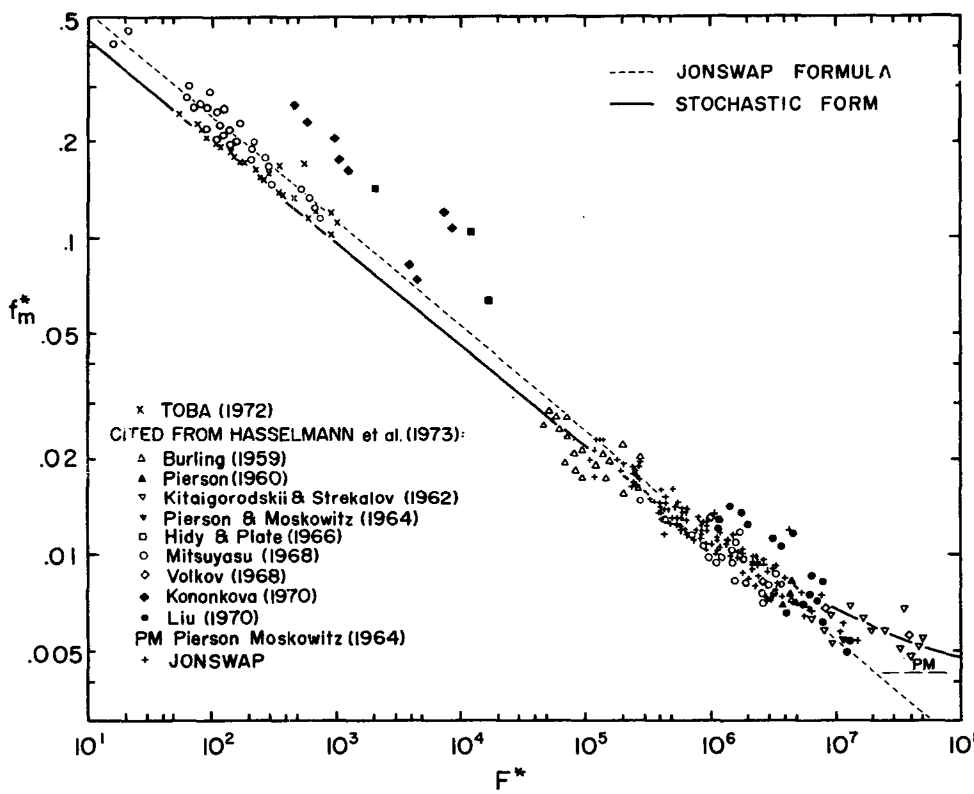

FIG. 4. Comparison of the integrated form of the fetch-limited prediction equation with data. The full line represents the integration of (3.29) with the substitution of (3.19) with (3.6) and (3.23); the dashed line is the JONSWAP formula (2.7). Most of the data points are reproduced from Fig. 2.6 of Hasselmann *et al.* (1973), and the author's wind-wave tunnel data are added.

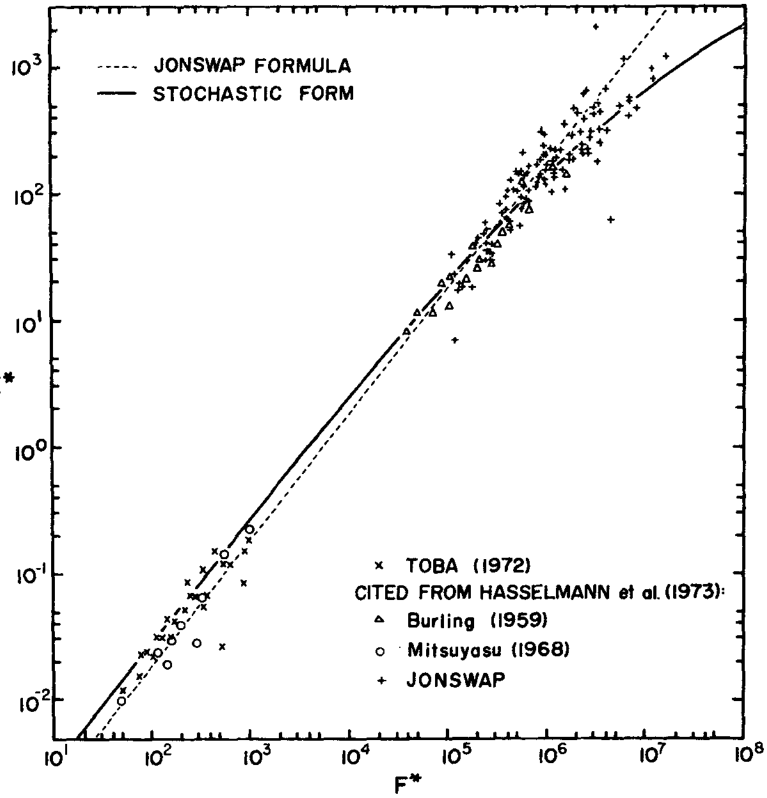

FIG. 5. As in Fig. 4 except that the ordinate is expressed by E^*. Most of the data points are reproduced from Fig. 2.10 of Hasselmann *et al.* (1973), and the author's wind-wave tunnel data are added.

From (3.8), the growth of fetch-limited wind waves may be expressed as

$$\frac{d(\sigma_m^{*-2})}{dF} = 2R_\sigma G\sigma_m^*. \qquad (3.29)$$

Substituting (3.19) into (3.29) and performing the required integration gives the fetch graph, as is shown by the full line of Fig. 4. The integration has been performed by a Runge-Kutta method in a logarithmic form, with an interval of $\Delta(\log F^*) = 0.5$, from the initial condition of $\sigma_m^* = 2.809$ at $F^* = 8.333$ which satisfy (3.16). The original formula of Wilson [eq. (3.16)] may be entered in Fig. 4 at almost entirely the same position with the full line up to $F^* \approx 10^8$. The integration is also compared with the integration of (3.29) with the substitution of (3.27), which is also (2.7), in Fig. 4. Most of the data points and the line of (2.7) are reproduced from Fig. 2.6 of Hasselmann *et al.* (1973) and the author's wind-wave tunnel data (Table 1 of Toba, 1972) have been added. It seems that the curve which represents the integration of (3.29), with the substitution of (3.19), corresponds well to the data points.

The author's wind-wave tunnel experiment data may be applied to (3.10) to obtain experimental values of G. Although the experimental range of T^* is limited, the average values and the standard deviation of G thus obtained are included in Fig. 3, and can be seen to coincide with the curve of (3.18). The dotted line seems to correspond to the mean value of the left half of the full line, the broken line to the mean value of the total of the full line, and the full line comes to approximately zero at $T^* = 250$. Consequently, it is assumed that calculated values of G, as expressed by the error function form, are reasonable.

The prediction equation may then be expressed by

$$\frac{\partial(\sigma_m^{*-2})}{\partial t^*} + \frac{1}{2\sigma_m^*}\frac{\partial(\sigma_m^{*-2})}{\partial F^*} = G_0 R_\sigma[1 - \mathrm{erf}(b_\sigma \sigma_m^{*-1})]. \qquad (3.30)$$

In the course of the derivation of (3.5), it was assumed that u_* was constant. If the time variation of the wind is not rapid, we may assume stepwise changes in the dimensionless variables for the purpose of the integration. However, it is considered that the quantity which is conserved during changes in the wind, and the resulting rapid adjustment of spectrum, should be the energy E of the wind wave field and not the peak frequency σ_m or T_m. Since we have the relation (2.3), it seems reasonable to convert (3.30) to the form

$$\frac{\partial(E^{*2/3})}{\partial t^*} + \frac{E^{*1/3}}{a}\frac{\partial(E^{*2/3})}{\partial F^*} = G_0R[1 - \mathrm{erf}(bE^{*1/3})], \quad (3.31)$$

where

$$\left.\begin{aligned} a &= 2B_\sigma^{1/3} = 0.74. \\ G_0R &= G_0R_\sigma B_\sigma^{2/3} = 2.4 \times 10^{-4}. \\ b &= b_\sigma B_\sigma^{-1/3} = 0.12. \end{aligned}\right\} \quad (3.32)$$

as the final form of the prediction equation, which is also applicable for a changing wind field.

The conversion of the thick line of Fig. 4 by use of (2.2), that corresponds to the integration of (3.31) for the fetch-limited case, is shown by the thick line in Fig. 5 in which most of the data points [and the line of (2.6) as the thin line] are reproduced from Fig. 2.10 of Hasselmann *et al.* (1973), and the remaining points are extracted from the author's wind-wave tunnel data (Toba, 1972), where the mean wave height $\bar{H}$ has been converted to H by $H = 1.6\ \bar{H}$ and used (2.5) to obtain E^*. For a sufficiently large value of F^*, the thick line approaches a maximum value for E^* of 3.7×10^3, which corresponds to the equilibrium peak frequency of $f_m^* = 3.8 \times 10^{-3}$.

4. Physical implications of power law, spectral form and stochastic growth equation

It has been shown in the preceding sections that there is an accurate yet simple relationship describing the overall structure of a growing wind-wave field which can be expressed by (2.3), (2.1) or (2.2). This enables us to construct a single-parameter prediction equation (3.31) for the growth of wind waves. What is the physical basis for the existence of the simple relationship? From the point of view that wind waves are dispersive, weakly nonlinear phenomena, Hasselmann *et al.* (1976) presented results of computations of wave-wave interactions showing that the characteristic relaxation time for the shape stabilization of the energy spectrum and the adjustment of the spectrum to its equilibrium level is reasonably short. However, as mentioned in the Introduction, we believe that wind waves are strongly nonlinear phenomena, very different from a simple superposition of component waves. It was from this point of view that the author (Toba, 1972, 1973a, 1974a,b) has previously pointed out the existence of the simple relation (2.1), and has tried to use macroscopic considerations in its interpretation.

By use of a wind-wave tunnel and flow visualization techniques, our recent experimental studies (Toba *et al.*, 1975; Okuda *et al.*, 1976; Kawai, 1977; Okuda *et al.*, 1977) have revealed several facts on the behavior of real wind waves. For example, immediately after a wind of, say, 6 m s^{-1} starts on a still surface of water, a surface skin flow is produced in a layer of up to 2 mm in thickness with a large vertical velocity shear. Shortly thereafter, regular waves of around 1.7 cm in wavelength are generated whose phase velocity is near the minimum seemingly produced by an instability mechanism. Within a couple of seconds after that, a downward thrust of the skin flow with consequent forced convection commences on the lee side of crest, and the transition of the surface layer to a turbulent state occurs. The growth of ordinary wind waves of irregular nature commences from this state (Okuda *et al.*, 1976, Kawai, 1977). At the surface of the wind waves, there is a strong variation of the tangential stress exerted by the wind. The stress value is several times larger than the average value of the wind stress at the windward side of the crest, and it becomes negligible at the lee side of the crest. Also, it seems that most of the wind stress is in the form of tangential stress (Okuda *et al.*, 1977). As long as the wind is blowing, wind waves appear to be accompanied by forced convections irrespective of whether or not the waves are breaking in the usual sense. Small particles of neutral buoyancy, placed just beneath the water surface prior to the start of the wind, are quickly dispersed into the water column by forced convection to a depth of one-half of the wavelength (Toba *et al.*, 1975).

An observed irregularity of wind waves, especially for smaller C/U, does not stem physically from a random superposition of free component waves of the dispersive nature, nor from the superposition of turbulence from some independent origins, in an expansible manner, on a wave system of small amplitudes, but the irregularity is a character *inherent* in wind waves.

It is believed that individual undulations at any location in the real wind-wave field have a specific structure, accompanied by phase distributions in the tangential stress of wind and the wind drift surface current, and consequently by the forced convection. Since the convection structure travels on the water surface together with the crests of the "individual waves", the motion of water is naturally characterized as turbulent. This is the origin of the irregularity of wind waves. In a limiting expression, wind waves for small values of C/U may be regarded as an ensemble of these "substantial waves", which have a specific internal structure which is unsteady and unclosed as mentioned above, and which have a lifetime longer than their period, but of the order of, say, several times the period, in a sense that they lose their own identity due to the strong nonlinearities. For large value of C/U also conditions will be the same for high-frequency parts.

Support of this view of wind waves is reinforced by the study of Rikiishi (1978) who carried out wind-wave tunnel experiments on the phase velocity of component waves. He developed an experimental technique for the determination of the directional structure of the phase velocity of component waves without pre-assuming a dispersion relation, and found that the phase velocities do not satisfy the dispersion relation of free-water waves. Rather they are independent of the local frequency and have the same value as that of the waves at the spectral maximum at respective fetches. Yefimov *et al.* (1972), Kato and Tsuruya (1974), Ramamonjiarisoa (1974) and Ramamonjiarisoa and Coantic (1976) also reported similar results in a study of one-dimensional phase velocities. These experimental results are interpreted as indicating that the assumption of wind waves as expansible into free-component waves with weak nonlinearity is not appropriate for growing wind waves. It should be added here that Grose *et al.* (1972) reported anomalous dispersion relations from the observation of wind waves in the Atlantic Ocean.

Since wind waves are presumed to be strongly nonlinear phenomena, having aspects of turbulence as well as of water waves, it is not possible to treat their motion purely analytically. However, we may assume a kind of similarity structure in the wind-wave field and seek a regularity in gross structure by invoking dimensional considerations.

There are several characteristic quantities near the air-water interface, such as the friction velocity u_* of the air, the average velocity u_s of the wind drift surface current, and the mass transport velocity u_0 at the surface, applicable to the motion of individual substantial waves. We may assume dimensionally that there is a proportional relation among these three velocities, as a result of the strong nonlinearity, of the form,

$$u_0 \propto u_s \propto u_* \quad \text{or} \quad u_0/u_* = \text{constant}. \tag{4.1}$$

This assumption includes the concept of similarity structure in the wind-wave field. In fact there are some experimental results showing that there is a proportional relation between u_s and u_* (e.g., Kondo *et al.*, 1974; Wu, 1975; Okuda *et al.*, 1976), the factor of proportionality of u_s to U being 2–5%. If we approximate u_0 of the individual waves of amplitude A and the frequency σ by that of the Stokes wave

$$u_0 = A^2\sigma^3/g + \text{higher order terms}, \tag{4.2}$$

and, if we neglect the higher order terms, we obtain immediately from (4.1) and (4.2)

$$A^2\sigma^3 = cgu_*, \tag{4.3}$$

where c is the proportionality factor. Although each individual wave exhibits some deviation from the conditions expressed by (4.3), it is expected that (4.3) holds statistically. If we transform (4.3) by use of the wave height H and period T of the individual waves, we have a dimensionless form,

$$H^* = BT^{*3/2} \tag{4.4}$$

which coincides with the form (2.1) first proposed for significant waves by the author with an empirically determined value of $B = 6.2 \times 10^{-2}$ (Toba, 1972). Eq. (4.4) has now been derived for individual waves. The value of B, however, may not necessarily coincide with the above value for significant waves.

The form of (4.3) may also be derived in a quite different manner (cf. Toba, 1974b). Namely, from the theory of turbulence, it has been well established from dimensional considerations that the size Λ of an eddy and the variation of its velocity component V over a distance of the order of Λ can be expressed by

$$V^3/\Lambda = \text{constant}. \tag{4.5}$$

In the theory of turbulence the constant in (4.5) is the rate of dissipation of the energy. In wind waves, there should be two directions in the energy flux—upward and downward. In any case, if we assume the form of (4.5) dimensionally for individual waves, and if we use $H(=2A)$ as Λ and $2u(=2A\sigma)$ as V from the orbital velocity of waves, we obtain $A^2\sigma^3$ = constant. Further, since we may assume that the constant is proportional to g and u_* and neglect viscosity, we obtain (4.3) immediately.

In order to relate the height of individual waves to the spectrum ϕ, we may assume the similarity structure and that

$$\mathscr{E}\int_\sigma^\infty \phi(\sigma)\,d\sigma = \frac{A^2}{s} \tag{4.6}$$

holds statistically for individual waves. This is in an analogy with (2.5) for significant waves, i.e.,

$$\int_0^\infty \phi\,d\sigma = \mathscr{E}\int_{\sigma_m}^\infty \phi\,d\sigma = \frac{H^2}{16}, \tag{4.7}$$

where $\mathscr{E}$ has entered since the lower limit of the integral has been taken as σ_m instead of zero, and $\mathscr{E}$ is a constant as long as the spectra have a similarity. If we then assume that the σ^{-4} spectral form is symmetrical on a logarithmic diagram with respect to the peak frequency, it is easily shown that $\mathscr{E} = 1.6$. The s in (4.6) is also a constant which might differ from 16 in (4.7). The combination of (4.6) with (4.3) immediately leads to the form of (2.13), i.e.,

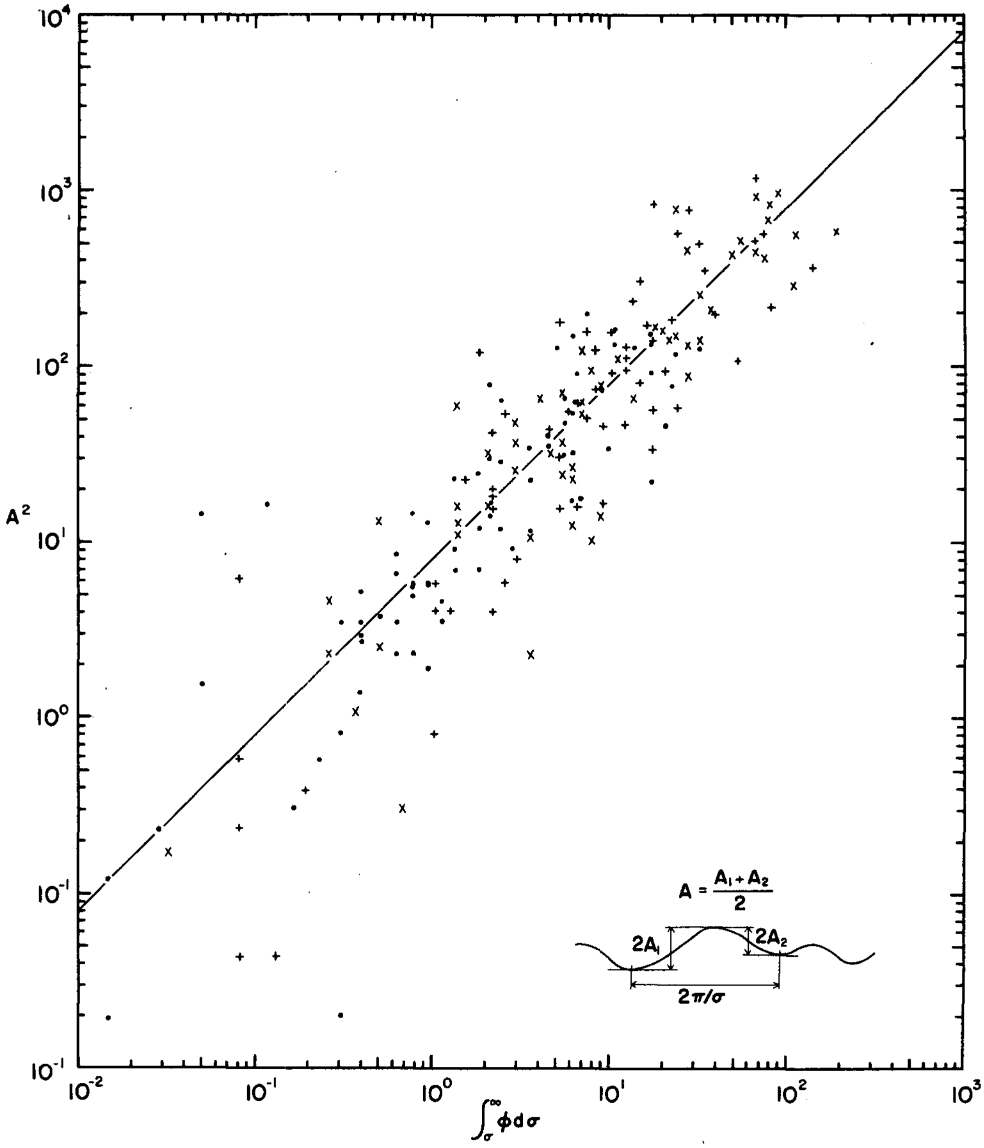

FIG. 6. Data support of the relation (4.6). Values of σ and A for each individual wave in three examples of growing wind-wave records obtained at sea are determined by the procedure illustrated by the inset, and values of A^2 are plotted against $\int_\sigma^\infty \phi d\sigma$ with ϕ given by (2.13) and (2.15). The three examples are shown by three kinds of marks, respectively.

$$\phi(\sigma) = \alpha_s g u_* \sigma^{-4}, \quad \alpha_s = 3c/\mathscr{E}s. \tag{4.8}$$

Fig. 6 presents data in support of the relation (4.6) using three examples of wind-wave records obtained during a field observation program (Kawai *et al.*, 1977). Values of A^2 and σ for each individual wave were determined by the procedure illustrated by the inset of Fig. 6. A^2 has been plotted as the ordinate, and

$$\int_\sigma^\infty \phi d\sigma$$

as the abscissa, where $\phi = \alpha_s g u_* \sigma^{-4}$ and $\alpha_s = 0.062$. The spectra ϕ obtained from the same records had the above form on the high-frequency side of σ_m as has been reported by Kawai *et al.* (1977). The plotted points are distributed along the 45° line shown in Fig. 6, and the value of the product $\mathscr{E}s$ is seen to be about 8.0. It is believed that (4.6) represents a link between the wind-wave spectra and the individual waves.

In Fig. 7 is shown an extension of the 3/2 power law to individual waves. The data source is the same as that of Fig. 6. The points for very small amplitudes, which are expressed as points for $H^* \lesssim 2$, may not be reliable, partly because the data were obtained from numerical wave records digitized at 0.1 s intervals. Thus, the 3/2 power law for individual waves appears to be established in a statistical sense. For comparison, points for significant waves are entered in Fig. 7 by marks enclosed by circles. These points represent the average of the upper one-third of the highest individual waves for each of the three cases.

Finally, a brief discussion is necessary on the physical interpretation of the stochastic growth of wind waves. As discussed above, the wind-wave field grows under the regulating condition expressed statistically by (4.1), (4.3) or (2.1), (2.3), and the rate of growth is expressed by (3.30) or (3.31). The implication of (3.30) or (3.31) is that the rate of growth of the dimensionless wind-wave field is determined by the deficit from the state of saturation. That the transition of E^* is expressed by a form of the error function makes us interpret the growth processes of the wind-wave field as a kind of stochastic processes, where E^* approaches a final state irrespective of the initial conditions, including a rapid internal adjustment of the state to satisfy the power-law relationships, as a result of the strong nonlinearities.

As already mentioned, Toba *et al.* (1975) and Okuda *et al.* (1976, 1977) have reported that wind waves are accompanied by forced convections relative to the instantaneous wave forms of the individual waves. These convections have the nature of more or less two-dimensional vortices with their axes parallel to the crest lines. Consequently, real wind waves may be regarded, in a sense, as an ensemble of various scales of somewhat two-dimensional convective eddies distributed along the water surface. When they come close to each other due to their different traveling velocities, a mutual coalescence of the eddies will occur as a result of the turbulent diffusion or dispersion of the two-dimensional vorticity. This may be one aspect of the mechanism of upward cascading of energy in wind waves.

Since the forced convection is caused by the action of the wind, strong nonlinearity is effectively produced when the phase velocities of waves are small compared to the wind speed, namely when C/U or T^* or σ_m^{*-1} is small. When the phase velocity of the wave of the peak frequency becomes comparable to the gust speed on the crest, convective eddies are not produced, and mutual coalescence does not occur. The condition of the termination of wave growth is given by (3.16) as

$$C/U \equiv (C_D)^{1/2}T^*/2\pi = 1.37,$$

which corresponds to a reasonable value of the gust factor near the wave crest (cf. Kondo *et al.*, 1972). It is considered that this is the physical

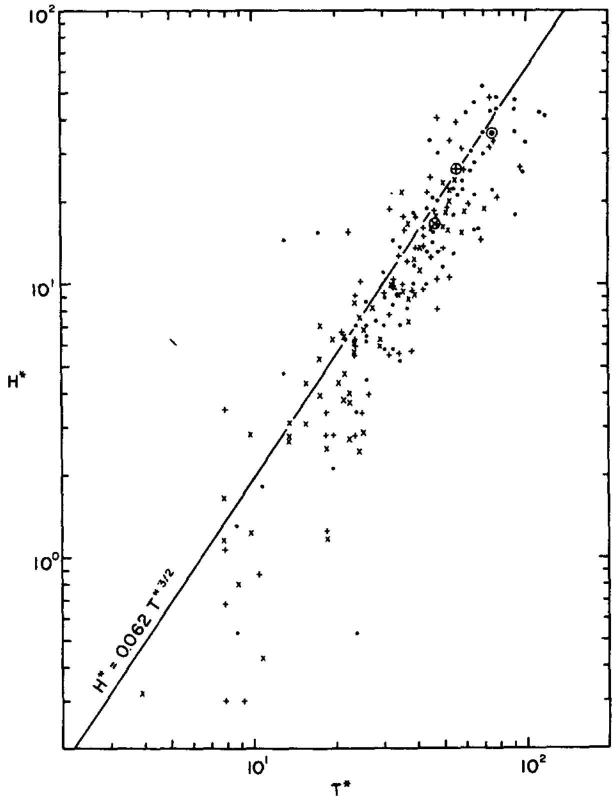

FIG. 7. Extension of the 3/2 power law (2.1) to individual waves in a statistical sense. The three kinds of marks corresponds to those in Fig. 6. Marks enclosed by circles represent significant waves for the three cases.

basis of the value of $2.5 \times 10^2 (= T_1^*)$ as the maximum value of T^*, and consequently of the value of b_T in (3.20).

Now we use $E_t(T^*)$ as a measure of the wind action which can bring about the upward cascading of the energy for the wind-wave field between T^* and $T^* + dT^*$, due to the total effect of the nonlinear processes. Since we are considering the net upward cascading, dissipation of energy is included in the total effect. Since various stochastic factors are included in the wind action, we assume a Gaussian distribution as the form of E_t, with a maximum at $T^* = 0$, and a negligible value at $T^* = T_1^*$ or $b_T T^* = 1.82$; namely

$$E_t \propto \exp[-(b_T T^*)^2].$$

The total wind action that brings about the growth of the wind-wave field will have a form proportional to the integral of E_t in the range of T^* greater than the present value of T^*, i.e.,

$$\int_{T^*}^{\infty} E_t dT^* \propto [1 - \mathrm{erf}(b_T T^*)],$$

since the wind-wave field represented by T^* can naturally grow by the wind action which is still effective for the wind waves greater than T^*. This will give the physical basis of the form of (3.18) or (3.19) and consequently the right-hand side of (3.31).

5. Summary

The 3/2 power law for growing wind waves [Eq. (2.1)], previously proposed by the author, may also be expressed as (2.2), (2.3) or (2.12). Although very simple, these expressions are supported by various available data including those compiled by Hasselmann *et al.* (1973).

As to the one-dimensional energy spectra of growing wind waves, it is shown that the Phillips' constant α is proportional to ν as expressed in (2.19); consequently, it is reasonable that the g^2f^{-5} form is replaced by the $g_*u_*f^{-4}$ form of Eq. (2.13) for the essential part of the high-frequency side of the spectra.

Starting from the premise of (2.3), the proportion G of that part of momentum which is retained as the momentum of waves to the total momentum transfered from the wind to the sea defined by (3.1), has been expressed by Eqs. (3.9) or (3.10). By applying (3.10) to existing empirical formulas and data, Eqs. (3.18) or (3.19) is proposed as the most reasonable form of G. The value of G decreases with increasing T^* or C/U in the form of an error function or in a simple stochastic form, and the maximum value G_0 is tentatively estimated as 0.062. The prediction equation (3.31) with (3.32) for the growth of wind waves, expressed by a single parameter E^*, is proposed.

By reviewing recent experimental studies of real wind waves especially by use of flow visualization techniques, emphasis is given to the fact that wind waves, especially for smaller values of C/U, are associated with strong nonlinearities, and the concept of substantial waves is proposed. From this point of view, a discussion has been presented as to the physical basis for the existence of a simple power law, spectral form and stochastic form of the growth equation. The power law, $g_*u_*f^{-4}$ form of the one-dimensional energy spectra are derived by invoking the assumption of self-similarity structure in the field of the wind waves and by dimensional considerations. In the course of the derivation, it is shown that (2.1), etc., may be extended to individual waves in a statistical sense. An idea is suggested that the upward cascading of the energy may be caused by the mutual coalescence of two-dimensional eddies of forced convections accompanying wind waves, and an attempt is made for derivation of the stochastic form of the growth of wind waves.

Acknowledgments. The author expresses his thanks to people who discussed the problem with him, on occasions, and gave valuable comments. The names include Professors H. Kunishi, K. Kajiura, H. Mitsuyasu, Y. Nagata, T. Asai, J. Kondo, T. Kotake and Dr. I. Isozaki in Japan, Professor K. Hasselmann in Germany, Professors F. MacIntyre and J. S. Turner in Australia, and members of the author's laboratory including Dr. T. Sugimoto, Messrs. M. Tokuda, K. Okuda and S. Kawai. His thanks are also extended to Mrs. F. Ishii for her kind assistance in the course of the present study.

This study was partially supported by the Grant-in-Aid for Scientific Research by the Japanese Ministry of Education, Science and Culture, Project Nos. 942004, 154104, 254114 and 202508.

REFERENCES

Banner, M. L., and O. M. Phillips, 1974: On the incipient breaking of small scale waves. *J. Fluid Mech.*, **65**, 647–656.

——, and W. K. Melville, 1976: On the separation of air flow over water waves. *J. Fluid Mech.*, **77**, 825–842.

Grose, P. L., K. L. Warsh and M. Garstang, 1972: Dispersion relations and wave shapes. *J. Geophys. Res.* **77**, 3902–3906.

Hasselmann, K., 1968: Weak-interaction theory of ocean waves. *Basic Developments in Fluid Dynamics*, Vol. 2, M. Holt, Ed. Academic Press, 117–182.

Hasselmann, K., T. P. Barnett, E. Bouws, H. Carlson, D. E. Cartwright, K. Enke, J. A. Ewing, H. Gienapp, D. E. Hasselmann, P. Kruseman, A. Meerburg, P. Müller, D. J. Olbers, K. Richter, W. Sell and H. Walden, 1973: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Deut. Hydrogr. Z.*, Suppl. A, **8**, No. 12, 95 pp.

——, D. B. Ross, P. Müller and W. Sell, 1976: A parametric wave prediction model. *J. Phys. Oceanogr.*, **6**, 200–228.

Imasato, N., and H. Ichikawa, 1977: Nonlinearity of the horizontal velocity field under wind-waves. *J. Oceanogr. Soc. Japan*, **33**, 61–66.

Iwata, N., W. Inada, T. Tanaka and I. Watabe, 1970: Ocean wave statistics and spectrum width parameter (I). *Rep. Nat. Res. Center Disas. Prevent.*, No. 4, 23–43.

——, —— and I. Watabe, 1971: Ocean wave statistics and spectrum width parameter (II). *Rep. Nat. Res. Center Disas. Prevent.*, No. 5, 81–87.

Kato, H. and K. Tsuruya, 1974: On the phase velocity of component waves of wind waves. *Proc. 21st Japan Conf. Coast. Eng.*, Japan Soc. Civil Eng. 255–259.

Kawai, S., 1977: On the generation of wind waves relating to the shear flow in water—A preliminary study. *Sci. Rep. Tohoku Univ.*, Ser. 5, *Geophys.*, **24**, 1–17.

——, K. Okada and Y. Toba, 1977: Support of the three-seconds power law and the $gu_*\sigma^{-4}$-spectral form for growing wind waves with field observational data. *J. Oceanogr. Soc. Japan*, **33**, 137–150.

Kitaigorodskii, S. A., 1961: Application of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. *Bull. Acad. Nauk SSSR Geophys. Ser.*, 105–117.

Kondo, J., 1975: Air-sea bulk transfer coefficients in diabatic conditions. *Bound.-Layer Meteor.*, **9**, 91–112.

——, Y. Fujinawa and G. Naito, 1972: Further analysis of wind fluctuation over the sea (in Japanese). *Umi to Sora*, **48**, 61–71.

——, G. Naito and Y. Fujinawa, 1974: Wind-induced current in the upper most layer of ocean (in Japanese). *Nat. Res. Center Disas. Prevent.*, No. 10, 67–82.

Kunishi, H., 1957: Studies on wind waves with use of wind flume (I)—On the shearing flow in the subsurface boundary layer caused by wind stress. *Ann. Disas. Prevent. Res. Inst. Kyoto Univ.*, **1**, 119–127.

——, 1963: An experimental study on the generation and growth of wind waves. *Disas. Prevent. Res. Inst., Kyoto Univ., Bull.*, No. 61, 41 pp.

Longuet-Higgins, M. S., 1952: On the statistical distribution of the heights of sea waves. *J. Mar. Res.*, **11**, 245–266.

Miles, J. W., 1957: On the generation of surface waves by shear flows. *J. Fluid Mech.*, **3**, 185–204.

Mitsuyasu, H., 1968: On the growth of the spectrum of wind-generated waves (I). *Rep. Res. Inst. Appl. Mech., Kyushu Univ.*, **16**, 459–482.

——, 1971: On the form of fetch-limited wave spectrum. *Coastal Eng. Japan*, **14**, 1–14.

——, 1977: Measurement of the high-frequency spectrum of ocean surface waves. *J. Phys. Oceanogr.*, **7**, 882–891.

Okuda, K., S. Kawai, M. Tokuda and Y. Toba, 1976: Detailed observation of the wind-exerted surface flow by use of flow visualization methods. *J. Oceanogr. Soc. Japan*, **32**, 51–62.

——, —— and Y. Toba, 1977: Measurement of skin friction distribution along the surface of wind waves. *J. Oceanogr. Soc. Japan*, **33**, 190–198.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. *J. Fluid Mech.*, **2**, 417–455.

——, 1958: The equilibrium range in the spectrum of wind-generated waves. *J. Fluid Mech.*, **4**, 426–434.

Pierson, W. J., Jr., 1952: A unified mathematical theory for the analysis, propagation and refraction of storm generated ocean surface waves, Parts I and II. Dept. Meteor. Oceanogr., NYU, prepared for the Beach Erosion Board, Dept. of the Army, and Office of Naval Res., Dept. of the Navy, 461 pp.

——, and L. Moskowitz, 1964: A proposed spectral form for fully developed wind sea based on the similarity theory of S. A. Kitaĭgorodskiĭ. *J. Geophys. Res.*, **69**, 5181–5190.

Ramamonjiarisoa, A., 1974: Contribution à l'étude de la structure statistique et des mécanismes de génération des vagues de vent. Thèse à L'Université de Provence Le Grade de Docteur ès Sciences, 160 pp.

Ramamonjiarisoa, A. and M. Coantic, 1976: Loi expérimentale de dispersion des vagues produites par le vent sur une faible longueur d'action. *C. R. Acad. Sci. Paris*, **B282**, 111–114.

Rikiishi, K., 1978: A new method of measuring directional spectrum for wind waves. II. Directional spectrum and dispersion relation for laboratory wind waves in the generating area. *J. Phys. Oceanogr.* to be published.

Shemdin, O. H., 1972: Wind-generated current and the phase speed of wind waves. *J. Phys. Oceanogr.*, **2**, 411–419.

Sverdrup, H. Y., and W. Munk, 1947: Wind, sea and swell. Theory of relations for forecasting. Publ. No. 601, U. S. Hydrogr. Office, Washington, D.C.

Toba, Y., 1961: Drop production by bursting of air bubbles on the sea surface (III). Study by use of a wind flume. *Mem. Coll. Sci. Univ. Kyoto*, **A29**, 313–344.

——, 1972: Local balance in the air-sea boundary processes, I. On the growth process of wind waves. *J. Oceanogr. Soc. Japan*, **28**, 109–120.

——, 1973a: Local balance in the air-sea boundary processes, II. Partition of wind stress to waves and current. *J. Oceanogr. Soc. Japan*, **29**, 70–75.

——, 1973b: Local balance in the air-sea boundary processes, III. On the spectrum of wind waves. *J. Oceanogr. Soc. Japan*, **29**, 209–220.

——, 1974a: Macroscopic principles on the growth of wind waves. *Sci. Rep. Tohoku Univ.*, Ser. 5, *Geophys.*, **22**, 61–73.

——, 1974b: Duality of turbulence and wave in wind waves. *J. Oceanogr. Soc. Japan*, **30**, 241–242.

——, 1976: Stochastic form of the growth of wind waves and its physical implications. *Book of Abstracts*, Joint Oceanogr. Assembly, Edinburgh, p. 192.

——, M. Tokuda, K. Okuda and S. Kawai, 1975: Forced convection accompanying wind waves. *J. Oceanogr. Soc. Japan*, **31**, 192–198.

Valenzuela, G. R., 1976: The growth of gravity-capillary waves in a coupled shear flow. *J. Fluid Mech.*, **76**, 229–250.

Wilson, B. W., 1965: Numerical prediction of ocean waves in the North Atlantic for December, 1959. *Deut. Hydrogr. Z.*, **18**, 114–130.

Wright, J. W. and W. C. Keller, 1971: Doppler spectra in microwave scattering from wind waves. *Phys. Fluids*, **14**, 466–474.

Wu, J., 1975: Wind-induced drift currents. *J. Fluid Mech.*, **68**, 49–70.

Yefimov, V. V., Yu. P. Solov'yev and G. N. Khristoforov, 1972: Observational determination of the phase velocities of spectral components of wind waves. *Izv. Atmos. Ocean. Phys.*, **8**, 246–251.